

# The Finite Element Analysis and the Optimum Geometric Design of Linear Motor

Tae-Won Lee<sup>1,#</sup>

<sup>1</sup> School of Mechanical Engineering, Kumoh National Institute of Technology, Gumi, South Korea  
<sup>#</sup> Corresponding Author / E-mail: twlee@kumoh.ac.kr, TEL: +82-54-478-7375, FAX: +82-2-478-7319

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*Linear motor has been considered to be the most suitable electric machine for linear control with high speed and high precision. Thrust of linear motor is one of the important factors to specify motor performance. Maximum thrust can be obtained by increasing the magnitude of current in conductor and is relative to the sizes of conductor and magnet. However, the magnitude of current and the size of conductor have an effect on temperature of linear motor. Therefore, it is practically important to find optimum design that can effectively maximize thrust of linear motor within limited range of temperature. Finite element analysis was applied to calculate thrust and numerical solutions were compared with experiments. The temperature of the conductor was calculated from the experimentally determined thermal resistance. The ADPL of ANSYS was used for the optimum design process, which is commercial finite element analysis software. Design variables and constraints were chosen based on manufacturing feasibility and existing products. As a result, it is shown that temperature of linear motor plays an important role in determining optimum design.*

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## NOMENCLATURE

$F$  = Thrust  
 $i$  = Magnitude of current  
 $N$  = Number of turns  
 $R$  = Electric resistance  
 $R_{th}$  = Thermal resistance  
 $T$  = Temperature of conductor  
 $x$  = Thickness of conductor  
 $y$  = Thickness of permanent magnet

## 1. Introduction

Linear motor can directly drive linear motion without auxiliary machine parts such as gears or cranks. It has been widely used in factory automation or office automation machines, because it doesn't generate noise or vibration. Since it is possible to make smaller motor with high power, it has been applied to small-sized precision products such as hard disk and CD player.

Generally, several methods were presented to evaluate thrust of linear motor. Equivalent magnetic circuits network method calculates thrust after dividing magnetic field into small regions and constructing magnetic network like electric circuit. There is the space harmonic analysis method using Fourier series expansion. Among these, finite element method has a solution that is close to experimental results and can model complicate magnetic field including nonlinear material properties.

Based on these electromagnetic analysis methods, many papers have been published on calculation of thrust and improvement of thrust for linear motor. Recently, optimization techniques have been used to improve thrust. Mizuno<sup>1</sup> explained that the size of permanent magnet affects static thrust and vertical force in linear synchronous motor. Yamada etc. all<sup>2</sup> obtained optimal design for the problem maximizing thrust. They defined turns of coil and thickness of magnet as design variables, and thickness of linear motor as a constraint. Hong etc. all<sup>3</sup> minimized a weight of mover by the response surface methodology. While it is indirectly to improve the thrust, Choi etc. all<sup>4</sup> selected a material and thickness of stator that increase the maximum magnetic flux density. Joo etc. all<sup>5</sup> obtained maximum thrust of linear motor using genetic algorithm.

Most of these researches have only focused on improvement of thrust without considering operation temperature. Ascending temperature in operation decreases performance of linear motor. In order to solve this problem, Eun<sup>6</sup> found heat source and improved thermal characteristic inserting insulation layer. Jung etc. all<sup>7</sup> tried design optimization considering temperature condition. Thrust as an object function was calculated by three dimensional finite element analysis. It required a lot of computation time. Hence, an approximated equation for thrust was induced by least square fitting. This method is not an optimization technique in the strict sense and it has a shortcoming that requires a new approximated equation for the simple change of allowable design range.

The present research suggests an optimization method for design of linear motor including temperature constraints. Using thermal resistance, temperature was formulated by simple equations. Thrust as an object function was directly calculated by finite element analysis and was iteratively obtained in optimization process.

## 2. Analysis for Thrust and Temperature

### 2.1 Thrust

In finite element analysis, the thrust is generally calculated by Maxwell stress or virtual work principle. The virtual work method is sensitive to shape and size of finite elements for air surrounding a mover. Since the shape of finite element changes due to design variations in optimization process, Maxwell stress method is more adequate. Therefore, this method was applied to evaluate thrust of linear motor and the commercial finite element analysis code ANSYS was used. To evaluate the validity of numerical analysis, numerical solutions were compared with experimental results.

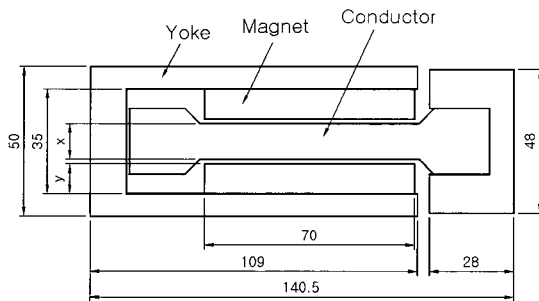


Fig. 1 Cross section of the linear motor

#### 2.1.1 Three Dimensional Analysis

Coreless linear motor, which was studied, was shown in Fig. 1. A yoke made of S45C and Nd-Fe-B magnet constructed stator. Magnets confront each other with reverse poles and are alternatively aligned with reverse poles in longitudinal direction. Eight conductors constituted a mover and connected by 4 series and 2 parallel circuits. A conductor has cross section with piles of copper wire and three phase sine wave current flows into conductors perpendicularly to magnetic flux.

Cross-sectional width of a conductor must be determined as 1/3 pitch. The size of permanent magnet was 25(W)x70(L) and the pole pitch was 30mm in linear motor of Fig. 1. Hence, the width of a conductor is automatically defined as 10mm. Since the mover has eight conductors, total thrusts are eight times of thrust from a conductor. From this fact, a conductor and magnets with two cycles were considered as shown in the finite element modeling of Fig. 2

In ANSYS, three dimensional finite element SOLID96 was selected to evaluate thrust. At each side of yoke, magnetic flux only exists in longitudinal direction of linear motor. Table 1 explained material properties of linear motor.

Table 1 Material properties for numerical analysis of linear motor

Relative Permeability of Air and Conductor	1
Relative Permeability of Permanent Magnet	1.034
Relative Permeability of Yoke	1000
Coercive Force of Magnet	1006000 A/m

Electromagnetic analysis must consider surrounding air region as well as linear motor. In this study, finite element model was defined as a cube including linear motor and ambient air with a great distance from yoke. Outside the defined air region, electromagnetic field is negligible. Total number of finite elements in modeling was 161,292. The finite element model that eliminate air regions was shown in Fig. 2.

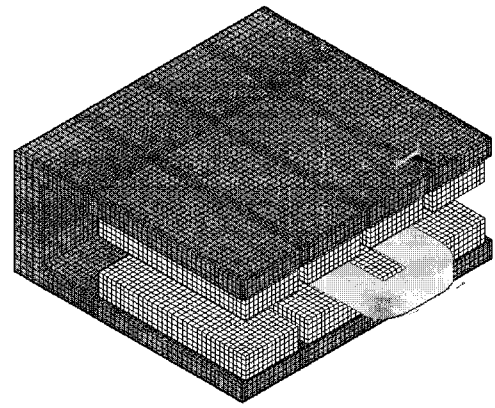


Fig. 2 Three dimensional finite element model

The thrusts have different values according to positions of a mover. Since this research aims at maximum thrust, the position having maximum thrust must be selected. This position of a mover was determined as a center of the finite element modeling in Fig. 2. Seeing analysis results, thrust only existed in moving direction of a mover and magnetic flux mainly distributed near permanent magnets.

#### 2.1.2 Validation by Experiments

To compare numerical solutions with experimental results, the conductors were manufactured by coiling copper wire with diameter of 0.6mm up to 230 number of turns. Then the thickness of a conductor had 9.41mm. The thickness of permanent magnet was 10mm. The magnetic flux density and thrust were measured by experiments and calculated by finite element analysis. To measure magnetic flux, transportation table with Gauss meter was slowly moved with velocity of 0.5m/sec between two magnets. In case of 1.85A current, maximum magnetic flux density of 0.63T was measured at the center of confronting magnets. The result by ANSYS was 0.61T at same position and had an error of 2.9% to experimental result.

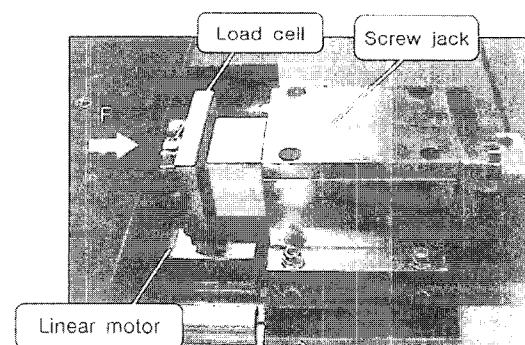


Fig. 3 Measurement of thrust

Thrust is classified as dynamic thrust and static thrust. Dynamic thrust is very important in application, but numerical or analytical solution has a large error to experimental results. Hence, static thrust was measured by load cell at center position, which has maximum thrust. Measured results were plotted in Fig. 4. Current in Fig. 4 denotes one that flow into a conduct and thrust means total thrust of conductors that were connected by two parallel and four series circuits. Namely, the thrusts in Fig. 4 were calculated after multiplying the result of finite element analysis by eight. Error was 7.8% for current of 5A, but 3.4% for current of 2A. Since magnitude of current had about 2A in products on the market, it can state that finite element analysis must make a good estimate the thrusts of linear motors.

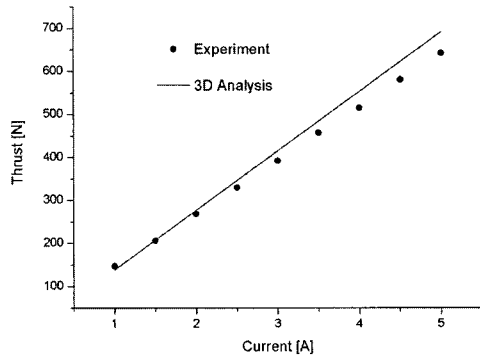


Fig. 4 Thrust according to current of conductor

### 2.1.3 Two Dimensional Analysis

Compared with experimental results, the three dimensional finite element analysis is recommended to calculate thrusts. In the viewpoint of optimal design, it requires excessive computation times, because the optimal solution is obtained through many iterations. To overcome this drawback, two dimensional analysis was applied in this paper.

As the yoke in linear motor has three dimensional shapes, it seems that two dimensional analysis is impossible. However, as mentioned above, the three dimensional analysis only produced the thrust in longitudinal direction and most of magnetic flux existed near magnets. Therefore, it is possible to consider two dimensional finite element modeling, which was only defined as the length and height of linear motor. Of course, the finite element modeling included air region surrounding linear motor. The two dimensional model excluding air region was described in Fig. 5.

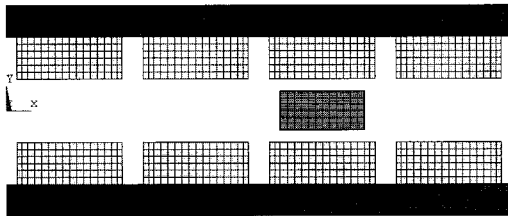


Fig. 5 Two dimensional finite element model

ANSYS was also used to calculate thrusts for two dimensional model. Finite element type PLANE53 was selected and total number of finite elements 3,520. If air region is deleted, it has 1,640 elements. Two dimensional model was defined as shown in Fig. 5. The symmetric part actually exists on the other side with respect to vertical centerline in this modeling, but it was excluded in this modeling. Instead, considering the direction of current and poles of magnets, symmetric condition on the of vertical centerline was given for magnetic flux. Two dimensional analysis results were similar to three dimensional analysis results from an angle of thrust. In two dimensional analysis, thrust was 145.85N for conductor thickness of 9.41 mm, magnet thickness of 10 mm and current of 1A. It had an error of 5.5% to three dimensional result of 138.30N. Note that the thrust from two dimensional analysis is always larger than three dimensional analysis result, because the yoke was simplified. Conclusively, the error grows larger in two dimensional analysis and has about 7.8% to experimental result at current of 2A. However, it is efficient to use two dimensional analysis for an optimal design of linear motor.

## 2.2 Temperature

To evaluate the temperature of a mover, two methods exist. Firstly, it can be calculated from finite element analysis. Secondly, after thermal resistance is determined by experiments, temperature can be obtained by dividing supplied heat into it. As a mover did not have symmetric shape and the position of wire in cross section of a conductor was slightly irregular, the calculation of temperature using finite element analysis

is impossible. Hence, thermal resistance was measured by experiments in this paper.

Temperature of conductors  $T$  is a function of ambient temperature  $T_{atm}$ , thermal resistance  $R_{th}$ , electric resistance  $R$  and magnitude of current  $i$  as follows;

$$T = T_{atm} + i^2 R R_{th} \quad (1)$$

Here ambient temperature is 20°C in case of this linear motor. Magnitude of current is given. Thermal resistance must be determined by experiment. Electric resistance of a conductor will discuss the following section and will numerically be calculated. Note that connection type of all conductors in a mover must be considered to evaluate total electric resistance  $R$ . If  $R_1$  denotes electric resistance of a conductor, there is  $R = 2 R_1$  in case of 4 series and 2 parallel circuits.

### 2.2.1 Electric Resistance of a Conductor at Normal Temperature

A shape of a conductor considered in this paper was shown in Fig. 6.

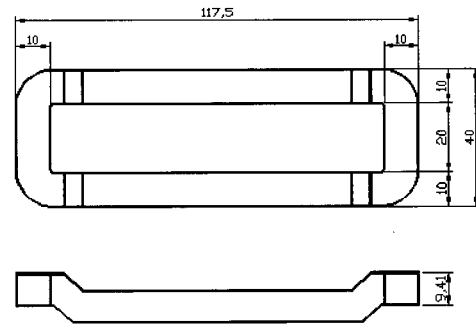


Fig. 6 Configuration of conductor

To explain electric resistance of a conductor, we defined electric resistance per unit length as  $k_r$ , which is specific resistance divided into cross section area of wire. The following relations were used to obtain the electric resistance  $R_1$  of a conductor.

$$\begin{aligned} N &= (1 - j) \cdot (A_{sect}/A_w) \\ L_c &= N L_{mean} \\ R_1 &= k_r L_c \end{aligned} \quad (2)$$

In the above,  $N$  denotes number of turns.  $A_{sect}$  means rectangular cross section of a conductor and  $A_w$  is a cross section of the coated wire.  $j$  is called as the coefficient of pore. The coefficient of pore has 0.13-0.22 in most cases.  $L_c$  is total length of coil and  $L_{mean}$  means average circular length in Fig.7.

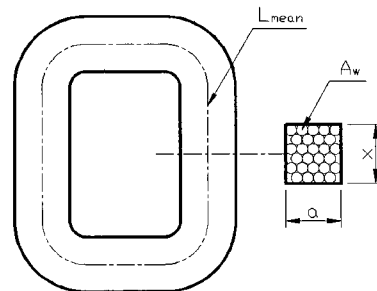


Fig. 7 Schematic of flat conduct and cross section

The specific resistance of copper wire was  $1.7241 \times 10^{-8} (\Omega / \text{m})$  and  $A_w$  was selected as maximum cross section for EIW Class 1 in Table 2.

Table 2 Class 1 polyester imide wire [unit :mm]

Coil Dia.	0.4	0.45	0.5	0.55	0.6
Max. Coated Coil Dia.	0.456	0.508	0.560	0.620	0.672

The conductor was manufactured with coil diameter of 0.55mm and 295 number of turns. When  $j=0.13$  was determined, the conductor thickness was 9.68mm. The measured electric resistance of this conductor was 6.1 $\Omega$  and numerical result from equation (2) was 6.07 $\Omega$  at 20 °C. As it had small error of 0.5%, the evaluation of electric resistance from an equation (2) is valid.

### 2.2.2 Thermal Resistance $R_{th}$

A conductor was molded in epoxy resin to protect impact and to isolate from electricity. When current  $i$  flows into conductors, heat  $i^2 R$  is generated because of electric resistance  $R$ . The heat is emitted in ambient air through epoxy resin. Since conductors are continuously arrayed along linear motor and are comparatively thin, most of heat are transferred forward the direction of magnets. Hence, conductors can be treated as one dimensional heat transfer problem.

Total heat transfer rate is equal to heat generation rate by current. If the heat transfer may be imagined as flow, all properties relating to conduction and convection can be treated as the resistance of this flow. It is thermal resistance  $R_{th}$  like electric resistance in electric circuit.  $R_{th}$  is obtained by the following equation.

$$R_{th} = \frac{T - T_{atm}}{i^2 R} [^{\circ}\text{C/W}] \quad (3)$$

To determine the thermal resistance, experiment as shown in Fig. 8 was done. When current flows into a mover from DC power supply with maximum current capacity 60A, temperatures of the mover increase sharply. After some times elapse, equilibrium temperature condition will prevail in the mover. Then temperature of the mover was measured by thermocouple and recorded by receiving from data acquisition board (HP3470A, 22bits)

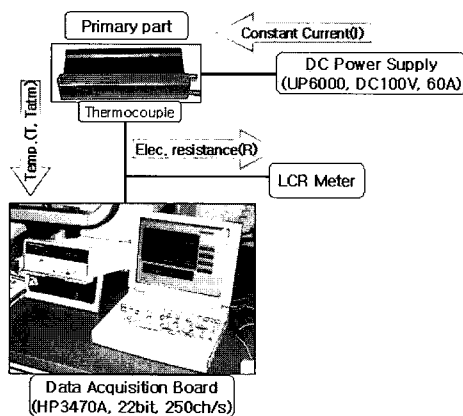


Fig. 8 Temperature Measurement

Experiments were performed for currents of 2A, 2.5A, 3A and 3.5A and steady state temperatures were measured after about 2-3 hours. Under the steady state criteria, thermal resistances for four magnitudes of current were explained in Table 3. The average thermal resistance was 0.456 $^{\circ}\text{C/W}$ . This  $R_{th} = 0.456^{\circ}\text{C/W}$  will be used to calculate temperature of conductors in optimal design of linear motor.

Table 3 Measured thermal conductivity of linear motor

$i$ [A]	$T_{atm}$ [°C]	$T$ [°C]	$R$ [ $\Omega$ ]	$R_{th}$ [°C/W]
2	21.0	36.1	8.3	0.455
2.5	21.7	47.0	8.7	0.462
3	21.4	59.1	9.1	0.460
3.5	20.3	72.8	9.6	0.447

### 2.2.3 Variation of Electric Resistance and Temperature

Electric resistance  $R$  and thermal resistance  $R_{th}$  were obtained from previous sections. Using these, the temperature of a mover can be evaluated from an equation (1) under given the thickness of conductor and given current. The electric resistance is a function of temperature. Electric resistance as  $R_0$  at normal temperature  $T_0$  was defined. When current flows into a mover, the temperature rises and this temperature variation changes the electric resistance.

$$R = \frac{234.5 + T}{234.5 + T_0} R_0 \quad (4)$$

Since temperature and electric resistance of a mover are coupled, equation (1) and (4) must be concurrently solved to obtain exact temperature of a mover. The temperatures of a mover were calculated at current of 2A, 2.5A, 3A and 3.5A. To show correctness of these equations, results by experiments were compared with numerical results. As shown in Table 4, numerical results were nearly same as experimental results with maximum error of 2.8%.

Table 4 Comparison between measured temperature and simulated temperature

$i$ [A]	Analysis T [°C]	Measured T [°C]	Error
2	36.4	36.1	0.8%
2.5	45.6	47.0	-2.8%
3	57.7	59.1	-2.4%
3.5	72.9	72.8	-0.1%

## 3. Optimal Design

### 3.1 Definition of Optimization Problem

Thrust is an important factor determining the performance of linear motor. Calculation of thrust using finite element analysis is reliable as proven by experiments. If magnitude of current, turns of coil and the size of permanent magnet increase, thrust may be improved. However, since the size of magnet and magnitude of current is limited because of standardization for linear motor, the turns of coil have been increased in most cases. Large number of turns raises electric resistance. It generates excessive heat and decreases the performance of linear motor. Hence, it is needed for the optimal design to maximize thrust within the range of operational temperature.

The presented research must determine magnitude of current, the size of a conductor and the size of magnets that maximize thrust under temperature limit of 80 °C. The global height of linear motor is 50mm and thickness of yoke is 7.5 mm. The gap distance between a conductor and a magnet is 2.5mm. Thickness of conductor ( $x$ ), thickness of magnet ( $y$ ) and magnitude of current ( $i$ ) are defined as design variables. The conductor is made from copper wire with diameter of 0.6mm and co-

efficient of pore of 0.13. Denoting thrust as  $F(x, y, i)$ , the given optimization problem is as follows;

$$\begin{aligned} &\text{Maximize } F(x, y, i) \\ &\text{Subject to } 6\text{mm} \leq x \leq 12\text{mm} \\ &\quad 6\text{mm} \leq y \leq 12\text{mm} \\ &\quad 1\text{A} \leq i \leq 3\text{A} \\ &\quad 18\text{mm} \leq x + 2y \leq 30\text{mm} \\ &\quad 147 \leq N \leq 295 \\ &\quad 20^\circ\text{C} \leq T \leq 80^\circ\text{C} \end{aligned} \quad (5)$$

Here turns of coil  $N$  is calculated from an equation (2) and temperature of conducts  $T$  is obtained from equations (1) and (4).

### 3.2 Optimization Method and Numerical Results

As the temperature constraint in an optimization problem (5) requires the solution of coupled equations (1) and (4), it is difficult to obtain analytical derivatives of this constraint with respect to design variables. Moreover, numerical derivative is not recommended because this method does not converge optimal solution well. The sub-problem approximation method among several methods in ANSYS, which does not use derivative, was selected as an optimization technique in order to get an optimal solution of this problem. The thrust defined as object function was iteratively calculated by finite element analysis code ANSYS during optimization process. Whenever the thickness of magnet and conductor are changed, new finite element modeling is always required. To overcome this major drawback, ADPL of ANSYS automatically generated finite element mesh.

In the optimization, initial values for design variables were  $x = y = 9\text{mm}$  and  $i = 1\text{A}$ . Then  $T = 35.625^\circ\text{C}$ ,  $N = 220.77$  and  $F = 123.63\text{N}$  were calculated. The optimal solution was obtained as  $x = 11.9516\text{mm}$ ,  $y = 9.0020\text{mm}$  and  $i = 1.638\text{A}$  after 11 iterations. The final maximum thrust was  $F = 266.75\text{N}$ . From optimum results,  $T = 79.384^\circ\text{C}$  and  $N = 293.17$  were determined. These design results satisfied all constraints of the design problem (5). Especially, the temperature constraint as well as the geometric constraint were tightly satisfied.

### 4. Conclusions

Considering operational temperature, the optimal design of linear motor finding maximum thrust was studied. Two dimensional finite element modeling was used to reduce the iterative computation times of thrust in optimization process. Experiments were performed on thrust and magnetic flux to confirm the validity of finite element analysis. The temperature of conductor affecting motor performance was calculated using the thermal resistance that was experimentally determined.

Thickness of a conductor, thickness of a permanent magnet and magnitude of current were defined as design variables. Number of turns was determined from an equation (2) with selected diameter of copper wire and given thickness of a conductor. Object function was thrust that was calculated from Maxwell stress. Including temperature and size constraints, optimal solution was obtained. Re-mesh of finite element was applied to this optimal design using the ADPL of ANSYS, because the geometry of linear motor is changed according to variations of design variables. Compared with initial design, the optimal design improved comparatively thrust. Then, temperature constraint plays a very important role in determining maximum thrust.

Though two dimensional electromagnetic analysis is used to evaluate thrust of linear DC motor, the present method does not put a limit to apply three dimensional analysis. In addition to, it was possible to get optimal design for other type of linear motor.

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