Constitutive Relation of Alloy Steels at High Temperatures

Youngseog Lee[1,#]

1 Department of Mechanical Engineering, Chung-Ang University, Seoul 156-756, Korea
Corresponding Author / E-mail: ysl@cau.ac.kr, TEL: +82-2-820-5256, FAX: +82-2-814-9476

KEYWORDS : Constitutive relation, Carbon equivalent model, Alloy steels, High temperature deformation, Hot rolling process

---

This paper presents a feasibility study whether Shida's constitutive equation being widely used for plain carbon steel in steel manufacturing industry can be extended to alloy steels with a due carbon equivalent model. The constitutive relation of the alloy steels (SAE9254, AISI52100 and AISI4140) is measured using hot deformation simulator (GLEEBLE 3500C) at high temperatures (800°C ~1000°C) within strain rates of 0.05~40 s^{-1}. It has been found the predicted flow stress behavior (constitutive relation) of AISI52100 steel is in agreement with the measured one. On the other hand, the measured flow stress behavior of SAE9254 and AISI4140 steel partly concords with the predicted one when material experiences relatively high strain rate (10~40 s^{-1}) deformation at low temperature (800°C). It can be deduced that, for AISI52100 steel, Shida's equation with the carbon equivalent model can be applicable directly to the roughing and intermediate finishing stand in hot rolling process for calculating the roll force and torque.

Manuscript received: August 17, 2004 / Accepted: March 17, 2005

NOMENCLATURE

C = Carbon (weight %)
C_{eq} = Equivalent carbon (weight %)
T= Temperature of material
T_p= Temperature for transformation
n = Strain hardening coefficient
m= Strain rate hardening coefficient
$\bar{\varepsilon}$ = Effective strain
$\dot{\bar{\varepsilon}}$ = Effective strain rate
$\bar{\sigma}$ = Flow stress

1. Introduction

In metal forming processes such as hot forging and rolling, the predictive capability of numerical simulation largely depends on constitutive relation (flow stress-strain relation) that describes the change in material response under external loading. Many researchers have proposed different types of constitutive equations.

Type of the equations basically can be divided into two groups. The first group expresses the equation in terms of macroscopic parameters such as strain, strain rate, temperature and chemical composition. Meanwhile, the equations in the second group are expressed as a function of temperature, strain, strain rate and chemical composition together with microscopic parameters such as recovery, recrystallization, grain growth, grain size, potential for precipitation, etc. The former usually has a well-structured formula and consequently does not have many material dependent constants. In the latter case, however, there are many material dependent constants to be determined through extensive test and subsequently we have a different constitutive equation for a different material. Hence, in manufacturing process such as hot rolling, the former is usually preferred because of efficiency in dealing with numerous different grades of materials.

There are several well-known constitutive equations belonging to the first group. Misaka and Yoshimoto [1] suggested a constitutive equation that specifies the mean resistance of deformation, which is an average value of flow stress during deformation, in terms of temperature, natural strain (reduction), strain rate and carbon content. This equation was derived purely from a curve-fitting/ regression of data acquired from a drop hammer test. Hence it might be useful for practical purpose only.

Johnson and Cook [2] reported a constitutive equation which assumes that the dependence of the stress on the strain, strain rate and temperature can be multiplicatively decomposed into three separate functions that include five constants to be determined. In this light, Johnson and Cook's equation is nothing but a form of constitutive equation that has five parameters to be determined by experimental data obtained for a material.

Shida [3] proposed a constitutive equation, giving the flow stress of carbon steels as a function of the strain, strain rate, temperature and carbon content. To the author's best knowledge, it is the first constitutive equation described in terms of strain, strain rate, temperatures and carbon contents based on physical meaning. Its biggest advantages are a wide range of thermo-mechanical parameters applicable to steel manufacturing process and no need to determine material constants through laboratory scale test. Hence it has been widely used for predicting the internal stress and strain of deforming material in many hot rolling processes and forging process. It is, however, applicable to plain carbon steel only, not to alloy steel. In this paper, we present a feasibility study whether Shida's equation can be applied to alloy steels with a due carbon equivalent model, which transforms the effect of various alloying components into a single carbon quantity. Steels chosen in this study are SAE9254, AISI52100 and AISI4140, widely being used for machine structural parts such as automobile suspension device, coil spring, bearing and other structural parts due to its high strength and fatigue resistance. This study is confined on the constitutive relation where strain hardening is dominant during deformation.

2. Experiment

The chemical components of the steels used in this study are listed in Table 1. They were prepared by a laboratory vacuum induction melting. Laboratory melt ingots were hot rolled to plate with 20 mm thickness. The specimens for compression test were machined from these plates and are of a cylindrical shape. The specimen axes are parallel to the traverse direction of the plates and the dimension of specimen is of 15mm length and 10mm diameter.

Table 1 Chemical composition (wt %) of the alloy steels used in this study

Steels	C	Mn	Si	Cr	Ni	Mo
SAE9254	0.55	0.75	1.40	0.75	0.70	-
AISI52100	1.00	0.50	0.25	-	1.45	0.08
AISI4140	0.40	0.725	0.25	1.05	-	0.15

The hot compression test has been performed to obtain flow stress-strain relation of the steels using GLEEBLE® 3500 thermo-mechanical simulator [4]. During test the specimen surface was protected from oxidation using argon. Figure 1 illustrates schematic of the hot compression test procedures. Samples were reheated to 1200°C at 5°Cs^{-1} and hold for 2 minutes before deformation using a direct resistance heating system inside the furnace. Specimens were reheated to 1200°C at 5°Cs^{-1} and hold for 2 minutes using direct resistance heating system inside the furnace (①-③). Specimens were then cooled to the isothermal test temperature. At the stage (④-⑤), specimens are deformed under various strain rates (0.05s^{-1}~40s^{-1}) and temperatures (800°C, 900°C and 1000°C).

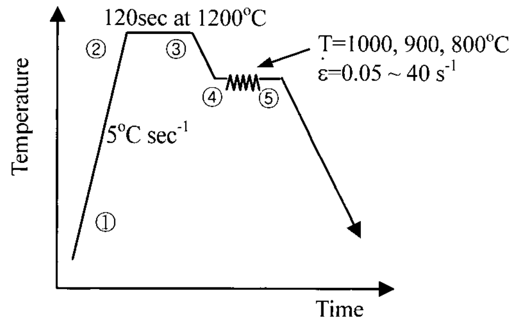

Fig. 1 Schematic of hot deformation test procedure

3. Shida's Constitutive Equation

To begin with, Shida's constitutive equation is investigated thoroughly. This is based on experimental data obtained from compression type of high temperature-high strain rate testing machines specifically suited for flow stress measurement, i.e., cam-plastometer and a drop hammer type of testing machine. This equation is applicable in the range of carbon content 0.07~1.2%, temperature 700~1200°C, strain up to 0.7 and strain rate up to 100s^{-1}. The experimental data were originally obtained by Suzuki *et al* [5]. Shida [3] then expressed the flow stress $\bar{\sigma}$ as a function of carbon content C, strain $\bar{\varepsilon}$, strain rate $\dot{\bar{\varepsilon}}$ and normalized temperature T as followings:

$$\bar{\sigma} = \sigma_d(C,T) \cdot f_w(\bar{\varepsilon}) \cdot f_r(\dot{\bar{\varepsilon}}) \cdot \tag{1}$$

The deformation resistance function, $\sigma_d(C,T)$ is given as follows:

$$\sigma_d = 0.28 \exp\left(\frac{5.0}{T} - \frac{0.01}{C+0.05}\right) \quad for\ T > T_p \tag{2}$$

or

$$\sigma_d = 0.28\, g(C,T) \exp\left(\frac{C+0.32}{0.19(C+0.41)} - \frac{0.01}{C+0.05}\right) \quad for\ T \le T_p \tag{3}$$

where $$T = \frac{T[°C]+273}{1000} \quad and \quad T_p = 0.95\frac{C+0.41}{C+0.32} \tag{4}$$

Equations (2) ~ (4) indicate that phase transformation occurs if T_p is greater than the normalized temperature, T. Equation (4) shows that the flow stress of steel in austenite phase appears to be almost independent upon the carbon content but heavily dependent on the temperature. As the temperature decreases, the flow stress of the steel increases exponentially. The flow stress behavior, however, would be different if material experiences phase transformation. Thus, $g(C,T)$ in Eq. (3) was expressed as

$$g(C,T) = 30.0(C+0.9)\left[T - 0.95\frac{C+0.49}{C+0.42}\right]^2 + \frac{C+0.06}{C+0.09} \tag{5}$$

The decrement of flow stress due to phase transformation was modeled using the second order parabolic function shown with blanket in Eq. (5). The flow stress decreases as the normalized temperature, T, does up to $0.95(C+0.49)/(C+0.42)$ and increases again as the normalized temperature is greater than it. Equation (5) implies that when the normalized temperature reaches the first ferrite regions, the flow stress falls sharply with a further temperature drop. After the end of the phase transformation, the flow stress increases again.

To describe the effect of the strain and strain rate on the flow stress, Shida introduced two functions, i.e., work hardening function, $f_w(\bar{\varepsilon})$ and strain rate hardening function, $f_r(\dot{\bar{\varepsilon}})$ (see Eq. (1)). The strain hardening function was expressed as following forms:

$$f_w(\bar{\varepsilon}) = 1.3\left(\frac{\bar{\varepsilon}}{0.2}\right)^n - 0.3\left(\frac{\bar{\varepsilon}}{0.2}\right) \tag{6}$$

$$n = 0.41 - 0.07C \tag{7}$$

Equation (6) shows that the material work hardening starts to decrease when the strain goes beyond 0.2, which is the reference strain for work hardening. Equation (7) implies that work hardening was modeled as a function of carbon content only and it has nothing to do with the variation of temperature and strain rate. Strain rate hardening function, $f_r(\dot{\bar{\varepsilon}})$ was given as

$$f_r(\dot{\bar{\varepsilon}}) = \left(\frac{\dot{\bar{\varepsilon}}}{10}\right)^m \tag{8}$$

We can know that the reference strain rate for the strain rate hardening was set as 10. It implies the strain rate hardening starts to increase when the strain rate of material is beyond 10. The coefficient of strain rate sensitivity, m, is as follows:

$$m=(-0.019C+0.126)T+(0.076C-0.05) \quad for\ T>T_p \qquad (9)$$

or

$$m=(0.081C-0.154)T+(-0.019C+0.207)+\frac{0.027}{C+0.32} \quad for\ T\le T_p \qquad (10)$$

Equations (9) and (10) illustrate that strain rate sensitivity is strongly reliant on temperature. Note that the effect of strain rate on the flow stress diminishes once the material undergoes phase transformation. Thus we can know that Shida's equation consists of three separate functions which can be multiplicatively decomposed and each function has a physical meaning to take into account thermo-mechanical parameter, carbons content and even phase transformation. Hence it has been used fruitfully to calculate constitutive relation of carbon steels in steel manufacturing industry.

4. Carbon Equivalent Model

Shida's model can be used only when the material is plain carbon steel. To extend its application range to steels that contain alloying element, we introduce a carbon equivalent model, which simplifies the effect of alloy components into a single carbon component. This simplification was made to facilitate the solution of this type of problem. Otherwise we may not deal with getting constitutive relation of alloy steels owing to complexity of the influence of alloying elements. The carbon equivalent C_{eq} may be calculated using the following model [6]

$$C_{eq}=[C]+[Mn]/6+([Cr]+[V]+[Nb])/12 \qquad (11)$$

Equation (11) takes account of the contribution of Mn, Cr, V and Nb to the resistance of the steel to deformation. The symbols in Eq. (11) indicate contents of elements in steel in weight %, and C_{eq} is the carbon equivalent, which replaces with C in Eqs. (2)~(10). It can be deduced that stress (strength) of alloy steel is always higher than that of plain carbon steel if the carbon component C is the same for both steels.

Another type of carbon equivalent model that includes more alloying elements than Eq. (11) is expressed as [7]

$$C_{eq}=[C]+[Mn]/6+([Ni]+[Cu])/15+ \\ ([Cr]+[Mo]+[V])/5 \qquad (12)$$

The differences between Eq. (11) and (12) are, first, the number of chemical components and secondly the assigned weighting function of the chemical components. In this study, Eq. (12) was used as the carbon equivalent model. It is worthy to note that, in both Eqs. (11) and (12), the effect of chemical component Si on the carbon equivalent was not included. This is because carbon activity of Si increases while its carbon diffusivity decreases. The other chemical components increase both carbons activity and carbon diffusivity. In addition, component Si does not lead to enhancing martensite structures through water quenching in comparison with other components. Hence Si might not play a role in carbon equivalent model and subsequently component Si is omitted in Eqs. (11) and (12).

5. Results and Discussion

Figures 2, 3 and 4 show the comparison between predicted flow stress-strain curves and measured ones for three alloy steels. Dotted lines indicate the experimentally measured flow stresses and solid lines the calculated ones. For SAE9254 steel shown in Fig. 2, the measured flow stresses are higher than the predicted ones on the whole. But the differences become small as the strain rate increases. Good agreement is noted at strain rate of $40s^{-1}$. At the strain rates of $0.05s^{-1}$ and $1.0s^{-1}$, the level of measured flow stress goes up or keeps the same level as the strain increases beyond approximately 0.4. This implies the barreling effect was not removed perfectly during the test. This is significant as the deformation temperature decreases, say 800°C.

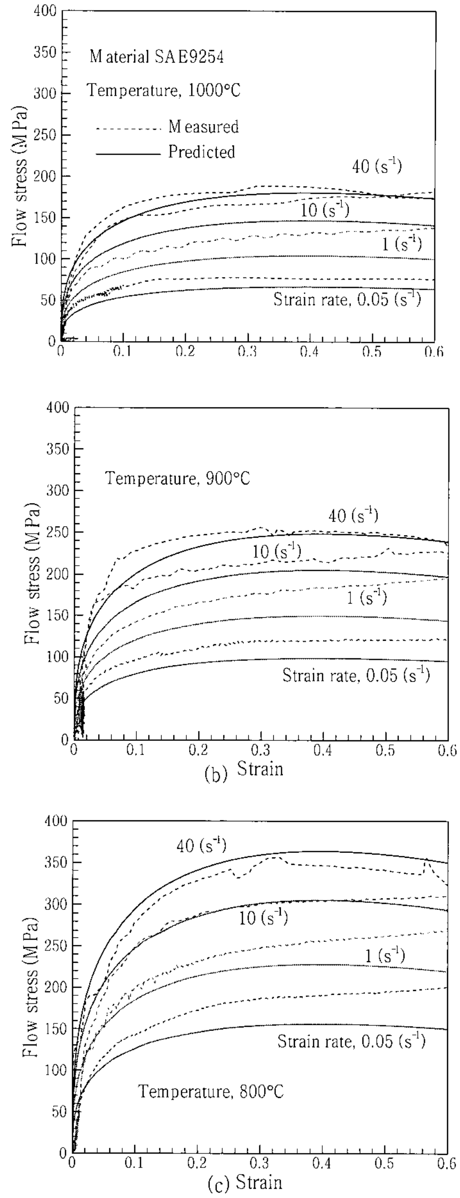

Fig. 2 Comparison of measured and predicted constitutive relations for SAE9254 steel at different temperatures and strain rates

Figure 3 shows that, except the barreling part, the predicted flow stress-strain curves of AISI4140 steel are in agreement with the measured ones for the temperatures of 1000°C and 900°C and strain rates of $10s^{-1}$ and $40s^{-1}$. The predicted values, however, overestimate appreciably the measured ones at 800°C, except the case that strain rate is $0.05s^{-1}$ and $1.0s^{-1}$. Note that at temperature of 800°C and low strain rates of $0.05s^{-1}$~$1.0s^{-1}$, large barreling is observed. For alloy steels (SAE9254 and AISI4140), on a certain case, the predicted flow stress behavior is in agreement with the measured one, but the measured flow stress overvalues the predicted ones at low strain rates and low temperature. This implies the carbon equivalent model should be revised such that it can take into account the effect of temperature and strain rate in addition to chemical components. But the carbon equivalent model may have limitations in considering the combination of many chemical components and thermo-mechanical parameters (temperature and strain rate) together. Alternatively we may use Johnson and Cook's constitutive equation [2] for each steel grade if test cost and time are not concerned.

Figure 4 illustrates the predicted flow stress-strain curves are in good agreement with the measured ones, except the part where barreling occurs and the flow stress-strain curves for the strain rate of 0.05 at 800°C. For the strain rates of $10s^{-1}$ and $40s^{-1}$, the predicted values overestimated significantly the measured ones. Hence we may use Shida's equation with the carbon equivalent model to predict constitutive relation of this steel at temperatures of 900°C and 1000°C.

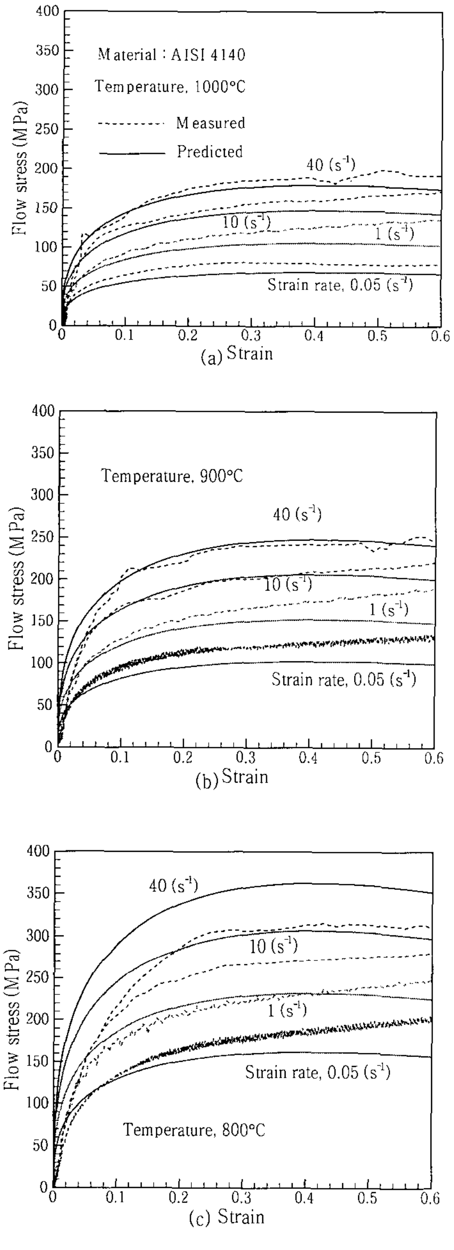

Fig. 3 Comparison of measured and predicted constitutive relations for AISI4140 steel at different temperatures and strain rates

6. Concluding Remarks

The impetus for this study was to investigate a number of constitutive equations that have been successfully used to describe the plastic behavior of carbon steels as a function of strain, strain rate, temperature and chemical composition. We have then investigated whether Shida's equation can be extended to predict the flow stress- strain relation of the three alloy steels (SAE9254, AISI52100 and AISI4140) by incorporating a carbon equivalent model. The conclusions are summarized as follows:

1) The carbon equivalent model for SAE9254 and AISI4140 steels should be revised when the deformation condition, i.e., temperature is low (800°C) and strain rate is low ($0.05\sim1.0s^{-1}$).
2) Shida's equation with the carbon equivalent model can be applicable directly to AISI52100 steel and this approach can be employed for predicting roll (separating) force and torque at the roughing and intermediate finishing train of rod mill, and roughing and finishing train of strip mill.

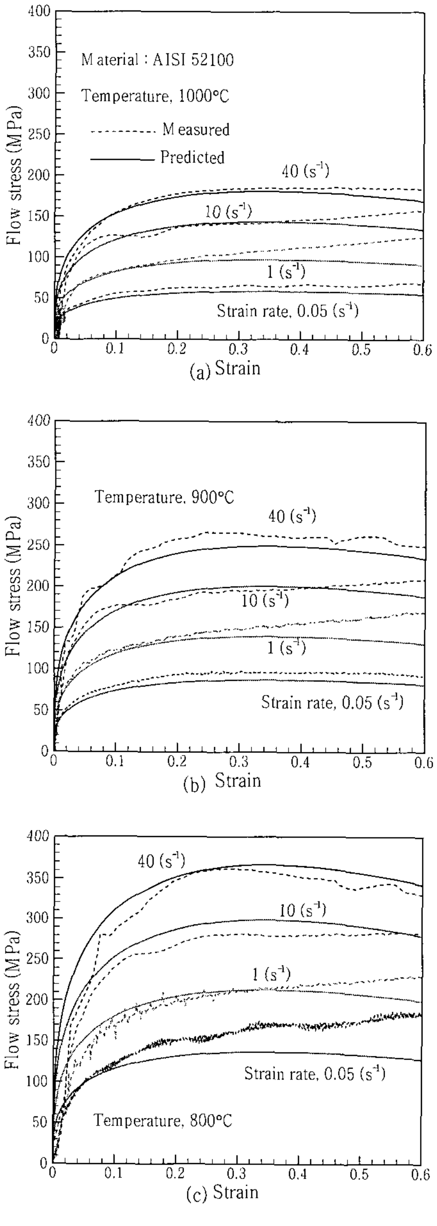

Fig. 4 Comparison of measured and predicted constitutive relations for AISI52100 steel at different temperatures and strain rates

ACKNOWLEDGEMENT

This research was supported by the Chung-Ang University Research Grants in 2004.

REFERENCES

1. Misaka, Y. and Yoshimoto, T., "Formulation of Mean Resistance of Deformation of Plain Carbon Steel at Elevated Temperature," J. Jpn. Soc. Technol. Plast. Vol. 8, pp. 414-422, 1967.
2. Johnson, G.R. and Cook, W.H., "A Constitutive Model and Data for Metals Subjected to Large Strains, High Strain Rates and High Temperatures," in Pro. 7th Intl. Sym. on Ballistics, Hague, Netherlands, pp. 541-547, 1983.
3. Shida, S., "Empirical Formula of Flow Stress of Carbon Steels – Resistance to Deformation of Carbon steels at Elevated Temperature," 2nd Report (in Japanese), J. JSTP, 10, pp. 610- 617, 1969.
4. Ferguson, H.S., "Recent Developments in Simulation and Testing Using Gleeble Systems," In: Proceedings of the 7th Inter. Sympo. on Physical Simulation of Casting, Hot Rolling and Welding, Suzuki HG et al. eds. Japan, 1-26. 1997.
5. Suzuki, H., Hashizume, S., Yabuki, Y., Ichihara, Y., Nakajima, S. and Kenmochi, K., "Studies on the Flow Stress of Metals and Alloys, Report of the Institute of Industrial Science," The University of Tokyo, 18, 1968
6. Lenard, J.G., Pietrzyk, M. and Cser, L., "Mathematical and Physical Simulation of the Properties of Hot Rolled Products," Elsevier Sci Ltd, Oxford, UK, 1999.
7. Laasraoui, A. and Jonas, J.J., "Prediction of Steel Flow Stress at High Temperatures and Strain Rates," Metall. Trans. A, 622A, pp.1545-1558, 1991.