# Adaptive Wavelet Analysis of Non-Stationary Vibration Signal in Rotor Dynamics

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*A rotor run-up or run-down process provide more useful information for modal analysis than normal operation conditions. A traditional difficulty associated with rotor run-up or run-down analysis is the non-stationary nature of vibration data. This paper compares Short-Time Fourier Transform (STFT) and the wavelets analysis in these non-stationary signal analyses. An Adaptive Wavelet Analysis (AWT) is proposed to analyze these signals. Although simulations and experiments in a simple rotor-bearing system show that both STFT and AWT can be used to analyze non-stationary vibration signals in rotor dynamics, proposed AWT provides better results than STFT analysis. From the amplitude-frequency curve obtained by AWT, the modal frequency and damping ratio are calculated. This paper also analyzes the characteristics of signals when the shaft touches the outer hoop in a run-up process. The AWT can give a good result in this complex dynamic analysis of the touching process.*

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## NOMENCLATURE

a = the dilation (scale) in wavelet transform
b = the translation in wavelet transform
$f_s$ = sampling frequency

## 1. Introduction

Modal testing is widely used for the dynamic test of rotor-bearing system in many industries. However, conventional modal identification procedures are limited to the force excitation under non-operational conditions[2]. Under operational conditions, the real loading status and environment change quite often. Therefore, the modal parameters under the working condition are different from ones obtained under non-operational conditions[3]. In rotor dynamics the dynamic behavior of the operating rotor–bearing system may differ significantly from the parameters obtained from a non-operational test because of the change of stiffness and damping of oil-film in bearings. Therefore, it is necessary to extract modal parameters from operational vibration signals. Rotor run-up or run-down process provides more information than stable working conditions because of the change of rotating speeds. The unbalance force excites the rotor-bearing systems as a swept sine exciting in run-up or run-down process. During these processes the rotor system may pass through the resonance frequency.

However, difficulty associated with a rotor-bearing system's run-up or run-down is the non-stationary nature of vibration signal. The power of classical Fourier analysis is affected by the various rotating speeds in a window length. Short-Time Fourier Transform (STFT) using shorter time window offers a better method for the study of time-varying signals. However, when the spectral components of the signal are changing rapidly it is difficult to find an appropriate short time window in which the signal is more or less stationary and thus allows to obtain an amplitude-frequency curve. We can extract the modal frequencies and damping ratios from the amplitude frequency curve.

Wavelets has been widely used in signal analysis[4,5]. The wavelet transform represents a signal as a sum of wavelets at different locations and scales, which has advantages to deal with non-stationary data. Most of the applications use its power to locate time-frequency distribution. In an entire run-up or run-down process, the rotating speed changes in a wide range, which requires the wavelet analysis with a large number of scales. However, because we try to use wavelet to locate amplitude-frequency distribution to extract modal frequencies and damping ratios, only a short time duration and a small range of scales related to rotating frequency needs to be considered around a specific time. Therefore, an Adaptive Wavelet Transform (AWT) is proposed for this task. A simple rotor-bearing system has been built to investigate the dynamic behaviour and the process of a shaft touching outer hoop.

## 2. Theory

### 2.1 Short-Time Fourier Transform

Fourier transform represents a signal as a superposition of sinusoids with different frequencies. The Fourier coefficients measure the contribution of the sinusoids at these frequencies. The basic idea of STFT is to divide the signal into small segments and to apply Fourier transform to analyze each segment to get the frequency components that exist in that segment[6]. To investigate the spectrum of signal $x(t)$ at time $t$, the signal is multiplied by a window function $h(t)$ centered at $t$:

$$x_t(\tau) = x(\tau)h(\tau - t) \tag{1}$$

In STFT

$$x_t(\tau) = \begin{cases} x(\tau) & \text{for } \tau \text{ near } t \\ 0 & \text{for } \tau \text{ far away from } t \end{cases} \tag{2}$$

The STFT of signal $x_t(\tau)$ is

$$X(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-j\omega\tau} x_t(\tau)d\tau \tag{3}$$

Because the short duration signals have inherently large bandwidths the frequency resolution is low. To obtain a higher resolution in practical calculation some zeros are added after each segment data. Since the windowed signal emphasizes the signal around time $t$ the STFT reflects the distrubution of frequency around the time. However, a signal shortened window, in general, cannot give good time and frequency localization. If the signal in one segment is non-stationary the STFT can only give an approximate result.

### 2.2 Adaptive Wavelet Transform

The continuous wavelet transform of signal x(t) is defined as[4]

$$(W_\Psi x)(a,b) = a^{-1/2}\int_{-\infty}^{+\infty} x(t)\overline{\Psi(\frac{t-b}{a})}dt \tag{4}$$

where $\Psi(\frac{t-b}{a})$ is the scaled wavelet, $\overline{\Psi(\frac{t-b}{a})}$ is the complex conjugate of $\Psi(\frac{t-b}{a})$, $a>0$, $b \in R$. In addition, $a$ is the dilation (scale), $b$ is the translation.

In this paper, the Morlet wavelet[7] is used which is defined as

$$\Psi(t) = e^{j\omega_0 t}e^{-\frac{t^2}{2}} \tag{5}$$

For a non-stationary signal analysis the frequencies of a signal in the total duration may change within a large range. The range of scale must be very large for a long time signal. If a fine scale is needed the calculation may become very time-consuming. Also it is not effective to extract fine characteristic from a very large scale range and time duration.

In a similar manner to STFT, we propose Adaptive Wavelet Transform (AWT). The basic idea of AWT is to break up the signal into small segments and analyze each time segment with wavelets to ascertain the scale (or frequency) that exists in the segment. In each segment calculation we use a smaller range of scales but finer resolution to calculate the wavelet transform of the signal at each segment. The range of scales in each segment is adapted to the signal automaticlly according to the former segment calculation. Equation (4) is changed into

$$(W_\Psi x)(a,b) = a^{-1/2}\int_{-\infty}^{+\infty} x_t(\tau)\overline{\Psi(\frac{\tau-b}{a})}d\tau \tag{6}$$

where $x_t(\tau)$ is same as equation (2). The wavelet transform involves scales rather than frequencies.

The basic frequency components in a rotor run-up process are rotating frequency and its harmonics. For extracting amplitude-frequency curve, the analyses are focused on the rotating frequency.

Consider a simple harmonic signal x(t) such that

$$x(t) = X_0 \sin(\omega_n t) \tag{7}$$

The wavelet transform[8] of x(t) is

$$(W_\Psi x)(a,b) = \sqrt{a}X_0 e^{-(a\omega_n - \omega_0)^2} e^{j\omega_n b} \tag{8}$$

In a simple rotor-bearing system, the vibration signals during run-up or run-down can be expressed as follows.

$$x(t) = D(t)\sin(\varphi(t)) \tag{9}$$

where amplitude $D(t)$ and phase $\varphi(t)$ are time varying. Noise is neglected.

We need to know the frequency at each time or the instantaneous frequency $\omega(t)$, which is the derivative of the phase $\varphi(t)$.[6]

$$\omega(t) = \frac{d\varphi(t)}{dt} \tag{10}$$

$\omega(t)$ is also time varied during a run-up or run-down process.

If we only consider a very small time range at time $t_0$, the instantaneous amplitude $D(t_0)$ and frequency $\omega(t_0)$ can be approximated as a constant. Equation (9) becomes

$$x(t) = D(t_0)\sin(\omega(t_0)t) \tag{11}$$

Only signals near time $t_0$ are calculated at one time of calculation so that the wavelet transform of x(t) is

$$(W_\Psi x)(a,b) = \sqrt{a}D(t_0)e^{-(a\omega(t_0)-\omega_0)^2} e^{j\omega(t_0)b} \tag{12}$$

The amplitude of the transform is

$$|(W_\Psi x)(a,b)| = \sqrt{a}D(t_0)e^{-(a\omega(t_0)-\omega_0)^2} \tag{13}$$

For a given time or translation, when

$$a\omega(t_0) - \omega_0 = 0 \tag{14}$$

Equation (13) gets its maximum value

$$|(W_\Psi x)(a,b)|_{\max} = \sqrt{a}D(t_0) \tag{15}$$

Because $\omega(t_0) = 2\pi f(t_0)$, from equations (14) and (15), we can obtain the instantaneous frequency and amplitude at time $t_0$ as follows.

$$f(t_0) = \frac{\omega_0}{2\pi a} \tag{16}$$

$$D(t_0) = |(W_\Psi x)(a,b)|_{\max} / \sqrt{a} \tag{17}$$

For digital calculation, the vibration signal is sampled at frequency $f_s$ and when the variable is index $k$, the signal near $k_0$ can be expressed as

$$x(k) = D(k_0)\sin(2\pi f(k_0)k / f_s) \tag{18}$$

The instantaneous frequency and amplitude can be calcutaled as follows.

$$f(k_0) = \frac{\omega_0 f_s}{2\pi a} \tag{19}$$

$$D(k_0) = \left|(W_\Psi x)(a,b)\right|_{\max} / \sqrt{a} \tag{20}$$

For practical calculation we divide each long time domain signal into many segments. 25% of each segment overlaps with the preceding segment. To ensure the quality of the results, we use only 50% of the results of wavelet transform at the center translation. After obtaining the amplitude-frequency curve we use the half power method to calculate the viscous damping ratios.

### 2.3 Analysis of a simulated signal

The response of a damped system under rotating unbalance as rotating speed is sweeping up linearly[9] is used to compare the STFT and AWT.

$$x(t) = X(t)\sin(\varphi(t) - q(t)) \tag{21}$$

Where

$$X(t) = X_0 \frac{(\omega(t)/\omega_n)^2}{\sqrt{\left[1-(\omega(t)/\omega_n)^2\right]^2 + (2\xi\omega(t)/\omega_n)^2}} \tag{22}$$

$$q(t) = \tan^{-1}\left(\frac{2\xi\omega(t)/\omega_n}{1-(\omega(t)/\omega_n)^2}\right) \tag{23}$$

$$\varphi(t) = (\omega_0 + 0.5ct)t \tag{24}$$

$$\omega(t) = \omega_0 + ct \tag{25}$$

where $c$ is a constant. Some data in the simulation calculation are as follows.

Start frequency: 50Hz
Stop frequency: 100Hz
Resonance frequency : 80 Hz
Damping ratio $\xi$ : 10%
Sampling frequency: 512Hz
$X_0 = 1$

Fig. 1 presents the result of STFT. Each segment has 256 sampling points. We have used different numbers of points for one segment, and 256 points per segment gave the best result at this situation.

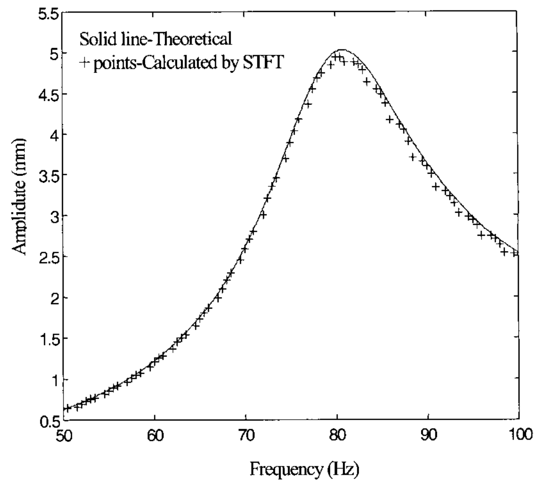



Fig. 1 The result of Short-Time Fourier Transform

Fig. 2 is the result of AWT. We used an adaptive scale range and scale resolution to fit each segment. The amplitude-frequency curve obtained by AWT is smoother and more accurate than the ones obtained by STFT. The curve calculated by AWT is almost the same as the theoretical one.

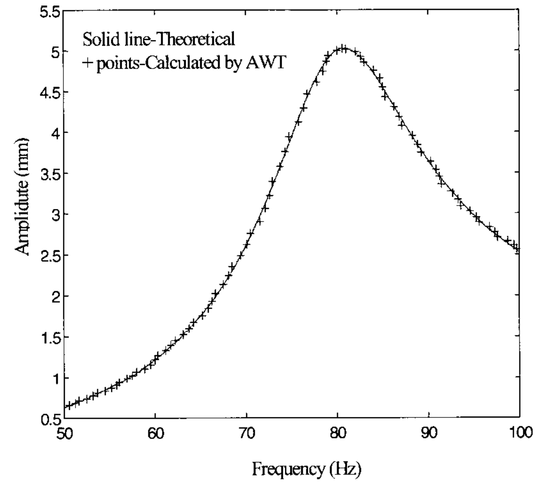
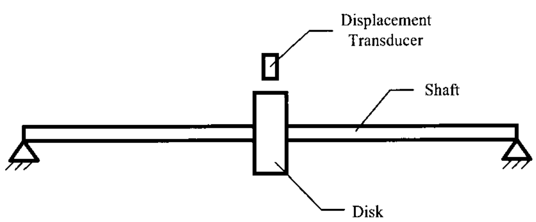


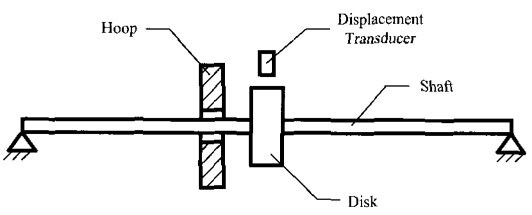

Fig. 2 The result of Adaptive Wavelet Analysis

## 3. Experiment and Results

In many rotating machines the rotor may be concentrated on one location on the shaft. In this paper, we consider a simple rotor-bearing system. The rotor mass is located at the center of the shaft. The both ends of the shaft are supported by oil-film bearings. The drawing of test rig is shown in Fig. 3. The displacement transducer is mounted on a supporter to measure the rotor displacement. The circular hoop is used to simulate the touching process of the shaft against the outer casing. The cirlular hoop is mounted near the rotor.




(a) Without circular hoop




(b) With circular hoop

Fig. 3 A simple rotor-bearing test rig

The experiment comprised of three parts: (1) impact method to test the frequency response function under non-rotating state, (2) measuring the response signals during the run-up process when the circular hoop is not mounted, (3) simulation of touching process. The hoop is mounted near the disk, and the gap between the shaft and the hoop is adjusted to an appropriate measure so that the shaft can touch the hoop when the rotating speed is near the resonant frequency. All the signals at the run-up process are recorded.

In the non-rotating state, we used the frequency domain modal analysis method[10] to extract the modal parameters from the frequency response function. The first bending modal frequency and viscous damping ratio were 84.8Hz and 1.36%, respectively, in the vertical direction measurement.

In rotating state, the rotor was controlled by a motor to run-up slowly. In this case, the rotating speed of shaft was time varying. To obtain an optimal result with less calculation, we used an adaptive scale adjustment in wavelet analysis so that the frequency has sufficient precision. With STFT, the shorter window length is fixed, then the frequency resulotion is fixed. It is difficult to adjust the window length to obtain a high precision result in STFT.

Fig. 4 shows the time domain displacement signal in the run-up process when no hoop was mounted. Fig. 5 and 6 are the amplitude-frequency curve obtained by STFT and AWT, respectively, during the run-up process when no hoop was mounted. The first resonance frequency is 78.3Hz and the viscous damping ratio is 16.2%.

Fig. 7 is the time domain displacement signal in the run-up process when a hoop was mounted and the shaft has touched the hoop near the resonance frequency. Fig. 8 and 9 are the amplitude-frequency curve obtained by STFT and AWT, respectively, from the signal shown in Fig. 7. The first resonance frequency is 76.7Hz and the viscous damping ratio is 4%.

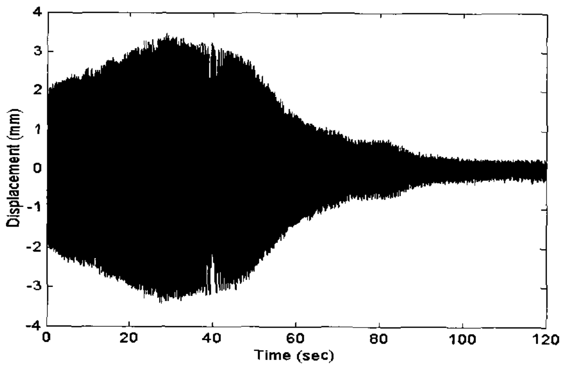



Fig. 4 Time domain signal in run-up process, no hoop mounted

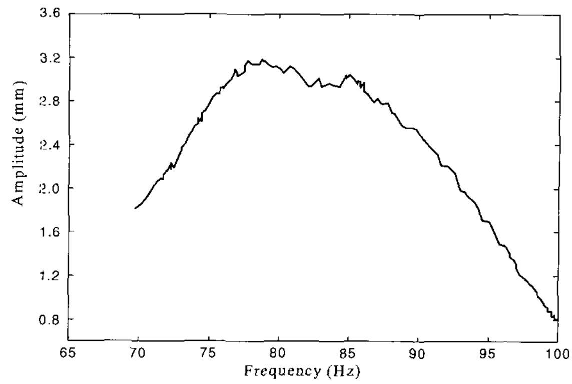



Fig. 5 Run-up process with STFT, no hoop mounted

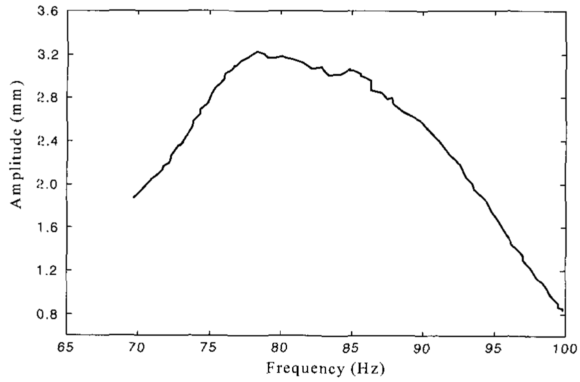



Fig. 6 Run-up process with AWT, no hoop mounted

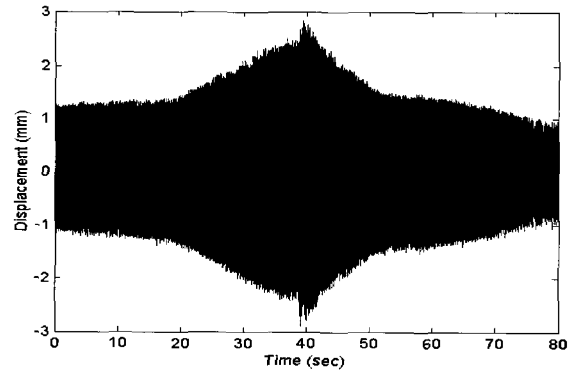



Fig. 7 Time domain signal in run-up process touching the hoop

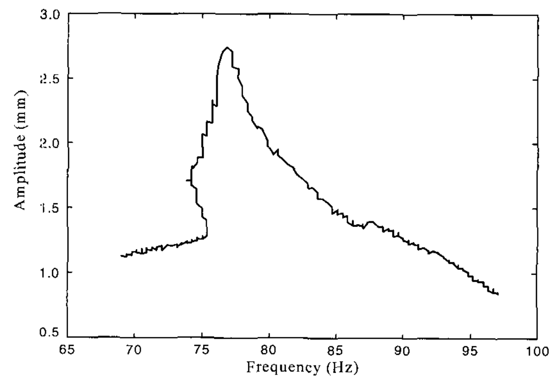



Fig. 8 Run-up process touching the hoop with STFT

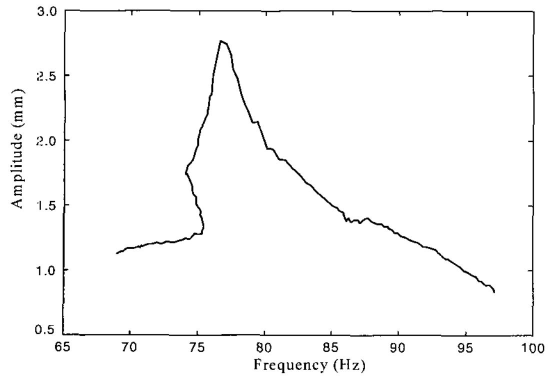



Fig. 9 Run-up process touching the hoop with AWT

It is obvious that the results obtained by AWT is better than those by STFT. During the touching process a unique behavior of the rotor has been observed. When the shaft runs up against the outer hoop the rotating speed of the shaft was decreased first, while the amplitude of vibration continues to grow higher. Then, after the first touching moment, the rotating speed increased. The damping ratio is much less than the case when there is no touching process.

## 4. Conclusions

The results from a simulated signal and a real rotor run-up process experiments show that both STFT and AWT can be used in non-stationary signal analysis. AWT gives better results than STFT to analyze the non-stationary data. The digital simulation results show that the maximum error of amplitude near the resonance frequency calculated by STFT and AWT are about 3% and 1%, respectively. The amplitude-frequency curve obtained by AWT is smoother than the curve obtained by STFT, which is helpful for further analysis. Therefore, the AWT method proposed in this paper is more useful to analyze non-stationary signals.

The experiment results also show that the modal frequency and damping ratio in operational states are significantly different from the results in non-operational states. This is why the stiffness and damping in running states show greater change than in static states. These results also indicate that it is necessary to analyze rotor dynamics using its run-up or run-down process.

When the shaft touches a hoop during the run-up process, the rotating speed is slowed down first and then it gradually picks up. The peak in amplitude-frequency curve is sharper than ones without hoop, which means the damping ratio becomes smaller by touching the hoop. In the touching process, the dynamics of the system is very complex, which is beyond the scope of this paper. However the amplitude-frequency curve indicates clearly the phenomenon of the touching process, which can be used as an indicator to monitor the condition of rotor-bearing system in a run-up or run-down process.

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