Electromagnetic Penetration into an Annular Aperture in a Thick Conductor

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Abstract

Electromagnetic penetration into an annular aperture in a thick conducting plane is investigated with the integral transform and eigen-function expansion method. The solution is analytic and is represented in rapidly-convergent series which is amenable to numerical analysis. Numerical computations shows that apertures with narrow annular gap have sharp transmit power peaks in frequency response.

Key words : Annular Aperture, Electromagnetic Wave Penetration.

Ⅰ. Introduction

Electromagnetic wave scattering from an annular aperture in a conducting plane is a canonical problem in diffraction theory. An annular aperture in a conducting plane is a basic element used as a frequency selective surface[1], a diffractionless beam-generating ring[2], and a slot antenna[3]~[5]. Extensive work has been performed to analyze annular slot antennas and cavity-backed annular antennas using numerical approaches[3],[5],[6]. Field leakage through an annular slot finds practical applications in electromagnetic interference, compatibility problems. Static potential penetration into an annular aperture in a conducting plane has been considered based on the Hankel transform[7],[8]. Far-field radiation from elliptic apertures and its annuli has been investigated[9]. We note that scattering from any arbitrarily-shaped aperture and slot can be also studied with the numerical methods-the moment method[10], the finite element method[11],[12], and the finite-difference time-domain method[13]. An understanding of electromagnetic scattering from an annular aperture requires the solution of a boundary-value problem dealing with an annular aperture in a thick conducting plane. A theoretical analysis for wave scattering from an annular aperture, however, seems to be lacking except for a high-frequency approximation[10], thus it is of interest to rigorously investigate its theoretical formulation.

An analytic formulation to be used in this paper is based on the integral transform and eigen-function expansion method. The analysis method allows us to represent the scattering solution as a rapidly convergent series which is amenable to numerical computations.

Ⅱ. Field Representation

The annular aperture is located in a thick conducting plane as shown in Fig. 1. As the geometry has z-axis as an axis of symmetry, we use a cylindrical coordinate system. The regions are divided into three areas according to the z-coordinates. Region I is the upper half space, region Ⅱ is the annular area in the conducting plane, and region Ⅲ is the lower half space. As the incident wave approaches the annular aperture from region Ⅲ, the wave meets discontinuity in region Ⅱ generating numerous higher order modes composed of TEM, TE and TM wave. We use Hertz potentials to represent thus generated higher order modes so as to

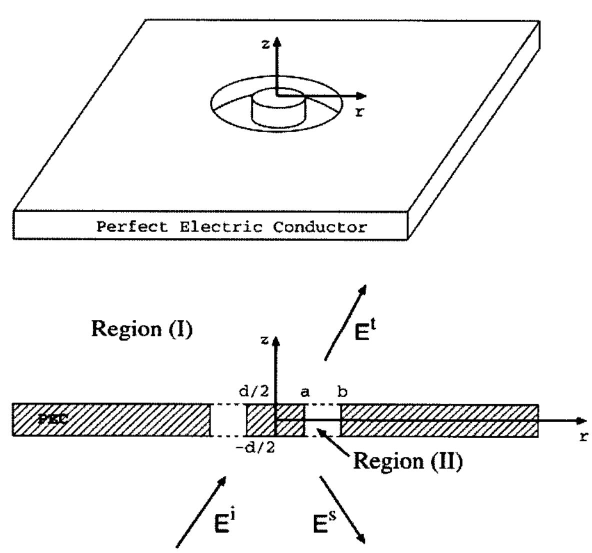

Fig. 1. Problem geometry. Plane waves are incident from region Ⅲ.

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simplify the representations[14].

In region I, field quantities are represented in integral forms as the area is unbounded and the eigen-modes have continuous spectrums. We use only transverse components of the electric and magnetic fields and they are as follows.

$$\mathbf{E}_{1t} = \sum_{\nu} \int_0^{\infty} A_m(\nu,\zeta)\,\mathbf{e}_m(\nu,\zeta)\, e^{-j\kappa z}\, d\zeta + \sum_{\nu} \int_0^{\infty} A_e(\nu,\zeta)\,\mathbf{e}_e(\nu,\zeta)\, e^{-j\kappa z}\, d\zeta \quad (1)$$

$$\mathbf{H}_{1t} = \sum_{\nu} \int_0^{\infty} A_m(\nu,\zeta)\,\mathbf{h}_m(\nu,\zeta)\, e^{-j\kappa z}\, d\zeta + \sum_{\nu} \int_0^{\infty} A_e(\nu,\zeta)\,\mathbf{h}_e(\nu,\zeta)\, e^{-j\kappa z}\, d\zeta \quad (2)$$

where $\mathbf{e}_m(\nu, \zeta)$ and $\mathbf{e}_e(\nu, \zeta)$ are TM and TE mode electric field eigen-functions and $\mathbf{h}_m(\nu, \zeta)$ and $\mathbf{h}_e(\nu, \zeta)$ are magnetic fields. Z_m and Z_e are TM and TE mode wave impedances.

The mode functions are represented via magnetic and electric Hertz potentials.

$$e_m(\nu,\zeta) = -j\kappa \nabla_t \Pi_m(\nu,\zeta) \quad (3)$$

$$e_e(\nu,\zeta) = -\hat{z}\times \mathrm{w} n_1 \nabla_t \Pi_e(\nu,\zeta) \quad (4)$$

$$h_m(\nu,\zeta) = -\hat{z}\times w n_1 \nabla_t \Pi_m(\nu,\zeta) \quad (5)$$

$$h_e(\nu,\zeta) = -j\kappa \nabla_t \Pi_e(\nu,\zeta) \quad (6)$$

$$\Pi_e(\nu,\zeta) = \Pi_m(\nu,\zeta) = J_\nu(\zeta r) e^{j\nu\phi} \quad (7)$$

where $\kappa = (\omega^2 n_1^2 - \zeta^2)^{1/2}$ and Π_m, Π_e, are magnetic and electric Hertz potentials respectively, satisfying Helmholtz equation. The subscript n_1 is a refractive index of the medium 1.

In region Ⅱ, the mode functions have discrete eigen-values to satisfy boundary conditions on the surface of electric conductor.

$$E_{2t} = (V_1^{tem+} e^{-jkz} + V_1^{tem-} e^{jkz}) e_{tem} + \sum_{\nu,i} (V_{\nu,i}^{m+} e^{-j\kappa_i z} + V_{\nu,i}^{m-} e^{j\kappa_i z}) e_m(\nu,i) + \sum_{\nu,i} (V_{\nu,i}^{e+} e^{-j\kappa'_i z} + V_{\nu,i}^{e-} e^{j\kappa'_i z}) e_e(\nu,i) \quad (8)$$

$$H_{2t} = (V_1^{tem+} e^{-jkz} - V_1^{tem-} e^{jkz}) h_{tem} + \sum_{\nu,i} (V_{\nu,i}^{m+} e^{-j\kappa_i z} - V_{\nu,i}^{m-} e^{j\kappa_i z}) h_m(\nu,i) + \sum_{\nu,i} (V_{\nu,i}^{e+} e^{-j\kappa'_i z} - V_{\nu,i}^{e-} e^{j\kappa'_i z}) h_e(\nu,i) \quad (9)$$

where

$$e_{tem} = \nabla_t \Pi_{tem},\ \Pi_{tem} = \ln r \quad (10)$$

$$e_m(\nu,i) = -j\kappa_{\nu,i} \nabla_t \Pi_m(\nu,i) \quad (11)$$

$$e_e(\nu,i) = -\hat{\mathbf{z}}\times \omega n_2 \nabla_t \Pi_e(\nu,i) \quad (12)$$

$$\Pi_m(\nu,i) = [J_\nu(\chi_{\nu,i} r/b) - P_{\nu,i} N_\nu(\chi_{\nu,i} r/b)] e^{j\nu\phi} \quad (13)$$

$$\Pi_e(\nu,i) = [J_\nu(\chi'_{\nu,i} r/b) - Q_{\nu,i} N_\nu(\chi'_{\nu,i} r/b)] e^{j\nu\phi} \quad (14)$$

where

$$P_{\nu,i} = J_\nu(\chi_{\nu,i} a/b) / N_\nu(\chi_{\nu,i} a/b) \quad (15)$$

$$Q_{\nu,i} = J'_\nu(\chi'_{\nu,i} a/b) / N'_\nu(\chi'_{\nu,i} a/b) \quad (16)$$

where $\chi_{\upsilon,i}$'s are roots of cylindrical function $J_\nu(x) N_\nu(x) - J_\nu(xa/b) N_\nu(xa/b) = 0$, and $\chi'_{\nu,i}$'s are roots of $J'_\nu(x) N'_\nu(x) - J'_\nu(xa/b) N'_\nu(xa/b) = 0$.

In region Ⅲ, the scattered fields are represented in the same manner as in region I.

$$E_{3t} = \sum_{\nu} \int_0^{\infty} B_m(\nu,\zeta) e_m(\nu,\zeta)\, e^{j\kappa z}\, d\zeta + \sum_{\nu} \int_0^{\infty} B_e(\nu,\zeta) e_e(\nu,\zeta)\, e^{j\kappa z}\, d\zeta \quad (17)$$

$$H_{3t} = -\sum_{\nu} \int_0^{\infty} B_m(\nu,\zeta) h_m(\nu,\zeta)\, e^{j\kappa z}\, d\zeta - \sum_{\nu} \int_0^{\infty} B_e(\nu,\zeta) h_e(\nu,\zeta)\, e^{j\kappa z}\, d\zeta \quad (18)$$

where the mode functions are defined as in the region I. Besides, there are incident field and specularly reflected field.

In the above equations, we have unknown quantities $A_m(\nu, \zeta)$, $A_e(\nu, \zeta)$, $B_m(\nu, \zeta)$, $B_e(\nu, \zeta)$ and $V_{\nu,i}^{m\pm}$, $V_{\nu,i}^{e\pm}$. The unknowns are found using tangential electric and magnetic field continuity conditions at the interfaces of each region.

First, we use electric field continuity condition at the interface of region I and region Ⅱ to get $A_m(\nu, \zeta)$.

$$\begin{aligned} \int_0^{\infty} E_{1t} \times h_m^*(\nu,\zeta')\cdot da &= j\omega\kappa\zeta' A_m(\nu,\zeta) \\ &= \langle E_{2t}(z=d/2) \,|\, h_m(\nu,\zeta') \rangle \\ &= (V_1^{tem+} e^{-jkd/2} + V_1^{tem-} e^{jkd/2}) \delta_{\nu,0} \langle e_{tem} \,|\, h_m(\nu,\zeta') \rangle \\ &+ \sum_{\nu,i} (V_{\nu,i}^{m+} e^{-j\kappa_i d/2} + V_{\nu,i}^{m-} e^{j\kappa_i d/2}) \langle e_m(\nu,i) \,|\, h_m(\nu,\zeta') \rangle \\ &+ \sum_{\nu,i} (V_{\nu,i}^{e+} e^{-j\kappa'_i d/2} + V_{\nu,i}^{e-} e^{j\kappa'_i d/2}) \langle e_e(\nu,i) \,|\, h_m(\nu,\zeta') \rangle \end{aligned} \quad (19)$$

Similarly, we get $A_e(\nu, \zeta)$

$$j\omega\kappa^* \zeta' A_e(\nu,\zeta') = \langle E_{2t}(z=d/2) \,|\, h_e(\nu,\zeta') \rangle \quad (20)$$

where we use the orthogonality properties of mode functions and the brackets in the expression is defined as follows.

$$<p|q>\equiv\int_S (p\times q^*)\cdot da=-<q|p>^* \tag{21}$$

In the same manner, we get $B_m(\nu,\ \zeta)$ and $B_e(\nu,\ \zeta)$ at the interface of region Ⅱ and region Ⅲ.

$$j\omega\kappa\zeta\, B_m(\nu,\zeta)=<E_{2t}(z=-d/2)|h_m(\nu,\zeta)> \tag{22}$$

$$j\omega\kappa^*\zeta\, B_e(\nu,\zeta)=<E_{2t}(z=-d/2)|h_e(\nu,\zeta)> \tag{23}$$

As shown above, field quantities in region I and Ⅲ are represented by the tangential electric field in region Ⅱ, although the still have unknown coefficients $V^{m\pm}_{\upsilon,i}$, $V^{e\pm}_{\upsilon,i}$.

We use magnetic field continuity condition to find out them.

$$<H_{1t}(z=d/2)|e_\alpha(\nu,i)>=<H_{2t}(z=d/2)|e_\alpha(\nu,i)> \tag{24}$$

$$<H_{3t}(z=-d/2)|e_\alpha(\nu,i)>=<H_{2t}(z=-d/2)|e_\alpha(\nu,i)> \tag{25}$$

where the subscripts α=tem, m, e.

Rewriting the above equations in the upper half space, it follows that

$$\begin{aligned}&(V^{\alpha+}_{\nu,q}e^{-jkd/2}-V^{\alpha-}_{\nu,q}e^{jkd/2})\delta_{p,q}<h_\alpha(\nu,p)|e_\alpha(\nu,q)>\\&=\int_0^\infty A_m(\nu,\zeta)<h_m(\nu,\zeta)|e_\alpha(\nu,q)>d\zeta\\&+\int_0^\infty A_e(\nu,\zeta)<h_e(\nu,\zeta)|e_\alpha(\nu,q)>d\zeta\end{aligned} \tag{26}$$

where $\kappa_{tem}=k$, $\kappa_m=\kappa_{\nu,i}=[\omega^2 n_2^2-(\chi_{\nu,i}/b)^2]1^{/2}$, $\kappa_e=\kappa'_{\nu,i}=[\omega^2 n_2^2-(\chi'_{\nu,i}/b)^2]^{1/2}$.

In the lower half space, the equations are as follows.

$$\begin{aligned}&(V^{\alpha+}_{\nu,q}e^{jkd/2}-V^{\alpha-}_{\nu,q}e^{-jkd/2})\delta_{p,q}<h_\alpha(\nu,p)|e_\alpha(\nu,q)>\\&=\frac{1}{\eta}<E_i-E_r|e_\alpha(\nu,q)>\\&-\int_0^\infty B_m(\nu,\zeta)<h_m(\nu,\zeta)|e_\alpha(\nu,q)>d\zeta\\&-\int_0^\infty B_e(\nu,\zeta)<h_e(\nu,\zeta)|e_\alpha(\nu,q)>d\zeta\end{aligned} \tag{27}$$

where E_i and E_r are incident electric field and reflected electric field, respectively. The incident field is a plane wave. For perpendicular polarization(TE), the electric field is as follows.

$$E_i=\hat{\mathbf{y}}\,e^{-jk(x\sin\theta_i+z\cos\theta_i)} \tag{28}$$

For parallel polarization(TM), E_I for TE substitutes for H_I for TM mode. Summarizing the equations, the following simultaneous matrix equations are made.

$$\begin{aligned}&(V^{\alpha+}_{\nu,q}e^{-jkd/2}-V^{\alpha-}_{\nu,q}e^{jkd/2})\delta_{p,q}<h_\alpha(\nu,p)|e_\alpha(\nu,q)>\\&=(V^{tem+}_1e^{-jkd/2}+V^{tem-}_1e^{jkd/2})\delta_{\nu,0}I_{1,\alpha}(\nu,q)\\&+\sum_{\nu,p}(V^{m+}_{\nu,p}e^{-j\kappa_p d/2}+V^{m-}_{\nu,p}e^{j\kappa_p d/2})I_{2,\alpha}(\nu,p,q)\\&+\sum_{\nu,p}(V^{e+}_{\nu,p}e^{-j\kappa'_p d/2}+V^{e-}_{\nu,p}e^{j\kappa'_p d/2})I_{3,\alpha}(\nu,p,q)\end{aligned} \tag{29}$$

$$\begin{aligned}&(V^{\alpha+}_{\nu,q}e^{jkd/2}-V^{\alpha-}_{\nu,q}e^{-jkd/2})\delta_{p,q}<h_\alpha(\nu,p)|e_\alpha(\nu,q)>\\&=\frac{1}{\eta}<E_i-E_r|e_\alpha(\nu,q)>\\&-(V^{tem+}_1e^{jkd/2}+V^{tem-}_1e^{-jkd/2})\delta_{\nu,0}I_{1,\alpha}(\nu,q)\\&+\sum_{\nu,p}(V^{m+}_{\nu,p}e^{j\kappa_p d/2}+V^{m-}_{\nu,p}e^{-j\kappa_p d/2})I_{2,\alpha}(\nu,p,q)\\&+\sum_{\nu,p}(V^{e+}_{\nu,p}e^{j\kappa'_p d/2}+V^{e-}_{\nu,p}e^{-j\kappa'_p d/2})I_{3,\alpha}(\nu,p,q)\end{aligned} \tag{30}$$

where

$$\begin{aligned}&I_{1,\alpha}(\nu,q)\\&=\frac{1}{j\omega}\int_0^\infty<e_{tem}|h_m(\nu,\zeta)><h_m(\nu,\zeta)|e_\alpha(\nu,q)>\frac{d\zeta}{\kappa\zeta}\\&+\frac{1}{j\omega}\int_0^\infty<e_{tem}|h_e(\nu,\zeta)><h_e(\nu,\zeta)|e_\alpha(\nu,q)>\frac{d\zeta}{\kappa^*\zeta}\end{aligned} \tag{31}$$

$$\begin{aligned}&I_{2,\alpha}(\nu,p,q)\\&=\frac{1}{j\omega}\int_0^\infty<e_m(\nu,p)|h_m(\nu,\zeta)><h_m(\nu,\zeta)|e_\alpha(\nu,q)>\frac{d\zeta}{\kappa\zeta}\\&+\frac{1}{j\omega}\int_0^\infty<e_m(\nu,p)|h_e(\nu,\zeta)><h_e(\nu,\zeta)|e_\alpha(\nu,q)>\frac{d\zeta}{\kappa^*\zeta}\end{aligned} \tag{32}$$

$$\begin{aligned}&I_{3,\alpha}(\nu,p,q)\\&=\frac{1}{j\omega}\int_0^\infty<e_e(\nu,p)|h_m(\nu,\zeta)><h_m(\nu,\zeta)|e_\alpha(\nu,q)>\frac{d\zeta}{\kappa\zeta}\\&+\frac{1}{j\omega}\int_0^\infty<e_e(\nu,p)|h_e(\nu,\zeta)><h_e(\nu,\zeta)|e_\alpha(\nu,q)>\frac{d\zeta}{\kappa^*\zeta}\end{aligned} \tag{33}$$

Integrands of the integrals are summarized in the appendix.

Ⅲ. Numerical Computations

The formulation derived in the previous section is useful for calculating admittance, polarizabilities, transmission coefficients and so on. In Fig. 2, input admittance and susceptance of an open-ended coaxial cable is calculated to verify our formulation. As the thickness of the conductor where an annular aperture is located increases, the annular core looks like open ended coaxial cable. If the annular core is excited by a TEM wave, input admittances are calculated using the reflection coefficients. The result matches exactly that of Irzinsky[15].

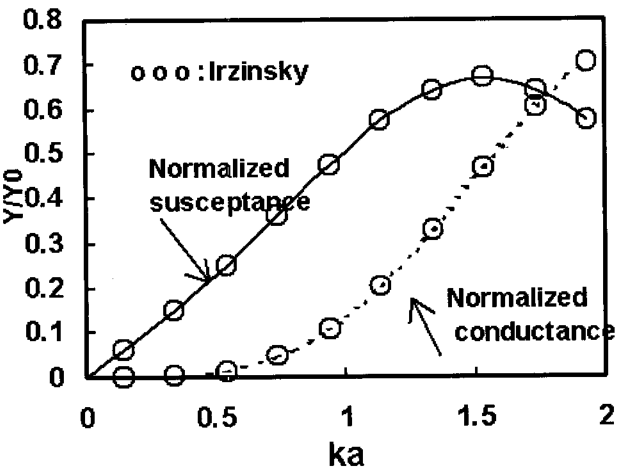

Fig. 2. Input admittances of an open-ended coaxial cable. As the thickness of the conductor increases to infinity, the annular aperture behaves like coaxial cable. The result exactly matches those of Irzinsky.

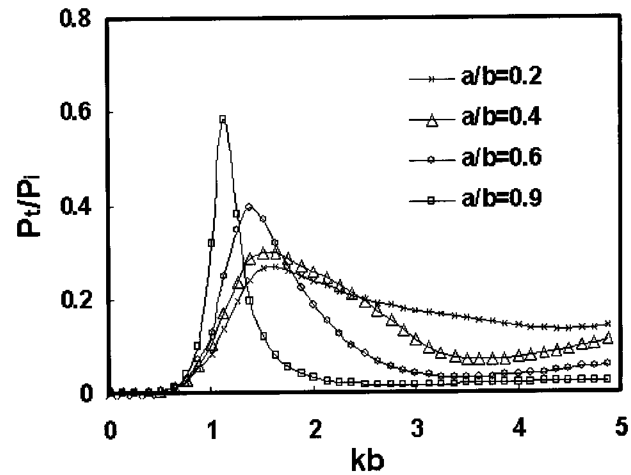

Fig. 4. Transmit power variation with the ratio of the inner to outer radius(a/b) changed. As the annular gap($b-a$) shrinks, transmission response has narrower peaks. The responses are nearly the same with a/b<0.2(Normal incidence, $P_i=\pi b^2 E_i H_i$).

In Fig. 3, magnitudes of transmitted electric fields are plotted as the outer radius increases. As is well-known from the small hole theory, the aperture behaves like equivalent electric or magnetic dipoles whose magnitudes are proportional to the cube of aperture radius and polarizabilities. In the figure, the incident field normally impinges on the conductor surface and a magnetic dipole suffices to calculate transmitted fields. When the radius is smaller than 0.1, our results match those of the small hole theory. These two examples validate our formulations.

Fig. 4 illustrates transmitted power variations with annulus witdh a/b changed. As shown in the figure, transmission occurs in narrow frequency bandwidth when the annular opening is small(a/b >1) and has sharp peak in frequency response. When the radii of the inner core are small compared with outer radius, variations of power transmission are rather small and has no frequency selective property. Transmitted power can be evaluated by Parseval's theorem as follows.

$$P_t = \mathrm{Re}\left\{\int_0^{2\pi}\int_0^{\infty} E_{1t} \times H_{1t}^{*} \cdot \hat{z}\, r dr d\phi\right\}$$
$$= \omega n_1 \sum_{v=-\infty}^{\infty} \mathrm{Re}\left\{\int_0^{\infty}\left[\kappa\left|A_m(v,\zeta)\right|^2 + \kappa^{*}\left|A_e(v,\zeta)\right|^2\right]\zeta d\zeta\right\} \tag{34}$$

Quantities A_m and A_e are represented by a series of modal coefficients in region Ⅱ. The number of terms needed in the calculation is five for each n to the accuracy of 0.1 %.

Figs. 5, 6 and 7 illustrate electric field strength maps

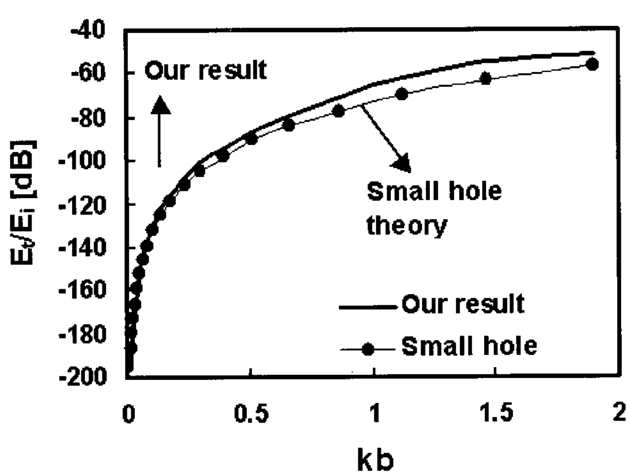

Fig. 3. Transmit field magnitude as obtained by our formulation and those of small hole theory. Note that the prediction of small hole theory matches our result well with small outer radius, but deviations become significant with the larger radius.

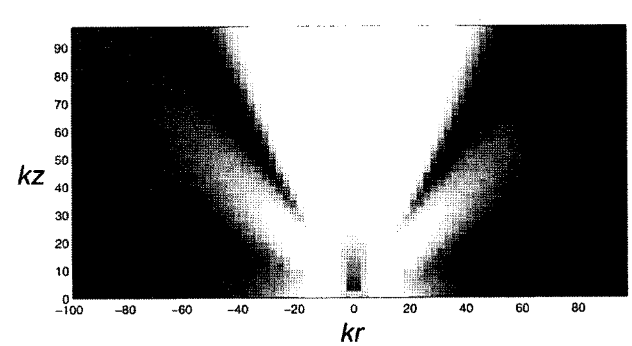

Fig. 5. Electric field strength map in region I, where horizontal axis runs from r=0 to r=100 b(a/b =0.1, kb=7.3).

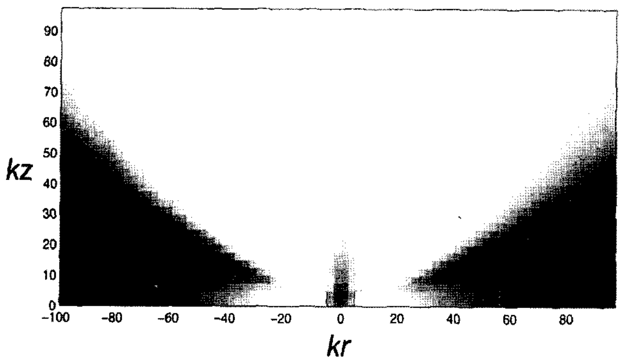

Fig. 6. Electric field strength map in region I(a/b=0.5, kb=7.3).

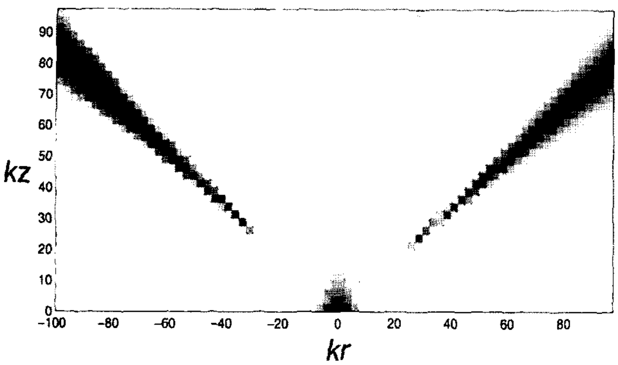

Fig. 7. Electric field strength map in region I(a/b=0.9, kb=7.3).

in region I. Note the variations of transmitted field with a/b changed. When kb is large, the scattered field shapes depend on a/b ratios. When $kb \leq 2$, however, the radiation pattern shapes are nearly the same independent of a/b.

Ⅳ. Conclusion

A closed form solution of scattering in an annular aperture is obtained. The problem geometry is divided into three regions and field in each region is represented by continuous or discrete modal functions. Using the condition of field continuity, fields with continuous spectrum can be expressed by sum of discrete modal functions, which facilitate numerical calculations. The obtained solution is fast converging and numerically efficient.

The transmission of electromagnetic wave through an annular aperture seems to insensitive to ratio of inner to outer radius, a/b, when $kb \leq 2$. It is noted that transmission of waves is frequency selective with annular gap $b-a$ shrinks. The smaller the gap, the narrower the bandwidth is obtained.

Appendix

The integrands in section Ⅱ are obtained by integrating products of modal functions in region Ⅱ and region I(or Ⅲ). They are tabulated below. Integrations can be performed using residue calculus or numerical integrations.

$$< e_{tem} \,|\, h_m(\nu,\zeta) >= \omega n_1 [J_0(\zeta a) - J_0(\zeta b)]\delta_{\nu,0} \tag{A1}$$

$$< e_m(\nu,i) \,|\, h_m(\nu,\zeta) > = \frac{j}{2\pi}\omega n_1 \kappa_{\nu,i} \frac{\zeta^2}{\zeta^2 - (\chi_{\nu,i}/b)^2} \cdot \left[\frac{J_\nu(\zeta a)}{N_\nu(\chi_{\nu,i} a/b)} - \frac{J_\nu(\zeta b)}{N_\nu(\chi_{\nu,i})} \right] \tag{A2}$$

$$< e_e(\nu,i) \,|\, h_m(\nu,\zeta) > = \frac{j}{2\pi}\omega^2 n_1 n_2 \frac{\nu}{\chi'_{\nu,i}} \cdot \left[\frac{J_\nu(\zeta b)}{N'_\nu(\chi'_{\nu,i})} - \frac{b J_\nu(\zeta a)}{a N'_\nu(\chi'_{\nu,i} a/b)} \right] \tag{A3}$$

$$< e_{tem} \,|\, h_e(\nu,\zeta) >= 0 \tag{A4}$$

$$< e_m \,|\, h_e(\nu,\zeta) >= 0 \tag{A5}$$

$$< e_e(\nu,i) \,|\, h_e(\nu,\zeta) > = \frac{j}{2\pi}\omega n_2 \frac{\chi'_{\nu,i}}{b} \frac{\kappa^* \zeta}{\zeta^2 - (\chi_{\nu,i}/b)^2} \cdot \left[\frac{J'_\nu(\zeta a)}{N'_\nu(\chi'_{\nu,i} a/b)} - \frac{J'_\nu(\zeta b)}{N'_\nu(\chi'_{\nu,i})} \right] \tag{A6}$$

$$< h_{tem} \,|\, e_{tem} >= j \ln(b/a) \tag{A7}$$

$$< h_m(\nu,i) \,|\, e_m(\nu,i) > = j\frac{2}{\pi^2}\omega n_2 \kappa^*_{\nu,i} \left[\frac{1}{N_\nu^2(\chi_{\nu,i})} - \frac{1}{N_\nu^2(\chi_{\nu,i} a/b)} \right] \tag{A8}$$

$$< h_e(\nu,i) \,|\, e_e(\nu,i) > = j\frac{2}{\pi^2}\omega n_2 \kappa'_{\nu,i} \left[\frac{1-\nu^2/{\chi'_{\nu,i}}^2}{N_\nu^2(\chi'_{\nu,i})} - \frac{1-\nu^2 b^2/(\chi'_{\nu,i} a)^2}{N_\nu^2(\chi'_{\nu,i} a/b)} \right] \tag{A9}$$

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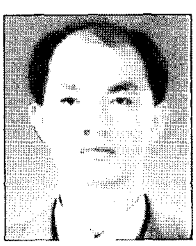

was born in Seoul, Korea, in 1971. He received B.Sc. degree in electronics engineering from Seoul National University in 1995, MS.c. and Ph.D. degrees in electrical engineering from Korea Advanced Institute of Science and Technology in 1997 and 2000. He worked for LG Electronics, digital media laboratory from 2000~2004. Then, he joined the department of electronics engineering at Sogang University, where he is currently assistant Professor. His main research interests include electromagnetic scattering, electromagnetic wave theory, and wave propagations.