Differential Code-Filtering Correlation Method for Adaptive Beamforming

Mostafa Hefnawi and Tayeb A. Denidni

Abstract: **An adaptive beamforming system based on code filtering and differential correlation approaches is proposed. The differential correlation method was originally proposed for time delay estimation of direct sequence code division multiple access (DS-CDMA) systems under near-far ratio conditions and the code filtering correlation algorithm, on the other hand, was proposed for array response estimation in DS-CDMA systems under perfect power control. In this paper, by combining differential correlation concept with the code filtering beamforming technology, an accurate estimate of the beamforming weights and an enhanced performance of DS-CDMA systems under sever near-far ratio conditions is achieved. The system performance in terms of beampattern and bit-error-rate (BER) shows that the proposed adaptive beamformer outperforms the conventional code filtering correlation technique.**

Index Terms: **Adaptive beamforming, array response estimation, code-filtering, differential correlation, DS-CDMA.**

I. INTRODUCTION

Adaptive beamforming is one of the most promising methods for improving the capacity of direct sequence code division multiple access (DS-CDMA) wireless systems by effectively reducing multipath and co-channel interference [1]–[7]. This is achieved by using an antenna array with digital signal processing to form a beampattern that adapts to the current channel and user characteristics. A summary of the application of antenna arrays to wireless communications, with focus on CDMA beamforming and directions of arrival estimation, is well documented in [8]. Traditional beamforming techniques generally require either knowledge of the array response vectors or the use of reference signals. Methods for array response estimation have been addressed in [9], [10] where a code-filtering correlation algorithm (CFCA) uses the baseband received signal and the post-correlation signal vector to estimate the array response vector for each path of DS-CDMA signals. It was shown that the CFCA works well when the system is under perfect power control, but its performance degrades rapidly as the near-far ratio increases. On the other hand, the algorithm proposed in [11] was found to be near-far resistant and was tolerating high system loading. A major limitation, however, is that this method cannot be extended to fading channels. This is mainly because of the need of long training sequence, during which the channel may have a mean too close to zero. In [12]–[14], the differential correlation method was proposed for time delay estimation in single antenna CDMA systems under multipath fading and near-far conditions and has shown improved performance compared to the non-differential correlation method [11], [15]. More accurate time delay estimation was achieved by using multiple antennas at the receiver in [16]. In this paper, we develop a new adaptive algorithm for CDMA smart antenna systems, which is based on a modified and significantly enhanced version of the conventional CFCA. When using CFCA, an update of the optimal beamforming weights requires an estimate of the interference-noise correlation matrix besides the array response [9], which is estimated as the principal eigenvector of the matrix pencil of the pre-despreading and post-despreading correlation matrices. In the proposed approach, by replacing correlation matrices with differential correlation ones, an estimate of the interference-noise correlation matrix is not needed and one can use the sub-optimum maximum signal-to-noise ratio (SNR) beamformer to get an accurate estimate of the beamforming weights and consequently an enhanced performance of the CDMA system in a multipath fading environment under severe near-far ratio conditions.

The remainder of the paper is organized as follows. Adaptive beamforming using code-filtering correlation approach is described in Section II. Section III introduces the differential code-filtering correlation method for array response estimation. The simulation results are presented in Section IV. Finally, the concluding remarks are given in Section V.

II. ADAPTIVE BEAMFORMING USING CODE-FILTERING CORRELATION METHOD

Fig. 1 shows the block diagram of the adaptive beamforming system based on the code-filtering correlation method. We consider an asynchronous CDMA up-link channel in which signals are transmitted from K mobile users to the base station equipped with a uniform linear array of P antenna elements with half-wavelength spacing. The array response vector can be written as

$$a_k(\theta) = [1 \quad e^{(-i\pi \sin\theta_k)} \quad \cdots \quad e^{(-i\pi(P-1)\sin\theta_k)}]^T \tag{1}$$

where θ_k is the direction of arrival (DOA) from the k-th user and the superscript T denotes the transpose. The received baseband signal at the p-th receiving antenna is given by

$$r^P(t) = \sum_{m=1}^{M}\sum_{k=1}^{K} b_k(m) a_k^P \sqrt{P_k}(t - mT_s - \tau_k T_c) + n^P(t) \tag{2}$$

where $b_k(m)$ is the data symbol at the m-th symbol interval of the k-th user, τ_k and P_k are the time delay and the power of

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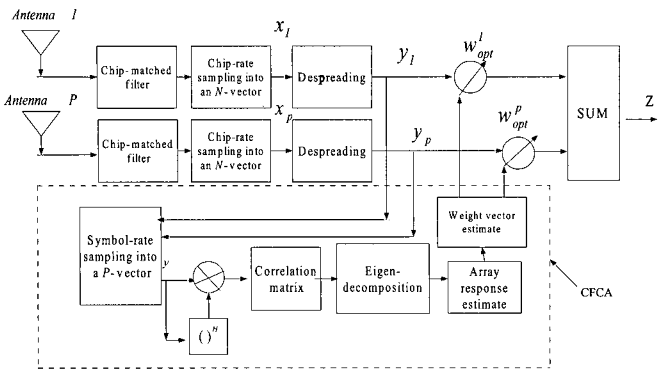

Fig. 1. Block diagram of adaptive beamforming using code-filtering correlation method.

the k-th user, respectively, T_s and T_c are the symbol and chip intervals, a_k^P is the p-th entity of the k-th user's array response vector $a_k(\theta)$, and $s_k(t)$ is the k-th user's spreading waveform given by

$$s_k(t) = \sum_{n=1}^{N} c_k(n)\Pi_{T_c}(t - nT_c) \tag{3}$$

where $c_k(n) \in \{1, -1\}$ is the k-th user's spreading code, N is the processing gain, and Π_{T_c} is a time-limited rectangular shape pulse with unit amplitude and duration T_c . The receiver front-end is a chip-matched filter following each receiver antenna. After chip-matched filtering and chip-rate sampling, the signal at the p-th element is collected into an N-vector as

$$x_p(m) = \sum_{k=1}^{K} a_k^P \sqrt{P_k} b_k(m) c_k(\tau_k) + n^P(m) \tag{4}$$

where $c_k(\tau_k)$ is the k-th user's signature vector, which is a function of its time delay τ_k and spreading code.

We form a PXN pre-despreading matrix $X(m)$ as follows

$$X(m) = [\ x_1(m) \quad x_2(m) \quad \cdots \quad x_P(m)]^T . \tag{5}$$

This pre-despreading matrix is despread by the desired user's signature waveform to form the post-despreading matrix $Y(m)$ as follows

$$Y(m) = X(m)c_1(\tau_1). \tag{6}$$

The pre-despreading correlation matrix, R_{XX}, and the post-despreading correlation matrix, R_{YY}, are defined by

$$R_{XX} = E\lfloor X(m)X^H(m)\rfloor \tag{7}$$

$$R_{YY} = E\lfloor Y(m)Y^H(m)\rfloor \tag{8}$$

where E denotes the expectation operator and the superscript H denotes the conjugate transpose.

Using the assumption that the data symbols $b_k(m)$ and the DS spreading waveform from different users are uncorrelated, the asymptotic values of R_{XX} and R_{YY}, for an infinite observation length, can be expressed as [10]

$$R_{XX} = P_1 a_1 a_1^H + \sum_{k\neq 1} P_k a_k a_k^H + \delta^2 I \tag{9}$$

$$R_{YY} = NP_1 a_1 a_1^H + \sum_{k\neq 1} P_k a_k a_k^H + \delta^2 I \tag{10}$$

where P_k represents the transmitted signal power from the k-th user, δ^2 is the noise power density, I is an identity matrix.

The received despread signals from each antenna element are scaled by the optimal complex weight $w_{opt} = [w_{opt}^1 \quad w_{opt}^2 \quad \cdots \quad w_{opt}^P]$ and summed to form the array output.

If the mobile users are transmitting signals with equal powers and if the CDMA processing gain is sufficiently large, the multiple-access interference (MAI) and noise are both temporally and spatially white, and it is only necessary to determine the maximum signal-to-noise ratio (SNR) beamforming weights as

$$w_{opt} = \hat{a}_1 \tag{11}$$

where $\hat{a}_1$ is the desired user's array response estimate that can be found as the principal eigenvector of R_{YY}. But if the near-far ratio is large, i.e., the differences between P_k's are large, or the processing gain is not large enough, this simple approximation is no longer appropriate. In such a case, a_1 should be estimated as the principal eigenvector of the matrix pencil of $R_{YY} - R_{XX}$ [9], [10] and the beamforming weights need to maximize the SINR, which is calculated using both the array response estimate and the interference-noise correlation matrix estimate $\hat{R}_{IN}$ [17]

$$w_{opt} = \hat{R}_{IN}^{-1}\hat{a}_1. \tag{12}$$

However, when a finite sample size is used to estimate R_{XX} and R_{YY}, the noise and interference will cause significant errors in their estimates. Therefore, it is of interest to investigate new approaches that can perform well under these conditions. In the following section, we introduce a new technique more suitable for CDMA with near-far problem, which is based on the differential correlation approach.

III. DIFFERENTIAL CODE-FILTERING CORRELATION APPROACH

The block diagram of the differential code-filtering correlation approach (DCFCA) for array response estimation is depicted in Fig. 2. The goal is to compute the differential correlation matrix of the post-despreading signal, which is defined as

$$R_{YY} = E(Y(m)Y^H(m + d)) \tag{13}$$

where d is an integer, corresponding to the time difference of the two received vectors used in the differential correlation. Due to the asynchronous transmission and the correlation property of CDMA spreading codes, the differential correlation coefficient d cannot be 1, it must satisfy $d \geq 2$ [12]. In Fig. 2, the DCFCA is shown for $d = 2$. We note that the non-differential correlation matrix R_{YY} can be viewed as a special case of the differential correlation with $d = 0$, i.e., $R_{YY}(0)$. It is assumed that an all-ones training sequence is available for the desired user for array response estimation, while the other users transmit randomly generated information symbols, which are uncorrelated with each other and with the desired user. Since the same user's

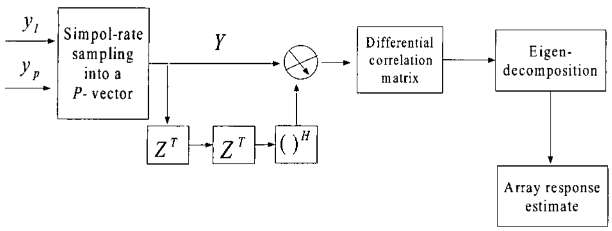

Fig. 2. Block diagram of the differential code-filtering correlation method for array response estimation.

data symbols at different symbol interval, $b_k(m)$ and $b_k(m+d)$, are uncorrelated, then the asymptotic value of $R_{YY}(d)$ can be expressed as

$$\begin{aligned} R_{YY}(d) &= NP_1 a_1 a_1^H + \\ &\quad \sum_{k\neq 1} P_k a_k a_k^H E\{b_k(m) b_k(m+d)\} + \delta^2 I \\ &= NP_1 a_1 a_1^H + \delta^2 I. \end{aligned} \tag{14}$$

The differential correlation matrix has only one dominant term, regardless of the near-far ratio and the processing gain of the CDMA system. We perform eigen-decomposition on the differential correlation matrix $R_{YY}(d)$ and the array response estimate, $\hat{a}_1$, is found as the principal eigenvector of $R_{YY}(d)$. So, when using differential correlation method, an estimate of the interference-noise correlation matrix is not needed and one can use the sub-optimum maximum SNR beamformer to get an accurate estimate of the beamforming weight.

The DCFCA has two advantages over the CFCA. First, when a perfect power control is applied and there is no near-far problem, the DCFCA may result in a shorter training sequence. Second, when there is a near-far problem, the DCFCA may still work while the CFCA may break down.

IV. NUMERICAL RESULTS

In the following simulation experiments, Figs. 3–6, we first evaluate how the proposed scheme behaves in different near-far ratio (NFR) conditions with the additive white Gaussian noise (AWGN) channel and compare its performance with the CFCA. The performance is evaluated in terms of the maximum SNR beamforming patterns with different training sequence lengths and different number of antennas. For all the simulations the number of users is $K = 9$, the processsing gain is $N = 64$, the desired signal's DOA is fixed at $20°$, and the interferers' DOAs are $78°, 74°, 71°, -43°, 40°, -71°, 12°$, and $-44°$. The desired user's SNR equals the nominal SNR, which is fixed at 10 dB, and the SNRs of the interfering users were set at 10 dB when the system is under perfect power control and 10 dB higher than the nominal SNR to create the near-far effect.

Fig. 3 compares the maximum SNR beamforming patterns of the DCFCA and the CFCA for an NFR $= 0$ dB, which corresponds to a CDMA system under perfect power control. The training sequence length is $M = 50$ and the number of receive antennas is $P = 3$. It is noted that the patterns obtained by both methods have their main lobes oriented toward the desired

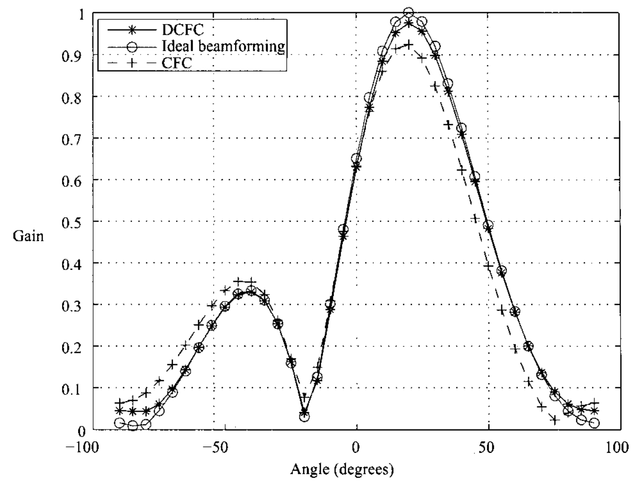

Fig. 3. Maximum SNR beamforming pattern for NFR $= 0$ dB with a 3-element antenna array ($P = 3$).

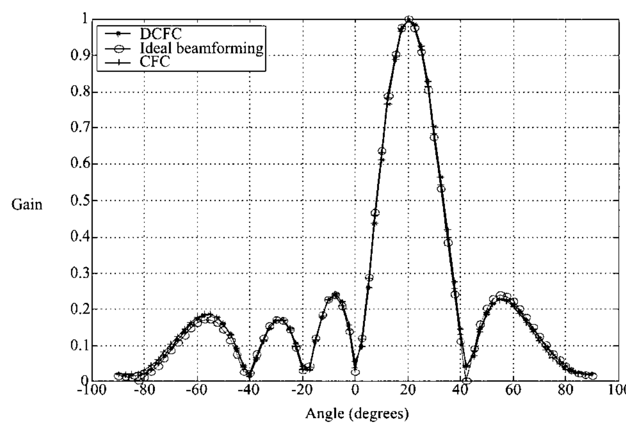

Fig. 4. Maximum SNR beamforming pattern for NFR $= 0$ dB with a 6-element antenna array ($P = 6$).

user's DOA, with a little performance improvement from CFCA to DCFCA. Fig. 4, on the other hand, shows the results when the number of antenna elements is changed to $P = 6$. It is seen that both methods almost have the same beampattern performance that approaches the theoretical array pattern. In Fig. 5 the NFR is fixed at 10 dB, the number of antennas at $P = 3$, and due to the severe MAI, the training sequence length is increased to $M = 500$. It is seen that the pattern difference between the DCFCA and the CFCA is more pronounced. Even with such a large observation length, the CFCA fails since it is not designed for such severe near-far ratio conditions. On the other hand, it is shown that the DCFCA still results in a beampattern, which is very close to ideal one. Also, when increasing the number of antenna elements to $P = 6$, Fig. 6 shows that the DCFCA has an array pattern performance identical to the ideal one and slightly better than that with $P = 3$. These results show that, under near-far conditions, the DCFCA has a much better beampattern performance without the need of an estimate of the interference-noise correlation matrix.

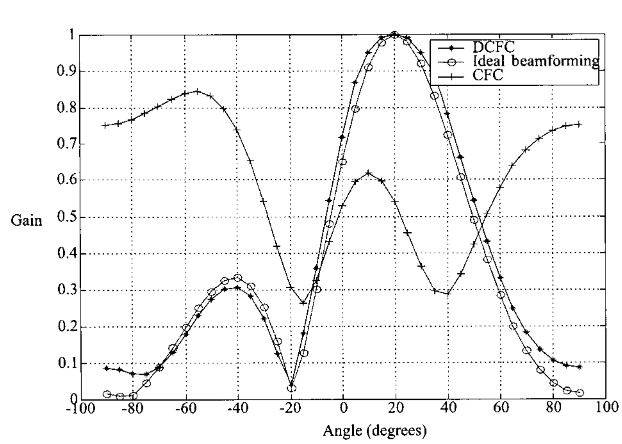

Fig. 5. Maximum SNR beamforming pattern for NFR = 10 dB with a 3-element antenna array ($P = 3$).

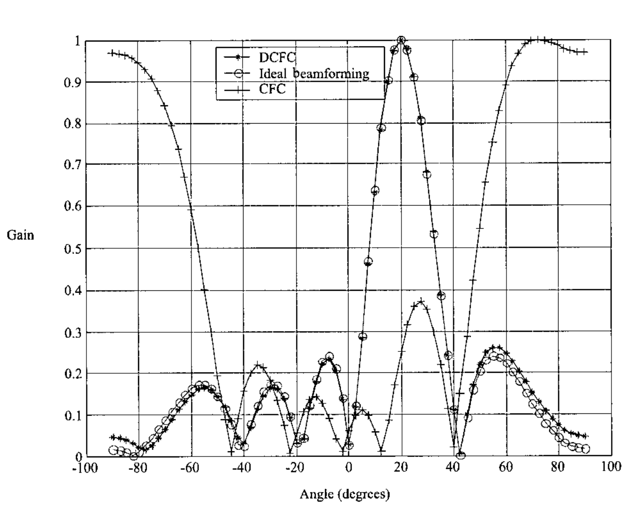

Fig. 6. Maximum SNR beamforming pattern for NFR = 10 dB with a 6-element antenna array ($P = 6$).

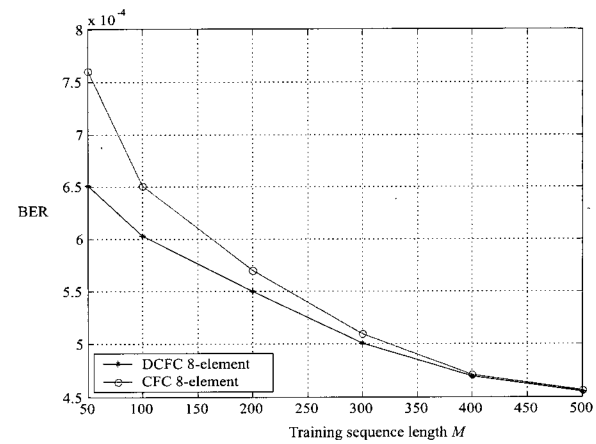

Fig. 7. BER performance as a function of the training sequence length M, NFR = 0 dB.

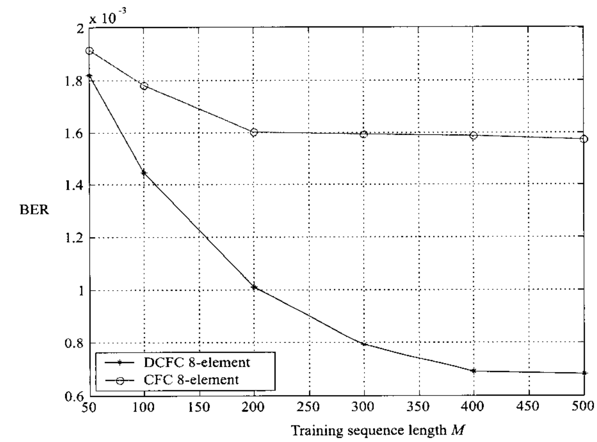

Fig. 8. BER performance as a function of the training sequence length M, NFR = 10 dB.

In order to show that our method can still work in multipath fading channels, the following results, based on a Monte Carlo simulation, compare the BER performance as a function of the training sequence length of the DCFCA and the CFCA methods in a slow flat fading channel. A slow flat fading channel was chosen because differential detection is an effective technique when the channel phase is approximately constant within at least two symbol intervals [12]. Therefore, in a time-varying fading channel the usefulness of differential detection is limited to low values of the Doppler spread. For these simulation experiments, the number of users is fixed at $K = 9$ and each user is assigned two multipaths with Raleigh distributions. For simplicity we have assumed that the two paths are coming from the same DOA with the relative time delays of 0 and $0.25T_c$ and the corresponding normalized average power of 0 dB and -3 dB. The normalized fading rate is set to 0.01. The number of antenna elements is $P = 8$, and the near-far ratio is 0 dB and 10 dB.

Fig. 7 shows the BER performance as a function of the training sequence length M for an NFR $= 0$ dB. It is noted that the performance gape between the DCFCA and the CFCA is reduced as the length of the training sequence is increased. It is also noted that with a length of $M = 500$ both methods provide almost identical performance.

On the other hand, the BER performance as a function of the training sequence length M for an NFR $= 10$ dB is shown in Fig. 8. It is noted that the performance gap between the two methods is more accentuated when the NFR $= 10$ dB. From this figure, it is also seen that a minimum BER of 0.00068152 is achieved with a training length of $M = 500$.

V. CONCLUSION

We have investigated the use of the differential correlation method for array response estimation and adaptive beamforming. Compared to the conventional code-filtering method, the differential code-filtering correlation method yields better array response estimate under perfect power control and under severe near-far conditions. Although the technique presented in this paper was applied to the code-filtering approach, it can be combined with many other pilot symbol assisted beamforming techniques to improve their performance. For example, with the sample matrix inversion algorithm (SMI) [18], [19], the covariance matrix of the received signal can be replaced with the differential correlation one to obtain an accurate update of the beamforming weights. The least mean square algorithm (LMS) [20] can also take advantage of this technique to update its optimal weight vector. In this case, since the estimate of the differential correlation matrix is not needed, the algorithm can update the weight vector by using the signals at the output of the differential correlation based matched filters.

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