Accounting for Extreme Values in GARCH Forecasts of Day-Ahead Electricity Prices

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Abstract - We employ a new technique to account for extreme values when using the generalized autoregressive conditionally heteroskedastic (GARCH) methodology to forecast day-ahead electricity prices in New York City.

Keywords: Electricity markets, forecasting, market clearing prices, GARCH, extreme values, time series analysis

1. Introduction

Electricity price time series violate the classical assumption of constant variance (homoskedasticity). They exhibit periods of large volatility and extreme values followed by periods of relative tranquility. These characteristics are due to periods of high demand, transmission congestion, and random outages of system components. [1]-[2]

As in [1] and [2], we employ the generalized autoregressive conditionally heteroskedastic (GARCH) methodology to forecast electricity prices in New York City (NYC) combined with a new technique to account for extreme observations. Readers not familiar with GARCH are referred to [3].

When extreme observations occur, including them in the time series methodology adversely affects the ability to forecast non-extreme values. We propose a technique that replaces extreme values with proxy values to improve the forecast of non-extreme values.

2. TIME SERIES ANALYSIS

2.1 The New York State Electricity Markets

New York State has a day-ahead market and a real-time market for energy based on locational marginal pricing. New York City comprises approximately 30% of the energy usage and 33% of the peak energy demand [4].

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We constructed time series of day-ahead, wholesale electricity prices in New York City at 2 pm along with six input fuel prices (three oil and three natural gas price streams at different locations). Oil or natural gas is likely to be the marginal fuel at 2 pm. Fuel costs are the major component of a fossil-fuel unit's variable costs. None of the oil price streams are statistically significant and only one natural gas price stream is statistically significant. It is denoted TRNY to indicate the Transcontinental Gas Pipe Line Corporation daily prices reported by DRI/ McGraw Hill.

The electricity price time series started on December 28, 2000 through March 31, 2003. Due to data limitations, the input fuel price data series started on December 28, 2000 but ended at the start of December 2002 and were only available for weekdays that are not holidays.

2.2 GARCH Forecasting Technique

Table I shows that our data set is stationary as indicated by Dickey-Fuller and Phillips-Perron tests at the 5% significance level.

Table 1 Dickey-Fuller and Phillips-Perron for Unit Root Test. (Dec. 28, 2000 to Nov. 25. 2002)

	NYC	TRNY
Dickey-Fuller	-7.22995	-3.91138
Phillips-Perron	-12.93464	-10.51210

We then investigate whether electricity price volatility can be modeled to capture the volatility variations in the electricity market. We run Lagrange multiplier test for ARCH and GARCH disturbances [5]. The purpose of this test is to determine whether ARCH or GARCH are appropriate by evaluating the correlation of the square of the residuals (variance) by regressing the square of the residuals on a constant and on one lag value.

Table 2 Lagrange Multiplier Test for ARCH of GARCH Errors(December-28-00 to November-25-02)

	NYC
Lagrange Multiplier	3.86
Significance Level	.049

As indicated by Table II, the null hypothesis that the squared disturbances are uncorrelated is rejected in favor of the alternative hypothesis of GARCH errors for the electricity prices in NYC at the 5% significance level.

We used a preferred model specified by GARCH(1,1) with one lag of natural gas prices (TRNY) whose coefficient is significant for values between 0.1 and 0.35. Our jointly estimated specification is as follows:

$$P_t = \alpha_1 + \alpha_2 P_{t-1} + \alpha_3 TRNY_{t-1} + \varepsilon_t \tag{1}$$

$$h_t = \beta_1 + \beta_2 h_{t-1} + \beta_3 \varepsilon_{t-1}^2 \tag{2}$$

Then, we utilize the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) algorithm to maximize the log likelihood function with respect to α_1, α_2, α_3, β_1, β_2, β_3, and β_4. [6]

We initially estimate the electricity prices over the first one hundred days extending from December-28-00 to May-21- 2001. Then, the one-day out-of-sample forecasting performance of our estimation technique is evaluated. Next, we add one day at a time to the ending date, and repeat the process of estimating and forecasting the electricity prices over the next 377 weekdays extending from May-23-01 to November-22-02. These rolling window estimations are more sensitive to including observations from the data set, which helps in locating any extreme observations that might mask the causality between the electricity prices and their determinants.

To detect the extreme observations that might be affecting the forecastability of the electricity prices, we propose an algorithm that locates these observations based on their effect on the kurtosis of the GARCH residuals. In line with [7] and [8], we propose the following procedure:

Step I: Conduct the forward rolling GARCH on the raw x_t process as discussed in the previous section. At each step calculate the Kurtosis statistics ($KURT_t$) and its absolute change [$abs\ (KURT_t - KURT_{t-1})$].

Step II: Construct a vector (δ) of arbitrary values to detect the sudden changes in the kurtosis. Typically, the upper boundary for the delta is the maximum value of [abs($KURT_t$-$KURT_{t-1}$)] calculated from step I. The rest of the deltas are usually determined by the following equation:

$$\delta_i = \text{Max}\ [\text{abs}(KURT_t - KURT_{t-1}] - .1(i) \quad \text{for } i = 1, 2, \ldots, n \tag{3}$$

For example, when Max [abs($KURT_t$-$KURT_{t-1}$] = 1.0, δ_5 = 1.0 - .1(5) =5.

Step III: If [abs ($KURT_t - KURT_{t-1}$)] > δ_i, replace x_t by its adjusted value (one-step forecast of x_t) or the lagged value of x_t) and repeat step I using the adjusted value at time t.

Step IV: Repeat the above procedure for different values of δ. Finally, choose δ which minimizes the excess kurtosis and skewness.

Step V: To account for the masking effect pointed out by [8], we conduct the backward rolling GARCH specified as follows: first we utilize adjusted series obtained through the first four steps to estimate the GARCH model over the sample period extending from observations 6/1/2002 to 11/22/2002. We then extend the sample by adding one month at a time to the starting dates. We repeat the process until we reach the beginning of the sample, where the starting and the ending date are 5/24/2001 and 11/22/2002 respectively. Ideally, the procedure should terminate when no further reduction of times we reject the three normality tests can be obtained.

Step VI: Finally, we account for the extreme observations by replacing them by the GARCH out-of-sample forecast.

2. Numerical Results and Conclusions

Our algorithm locates four extreme observations occurred on August 7-10, 2001. Once these observations are adjusted, the non-normality of the excess returns decreases dramatically. Although, such observations are of important value, their nature and the probability of their occurrence seem to be unique and non-repetitive. Therefore, including these observations with such unusual high values may produce bias in parameter estimates and hence may deteriorate the efficiency of our forecasts.

Table III reports the performance of our specification both including and replacing extreme values (EV). We use four standard measures of the performance of the four forecasting techniques: mean forecasting error (MFE), mean absolute forecasting error (MAFE), root mean squared forecasting error (RMSFE), and the Theil U statistics. As Table III indicates, accounting for the few extreme values in a GARCH process improves the price forecast

Table 3 Performance of Out-of-Sample Forecasting of GARCH Specification

(May 23, 2001 to Nov. 22, 2002)

	MFE	MAFE	RMSFE	U Theil
GARCH	2.2477	12.6833	53.3903	0.3355
Replace EV	0.8401	8.4446	14.9385	0.1170

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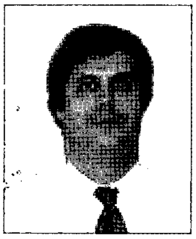

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