Journal of the Korean Statistical Society (2005), **34**: *3, pp 219–233*

l-STEP GENERALIZED COMPOSITE ESTIMATOR UNDER 3-WAY BALANCED ROTATION DESIGN†

K. W. KIM[1], Y. S. PARK[2] AND N. Y. KIM[2]

ABSTRACT

The 3-way balanced multi-level rotation design has been discussed (Park Kim and Kim, 2003), where the 3-way balancing is done on interview time, in monthly sample and rotation group and recall time. A greater advantage of 3-way balanced design is accomplished by an estimator. To obtain the advantage, we generalized previous generalized composite estimator (GCE). We call this as l-step GCE. The variance of the l-step GCE's of various characteristics of interest are presented. Also, we provide the coefficients which minimize the variance of the l-step GCE. Minimizing a weighted sum of variances of all concerned estimators of interest, we drive one set of the compromise coefficient of l-step GCE's to preserve additivity of estimates.

AMS 2000 subject classifications. Primary 62G08; Secondary 62G20.
Keywords. Rotation sampling design, Multi-level rotation design, Generalized composite estimator, Variance, Optimal coefficients, Efficiency.

1. INTRODUCTION

A rotation sampling design has been used to estimate the monthly level or changes efficiently and to reduce the respondents burden. Its efficiency arises from correlations between measurements observed at several survey months. Rotation sampling designs are classified into one-level and multi-level rotation sampling designs according to their reported number of months of information at each survey month. The term "level" of a design indicates the number of months for which information is solicited in one interview. In an one-level rotation design,

Received March 2004; accepted July 2005.
†Supported by a Korea University Grant.
[1]Department of Informational Statistics, Korea University, 208, Seochang-Ri, Jochiwon-Eup, Yeonki-Gun, Chung-Nam, 339-700, Korea (e-mail : korpen@korea.ac.kr)
[2]Department of Statistics, Korea University, 1, 5-Ka, Anam-dong Sungbuk-ku, Seoul, 136-701 Korea (e-mail : yspark@korea.ac.kr)

the respondents report only the information of the interview month while the information of the interview month as well as a certain number of previous months in a multi-level rotation design. It is obvious that the multi-level rotation sampling design may take less cost and response burden than the one-level rotation design. According to this classification, the the U.S. Current Population Survey (CPS), the Canadian Labor Force Survey (CLFS) and Australian Labor Force Survey (ALFS) belong to an one level rotation sampling design and the U.S. Monthly Retail Trade Survey (MRTS) and the Survey of Income and Program Participation (SIPP) belong to a multi-level rotation design.

The most recent work about one-level rotation sampling design was done by Park, Kim and Choi (2001). They introduced the 2-way balanced semi one-level rotation sampling schemes, called as $r_1^m - r_2^{m-1}$. Their design is balanced on interview time horizontally and vertically in any survey month, and contains most existing one-level designs such as the CPS, CLFS and ALFS as special cases.

Cantwell and Caldwell (1998) dealt with the two-level MRTS and compared it with a fixed-panel design. They examined the issues of sampling variability, revisions, panel imbalance, response bias, cost, respondent burden and data quality. They demonstrated biases from unbalanced rotation groups and recall times, and showed that those biases overwhelm the mean squared error.

There are two ways to overcome the bias problems. The first method is to develop new design and the second is to contrive new estimation method to minimize effects of biases. The new design that minimize or reduce the effects of biases is introduced already by Park Kim and Kim (2003) called as 3-way balanced multi-level design call as $r_1^m(l) - r_2^{m-1}$ design. This design is accomplished not only on interview time in monthly sample and rotation group but also on recall time as well. The second method is our main point in this paper. The remainder of this paper is divided into 4 sections. In Section 2, we briefly introduce the $r_1^m(l) - r_2^{m-1}$ design. Newly extended generalized composite estimator (l-step GCE) is suggested in Section 3. In Section 4, We also derive the variance of the l-step GCE. To preserve the consistency in total, we derive one set of compromised coefficients minimizing a weighted sum of variances of l-step GCE's for all characteristics of interest. In Section 5, we compare the efficiency of l-step GCE relative to simple estimator.

2. 3-WAY BALANCED $r_1^m(l) - r_2^{m-1}$ DESIGN

The multi-level rotation system can be expressed as l-level rotation system. In l-level rotation design, each sample unit reports the information for the current month as well as for the $l-1$ previous months. When the sample unit returns to the sample for every lth month, it again provides l months information. One-level $r_1^m - r_2^{m-1}$ rotation system is generalized to "multi-level" rotation system : once a sample unit is selected from each rotation group, the sample unit returns to the sample for every l-th month until its r_1-th interview. Then no information is obtained from the sample unit for the next r_2 months; this procedure is repeated until this sample unit returns to the sample for its final mr_1-th interview. This rotation system is called as $r_1^m(l) - r_2^{m-1}$ for the l-level design. Under the some conditions, $r_1^m(l) - r_2^{m-1}$ design is balanced in 3-ways by interview time, by rotation group and by recall time. Rotation designs do not satisfy 3-way balancing even if it can be expressed as $r_1^m(l) - r_2^{m-1}$. The necessary and sufficient condition to satisfy 3-way balancing in $r_1^m(l) - r_2^{m-1}$ is as following (see Park Kim and Kim, 2003).

THEOREM 2.1. *Suppose that the $r_1^m(l) - r_2^{m-1}$ design is balanced in 3 ways. For each given i, $i = 0, 1, \cdots, r_1 - 1$, there is an unique integer m_i^*, $1 \le m_i^* \le mr_1$ satisfying*

$$mod_{mr_1}\left\{m_i^* + i + (m_i^* - 1)(l-1) + \left[\frac{m_i^* - 1}{r_1}\right]r_2\right\} = 0 \tag{2.1}$$

where $[\cdot]$ is the integer operator.

This 3-ways balancing ensures that all rotation groups are included in the monthly sample at any survey month and the rotation pattern of a sample unit depends only on its interview time regardless of its rotation group and recall time.

The U.S. CEX uses $5^1(3) - 0^0$ design which satisfies all properties of 3-way balancing. We illustrate this design in Figure 2.1. In the $5^1(3) - 0^0$ design, a sample unit is in the sample every third month for a total 5 times. The notation (α, g) in Figure 2.1 are the symbols for the αth unit in the gth group and u_i is for the αth unit interviewed in the i-th time.

In spite of the two different times of the observation, u_4 in month t and u_5 in month $t+3$ are the same sample unit since both are indexed by $(1, 4)$; but u_4 in month t and u_4 in month $t+1$ are different units, $(1, 4)$ and $(1, 5)$ although they are interviewed for the same time in two different months. The symbols "ı" and "ıı" above the sample unit u_i denote the same sample unit which provides

FIGURE 2.1 *The Three-way balanced* $5^1(3) - 0^0$ *design*

α	1					2					3					4					5					6		
g	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3
t	u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II													
t+1		u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II												
M t+2			u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II											
O t+3				u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II										
N t+4					u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II									
T t+5						u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II								
H t+6							u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II							
t+7								u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II						
t+8									u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II					
t+9										u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II				
t+10											u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II			
t+11												u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II		
t+12													u_5	I	II	u_4	I	II	u_3	I	II	u_2	I	II	u_1	I	II	

the information of the previous 2 months. The recall time of the unit u_i is 0 at the survey month, 1 at "I" for one month before the survey month and 2 at "II" for two months before the survey month.

3. l-STEP GENERALIZED COMPOSITE ESTIMATOR

Some sample units are used repeatedly for a pre-determined number of months according to their rotation pattern. Efficient estimators have been suggested to use this longitudinal information from the same sample units. The current composite estimator by Rao and Graham (1964), the A-K composite and the generalized composite estimators by Breau and Ernst (1983) are such estimators. Among them, we consider the generalized composite estimator (GCE) since the GCE contains all previous composite estimators as special cases. Let $x_{t,i}$ be a simple estimator for a characteristic from the sample unit interviewed for the i-th time at month t. Then the GCE at month t is defined as

$$y_t = \sum_{i=1}^{M} a_i x_{t,i} - \omega \sum_{i=1}^{M} b_i x_{t-1,i} + \omega y_{t-1} \tag{3.1}$$

where M is the number of rotation groups, $0 \leq \omega < 1$ and $\sum_{i=1}^{M} a_i = \sum_{i=1}^{M} b_i = 1$.

By (3.1), the GCE is a linear combination of the current and all previous information. Since there are multiple $x_{t,i}$'s by recall times in a multi-level rotation design, it is natural that the multi-level version of (3.1) is

$$\begin{gathered} y_t = \sum_{i=1}^{M} \sum_{j=0}^{l-1} a_{ij} x_{t,i}^{(j)} - \omega \sum_{i=1}^{M} \sum_{j=0}^{l-1} b_{ij} x_{t,i}^{(j)} + \omega y_{t-1}, \\ 0 \leq \omega < 1, \quad \sum_i \sum_j a_{ij} = \sum_i \sum_j b_{ij} = 1 \end{gathered} \tag{3.2}$$

where $x_{t,i}^{(j)}$ is a simple estimator from the sample unit with the ith interview time and the jth recall time at month t. The $x_{t,i}^{(j)}$ will be obtained at month $t+j$, $j=0,\cdots,l-1$. This implies that y_t in (3.2) (the GCE of month t) can not be calculated until the future month $t+l-1$. To eliminate this discrepancy between the survey month and the month the GCE is obtained, we propose the following l-step generalized composite estimators. For $k=0,\cdots,l-1$

$$y_t^{(k)} = \sum_{i=1}^{M}\sum_{j=0}^{k} a_{ij}^{(k)} x_{t,i}^{(j)} - \omega \sum_{i=1}^{M}\sum_{j=0}^{k+1} b_{ij}^{(k)} x_{t-1,i}^{(j)} + \omega y_{t-1}^{(k)} \tag{3.3}$$

where $x_{t,i}^{(j)}$, ω are the same as in (3.2), $\sum_{i=1}^{M}\sum_{j=0}^{k} a_{ij}^{(k)}=\sum_{i=1}^{M}\sum_{j=0}^{k+1} b_{ij}^{(k)}=1$ for each $k=0,\cdots,l-1$ where $b_{i,l}^{(l-1)}\equiv 0$ for all $i=1,\cdots,M$.

Therefore, we have l GCE's for the month t : $y_t^{(0)}$ is obtained at the survey month t, and updated by $y_t^{(1)}$ at one month later; the updated $y_t^{(1)}$ is again updated by $y_t^{(2)}$ at month $t+2$ and so on until we have $y_t^{(l-1)}$ which is nothing but (3.2). Therefore $y_t^{(k)}$ in (3.3) uses all possible information of month t obtained for $k+1$ months from month t to month $t+k$ and can be interpreted as an intermediate estimator of a characteristic at month t finally to have $y_t^{(l-1)}$.

4. VARIANCES OF l-STEP COMPOSITE ESTIMATORS IN $r_1^m(l) - r_2^{m-1}$ DESIGN

In this section, we derive variances of l-step composite estimators for 4 types of characteristics in the 3-way balanced design: the current level, level change over a certain length of months such as month-to-month change and year-to-year change, aggregate levels and change such as the year sum and change of the quarter sum. For notation simplicity, let $\mathbf{a}(j)=(a_{1j}^{(k)},a_{2j}^{(k)},\cdots,a_{Mj}^{(k)})'$ and $\mathbf{a}_k=(\mathbf{a}'(0),\mathbf{a}'(1),\cdots,\mathbf{a}'(k),0'_{1\times M})'$. Similarly, $\mathbf{b}(j)=(b_{1j}^{(k)}, b_{2j}^{(k)}, \cdots, b_{Mj}^{(k)})'$ and $\mathbf{b}_k=(\mathbf{b}'(0),\mathbf{b}'(1),\cdots,\mathbf{b}'(k),\mathbf{b}'(k+1))'$. Finally, $\mathbf{x}_t(j)=(x_{t,1}^{(j)},\cdots,x_{t,M}^{(j)})'$ and $X_{t,k}=(\mathbf{x}'_t(0), \mathbf{x}'_t(1), \cdots, \mathbf{x}'_t(k+1))'$ for each $k=0,1,\cdots,l-1$. Then (3.3) is written as

$$y_t^{(k)} = \mathbf{a}'_k X_{t,k} - \omega \mathbf{b}'_k X_{t-1,k} + \omega y_{t-1}^{(k)},\ k=0,1,\cdots,l-1 \tag{4.1}$$

where $\mathbf{a}'_k\mathbf{1}=\mathbf{b}'_k\mathbf{1}=1$ and $0\le\omega<1$. Note that $y_t^{(k)}$ is the simple estimator when $\omega=0$ and all $\mathbf{a}_k$ are same.

The repeated interviews of the same sample units are more likely to be correlated and we call this correlation as the first-order correlation (or time correlation). Since sample units in the same rotation group are usually close to each other regionally, the sample units in a rotation group are also rather dependent. We call this type of correlation as the second-order correlation (or spatial correlation). These two types of correlations are incorporated into our variance estimation. Previous works (Rao and Graham, 1964; Cantwell, 1990; Yansaneh and Fuller, 1992) ignored the second-order correlation for the calculation of variances. However, Kumar and Lee (1983) and Park Kim and Choi (2001) showed that the variance of the GCE is seriously underestimated when the second-order correlation is ignored even for a small value of the second-order correlation.

The interview and recall times of a sample unit may also have some influence on its variance. Hence we allow that the variance does not remain the same, but rather varies over the course of interviewing and recalling sample units. We assume that there is at least two moments so that $E\left(x_{t,i}^{(j)}\right) = \mu_t$ for all $i = 1, 2, \cdots, M$ and $j = 0, 1, \cdots, l-1$ and $Var\left(x_{t,i}^{(j)}\right) = \sigma_{ij}^2$ for all t where $\sigma_{i0}^2 \leq \sigma_{i1}^2 \leq \cdots \leq \sigma_{i,l-1}^2$ is often assumed to reflect the recall time variability.

The following covariance of $x_{t,i}^{(j)}$ and $x_{t+t',i'}^{(j')}$ summarizes the above argument.

$$Cov(x^j{}_{t,i}, x^{j'}{}_{t+t',i'}) = \begin{cases} \sigma_{ij}^2 & \text{if } t' = 0,\ i = i' \text{ and } j = j', \\ \rho_{1t'}\sigma_{ij}\sigma_{i'j'} & \text{if both are from the same unit.} \\ \rho_{2t'}\sigma_{ij}\sigma_{i'j'} & \text{if both are from different unit} \\ & \quad \text{but from the same group.} \\ 0 & \text{otherwise.} \end{cases} \tag{4.2}$$

where $\rho_{1t'}$ is the first-order correlation and $\rho_{2t'}$ is the second-order correlation between months t and $t+t'$. To accommodate this correlation structure into the variance of $y_t^{(k)}$ in (4.1), we need to identify which two sample units are from the same rotation group and which two $x_{t,i}^{(j)}$ and $x_{t+t',i'}^{(j')}$ come from the same unit based only on interview times and recall times. Let L be $mr_1 \times mr_1$ with the (i,j)th element

$$(L)_{i,j} = \begin{cases} 1 & \text{if } i = I_{0,k_2} \text{ and } j = k_2 r_1 + 1 \text{ or } i = I_{k_1,k_2} \text{ and } j = I_{k_1-1,k_2}, \\ 0 & \text{otherwise.} \end{cases} \tag{4.3}$$

where $I_{k_1,k_2} = mod_{mr_1}(m_{k_1}^* + k_2 r_1)$ for $k_1 = 1, 2, \cdots, r_1 - 1$, $k_2 = 0, 1, \cdots, m-1$, and we replace $I_{k_1,k_2} = mr_1$ if $I_{k_1,k_2} = 0$.

This L matrix is another expression of the rotation pattern of the 3-way balanced design. Define $L^0 = I$ and $L^t = L^{t-1} \cdot L$. $L^{t'}$ matrix identifies that which two rotation groups surveyed in month t and $t+t'$ are same based only on interview times.

As an example, consider $5^1(3) - 0^0$ design in Figure 2. The matrix L for this $5^1(3) - 0^0$ design is $(L)_{ij} = 1$ if $(i,j) \in \{(1,3),\ (2,4),\ (3,5),\ (4,1),\ (5,2)\}$ and 0, otherwise. Thus $(L^3)_{ij} = 1$ if $j = i+1$ for $i = 1, \cdots, 4$ or $i = 5$ and $j = 1$ and $(L^3)_{ij} = 0$, otherwise. That is, the rotation group interviewed for the i-th time with recall time 0 at month t is the same as that interviewed for the $(i+1)$th time with recall time 0 at month $t+3$ for $i = 1, 2, \cdots, 4$. Similarly, the two rotation groups with interview time 5 and recall time 0 at month t, and with interview time 1 and recall time 0 at month $t+3$ are the same.

One can easily see from Figure 2.1, the construction of the 3-way balanced design, the sample units interviewed at month $t+1$ are determined by shifting one step to the right from the sample units at month t. Hence, the overlapped sample units between months t and $t+1$ are the same sample units. This can be expressed by the forward matrix of $(mr_1l + (m-1)r_2) \times (mr_1l + (m-1)r_2)$ L_s matrix with $(L_s)_{i,j} = 1$ if $j = i+1$ and 0, otherwise.

Since we only concerns on the overlapped sample units with recall time 0 at both months t and $t+1$, let $A_0 = \{i : i = 1 + (n_1 - 1)l + (n_2 - 1)(r_1l + r_2)$ for $n_1 = 1, 2, \cdots, r_1$ and $n_2 = 1, 2, \cdots, m\}$. Then we remove all rows and columns not in A_0. We denote the remaining L_s matrix by L_1 whose size is $mr_1 \times mr_1$. In this L_1 matrix, the ith row indicates the sample unit interviewed for the ith time with recall time 0 at month t and the jth column indicate the sample unit interviewed for the jth time with recall time 0 at month $t+1$.

This L_1 matrix is now used to identify that which two sample units at two months t and $t+1$ are same. If $(L_1)_{ij} = 1$ then the sample unit with the ith interview time and 0 recall time at month t and the unit with the jth interview time and 0 recall time at month $t+1$ are the same unit while if $(L_1)_{ij} = 0$ they are different. In general, define L_1^t to be the $mr_1 \times mr_1$ matrix after removing the rows and columns from L_s^t in which the rows and columns are not in A_0. Here $L_s^t = L_s \cdot L_s^{t-1}$ with $L_s^0 = I$. If $(L_1^{t'})_{ij} = 1$, two sample units at month t and $t+t'$ with recall time 0 and respective interview times i and j are same but if $(L_1^{t'})_{ij} = 0$ the corresponding two sample units are different.

Finally, define another $mr_1 \times mr_1$ matrix $L_2^t = L^t - L_1^t$. Since L matrix is used for identification of the same rotation group and L_1 is used for identification of the same sample unit, L_2^t can be interpreted as follows. If $(L_2^{t'})_{ij} = 1$, two sample units at month t and $t+t'$ with recall time 0 and respective interview times i and j are different but from the same rotation group. Therefore $L_1^{t'}$ and $L_2^{t'}$ matrices completely identify two sample units at respective survey months t and $t+t'$ only by their interview times : if $(L_1^{t'})_{ij} = 1$ and $(L_2^{t'})_{ij} = 0$ then two sample units with respective interview times i and j at months t and $t+t'$ are the same unit, and they are different but from the same rotation group if $(L_1^{t'})_{ij} = 0$ and $(L_2^{t'})_{ij} = 1$.

In $5^1(3)-0^0$ design, for example, $A_0 = \{1, 4, 7, 10, 13\}$. Thus we have $(L_1^3)_{ij} = 1$ if $j = i+1$ for $i = 1, 2, 3, 4$ and $(L_1^3)_{ij} = 0$ otherwise. Since $L_2^3 = L^3 - L_1^3$, we have $(L_2^3)_{ij} = 1$ if $(i = 5, j = 1)$. Using these L_1 and L_2 matrices, first we show

LEMMA 4.1. *Suppose that a multi-level rotation design is balanced in 3-ways with $r_2 = k_1 r_1$ for $k_1 = 0, 1, \cdots$. Then under the covariance structure given in (4.2), we have*

$$Cov\left(X_t^{(k)},\ X_{t+t'}^{(k)}\right) = V_{t',k},\ t' = 0, 1, \cdots$$

in which the $mr_1 \times mr_1$ $(i+1, j+1)$th block matrix of $V_{t',k}$ is

$$Cov(\mathbf{x}_t(i), \mathbf{x}_{t+t'}(j)) \equiv Q_{t',i,j} = \rho_{1t'}\Lambda_i L_1^{t'-i+j}\Lambda_j + \rho_{2t'}\Lambda_i L_2^{t'-i+j}\Lambda_j$$

where $\Lambda_j = diag(\sigma_{1j},\ \sigma_{2j},\ \cdots,\ \sigma_{Mj})$ for $i, j = 0, 1, \cdots, k+1$, $k = 0, 1, \cdots, l-1$.

PROOF. For $i, j = 0, 1, \cdots, l-1$ and $k_1, k_2 = 1, 2, \cdots, mr_1$, the covariance structure given in (4.2) gives $Cov\left(x_{t,k_1}^{(i)}, x_{t+t',k_2}^{(j)}\right) = \rho_{1t'}\sigma_{k_1,i}\sigma_{k_2,j}$ if the two are from the same sample unit. Since $x_{t,k_1}^{(i)}$ and $x_{t+t',k_2}^{(j)}$ are reported by the respective two sample units at months $t+i$ and $t+t'+j$ by the definition of the l-level rotation system, the identification matrix of L_1 between $x_{t,k_1}^{(i)}$ and $x_{t+t',k_2}^{(j)}$ is the same as that between $x_{t+i,k_1}^{(0)}$ and $x_{t+t'+j,k_2}^{(0)}$. By the construction of L_1, $(L_1^{t+t'+j-t-i})_{k_1,k2}$ $= (L_1^{t'+j-i})_{k_1,k_2} = 1$ implies that $x_{t+i,k_1}^{(0)}$ and $x_{t+t'+j,k_2}^{(0)}$ come from the same sample unit and, in turn, $x_{t,k_1}^{(i)}$ and $x_{t+t',k_2}^{(j)}$ are also the measurements from the same sample unit. Therefore, we have

$$Cov\left(\mathbf{x}_t(i), \mathbf{x}_{t+t'}(j)\right) = \rho_{1t'}\Lambda_i L_1^{t'+j-i}\Lambda_j \tag{4.4}$$

whenever any pair of $\mathbf{x}_t(i)$ and $\mathbf{x}_{t+t'}(j)$ are from the same sample unit. Similarly, $Cov(x_{t,k_1}^{(i)}, x_{t+t',k_2}^{(j)}) = \rho_{2t'}\sigma_{k_1,i}\sigma_{k_2,j}$ if $(L_2^{t'+j-i})_{k_1,k_2} = 1$. This gives

$$Cov\left(\mathbf{x}_t(i), \mathbf{x}_{t+t'}(j)\right) = \rho_{2t'}\Lambda_i L_2^{t'+j-i}\Lambda_j \tag{4.5}$$

whenever any pair of $\mathbf{x}_t(i)$ and $\mathbf{x}_{t+t'}(j)$ are from different sample units but from the same rotation group. Combining (4.4) and (4.5), we have the desired result. □

Define $B_{t_0,n}(k) = \sum_{j=0}^{\infty} \omega^{j+n} V_{t_0+j-n,k}$ for $t_0 \geq n$. Then we now obtain the variance of $y_t^{(k)}$.

THEOREM 4.1. *Under the same assumptions given in Lemma 4.1, the variance of* $y_t^{(k)}$ *for* $k = 0, 1, \cdots, l-1$ *is*

$$(1-\omega^2)Var(y_t^{(k)}) = \mathbf{a}_k'(V_{0,k} + 2\omega B_{1,0}(k))\mathbf{a}_k + \omega^2\mathbf{b}_k'(V_{0,k} + 2\omega B_{1,0}(k))\mathbf{b}_k - 2\omega\mathbf{b}_k'(B_{1,0}(k) + B_{1,1}'(k))\mathbf{a}_k.$$

PROOF. Recursively solving (4.1), we have $y_t^{(k)} = \sum_{j=0}^{\infty}\left(\omega^j\mathbf{a}_k'X_{t-t_0-j}^{(k)} - \omega^{j+1}\mathbf{b}_k' X_{t-t_0-j-1}\right)$. By Lemma 4.1, for $n \le t_0$,

$$\begin{aligned}
&Cov(y_{t-t_0}^{(k)},\ \omega^n\mathbf{a}_k'X_{t-n}^{(k)})\\
&= \sum_{j=0}^{\infty}\mathbf{a}_k'Cov(X_{t-t_0-j}^{(k)}, X_{t-n}^{(k)})\mathbf{a}_k - \sum_{j=0}^{\infty}\omega^{j+1+n}\mathbf{b}_k'Cov(X_{t-t_0-j-1}^{(k)}, X_{t-n}^{(k)})\mathbf{a}_k\\
&= \mathbf{a}_k'\sum_{j=0}^{\infty}\omega^{j+n}V_{t_0+j-n,k}\mathbf{a}_k - \mathbf{b}_k'\sum_{j=0}^{\infty}\omega^{j+n+1}V_{t_0+j+1-n,k}\mathbf{a}_k\\
&= \mathbf{a}_k'B_{t_0,n}^{(k)}\mathbf{a}_k - \omega\mathbf{b}_k'B_{t_0+1,n}^{(k)}\mathbf{a}_k.
\end{aligned} \tag{4.6}$$

Similarly, for $n \le t_0$

$$Cov(y_{t-t_0}^{(k)},\ \omega^n\mathbf{b}_k'X_{t-n}^{(k)}) = \mathbf{a}_k'B_{t_0,n}^{(k)}\mathbf{b}_k - \omega\mathbf{b}_k'B_{t_0+1,n}^{(k)}\mathbf{b}_k. \tag{4.7}$$

$Cov\left(X_t^{(k)},\ X_{t+t'}^{(k)}\right)$ depends only on the time lag t' by Lemma 4.1, we obtain that

$$\begin{aligned}
(1-\omega^2)Var(y_t^{(k)}) =& \mathbf{a}_k'Var(X_t^{(k)})\mathbf{a}_k + \omega^2\mathbf{b}_k'Var(X_{t-1}^{(k)})\mathbf{b}_k\\
&- 2\omega\mathbf{b}_k'Cov(X_{t-1}^{(k)}, X_t^{(k)})\mathbf{a}_k\\
=& 2\omega Cov(y_{t-1}^{(k)}, \mathbf{a}_k'X_t^{(k)}) - \omega Cov(y_{t-1}^{(k)}, \omega\mathbf{b}_k'X_{t-1}^{(k)}).
\end{aligned}$$

The two equations (4.6) and (4.7) produce

$$\begin{aligned}
(1-\omega^2)Var(y_t^{(k)}) =& \mathbf{a}_k'(V_{0,k} + 2\omega B_{1,0}(k))\mathbf{a}_k + \omega^2\mathbf{b}_k'(V_{0,k} + 2\omega B_{1,0}(k))\mathbf{b}_k\\
&- 2\omega\mathbf{b}_k'(V_{1,k} + \omega B_{2,0}(k) + B_{1,1}'(k))\mathbf{a}_k.
\end{aligned}$$

Finally, since $V_{1,k} + \omega B_{2,0}(k) = B_{1,0}(k)$, we have the results. □

For each $k = 0, 1, \cdots, l-1$ and $t_0, t' > 0$, we are often interested in variances for the change of $y_t^{(k)} - y_{t-t_0}^{(k)}$, the aggregate of the $y_t^{(k)}$ expressed by $S_t^{t_0}(k) = \sum_{t'=0}^{t_0-1} y_{t-t'}^{(k)}$, and the difference of two aggregates expressed by $S_t^{t_0}(k) - S_{t-t_1}^{t_0}(k) = \sum_{i=0}^{t_0-1} y_{t-i}^{(k)} - \sum_{i=0}^{t_0-1} y_{t-t_1-i}^{(k)}$ for $t_1 \ge t_0$. For variances of these three estimators, let $P_{1k} = (1-\omega^2)^{-1}(V_{0,k} + 2\omega B_{1,0}(k))$, $P_{2k} = -2(1-\omega^2)^{-1}\omega(B_{1,0}(k) +$

$B'_{1,1}(k))$ and $P_{3k} = \omega^2 P_{1k}$ which are from $Var(y_t)$ given in Theorem 4.1. Further, let $P_{1k}(t^*) = 2(1-\omega^{t^*})P_{1k} - 2\sum_{n=0}^{t^*-1} B_{t^*,n}(k)$, $P_{2k}(t^*) = 2(1-\omega^{t^*})P_{2k} + 2\omega\sum_{n=0}^{t^*-1}\big(B_{t^*+1,n}(k) + B'_{t^*,n+1}(k)\big)$ and $P_{3k}(t^*) = \omega^2 P_{1k}(t^*)$ for $t^* \geq 1$. Then the following variances are obtained.

THEOREM 4.2. *Under the same conditions in Theorem 4.1,*

$$\begin{aligned} Var\big(y_t^{(k)} - y_{t-t_0}^{(k)}\big) &= \mathbf{a}_k' P_{1k}(t_0)\mathbf{a}_k + \mathbf{b}_k' P_{2k}(t_0)\mathbf{a}_k + \mathbf{b}_k' P_{3k}(t_0)\mathbf{b}_k, \\ Var\big(S_t^{t_0}(k)\big) &= \mathbf{a}_k'\big(t_0 P_{1k} - \sum_{t^*=1}^{t_0-1}(t-t^*)P_{1k}(t^*)\big)\mathbf{a}_k \\ &\quad + \mathbf{b}_k'\big(t_0 P_{2k} - \sum_{t^*=1}^{t_0-1}(t-t^*)P_{2k}(t^*)\big)\mathbf{a}_k \\ &\quad + \mathbf{b}_k'\big(t_0 P_{3k} - \sum_{t^*=1}^{t_0-1}(t-t^*)P_{3k}(t^*)\big)\mathbf{b}_k \end{aligned}$$

and

$$\begin{aligned} &Var\big(S_t^{t_0}(k) - S_{t-t_1}^{t_0}(k)\big) \\ &= \mathbf{a}_k'\Big(\sum_{t^*=-t_0+1}^{t_0-1}(t_0 - |t^*|)P_{1k}(t_1 - t^*) - 2\sum_{t^*=1}^{t_0-1}(t_0 - t^*)P_{1k}(t^*)\Big)\mathbf{a}_k \\ &+ \mathbf{b}_k'\Big(\sum_{t^*=-t_0+1}^{t_0-1}(t_0 - |t^*|)P_{2k}(t_1 - t^*) - 2\sum_{t^*=1}^{t_0-1}(t_0 - t^*)P_{2k}(t^*)\Big)\mathbf{a}_k \\ &+ \mathbf{b}_k'\Big(\sum_{t^*=-t_0+1}^{t_0-1}(t_0 - |t^*|)P_{3k}(t_1 - t^*) - 2\sum_{t^*=1}^{t_0-1}(t_0 - t^*)P_{3k}(t^*)\Big)\mathbf{b}_k. \end{aligned}$$

PROOF. Since $y_t^{(k)} = \sum_{n=0}^{t_0-1}(\omega^n \mathbf{a}_k' X_{t-n}^{(k)} - \omega^{n+1}\mathbf{b}_k' X_{t-n-1}^{(k)}) + \omega^{t_0} Var(y_t^{(k)})$, we have

$$\begin{aligned} Var(y_t^{(k)} - y_{t-t_0}^{(k)}) &= 2Var(y_t^{(k)}) - 2Cov(y_{t-t_0}^{(k)}, y_t^{(k)}) \\ &= 2(1-\omega^{t_0})Var(y_t^{(k)}) - 2\sum_{n=0}^{t_0-1} Cov(y_{t-t_0}^{(k)}, \omega^n \mathbf{a}_k' X_{t-n}^{(k)} \\ &\quad + 2\sum_{n=0}^{t_0-1} Cov(y_{t-t_0}^{(k)}, \omega^{n+1}\mathbf{b}_k' X_{t-n-1}^{(k)}). \end{aligned}$$

Thus we have the first claim by (4.6) and (4.7).

Observe that

$$\begin{aligned} Var(S_t^{t_0}(k)) &= \sum_{t'=0}^{t_0-1} Var(y_{t-t'}^{(k)}) + 2\sum_{i=0}^{t_0-2}\sum_{i'=i+1}^{t_0-1} Cov(y_{t-i}^{(k)}, y_{t-i'}^{(k)}) \\ &= t_0 Var(y_t^{(k)}) + 2\sum_{t'=1}^{t_0-1}(t_0-t')Cov(y_t^{(k)}, y_{t-t'}^{(k)}). \end{aligned}$$

Now, since $Cov(y_t^{(k)}, y_{t-t'}^{(k)}) = Var(y_t^{(k)}) -(1/2)Var(y_t^{(k)} - y_{t-t'}^{(k)})$, we have the second claim.

By definition of $S_t^{t_0}(k) - S_{t-t_1}^{t_0}(k)$ for $t_1 \geq t_0$, $Var\big(S_t^{t_0}(k) - S_{t-t_1}^{t_0}(k)\big)$ is expressed as

$$\begin{aligned} &\sum_{i=0}^{t_0-1}\Big(Var(y_{t-i}^{(k)}) + Var(y_{t-t_1-i}^{(k)})\Big) + 2\Bigg[\sum_{i=0}^{t_0-2}\sum_{i'=i+1}^{t_0-1}\Big(Cov(y_{t-i}^{(k)}, y_{t-i'}^{(k)}) \\ &\quad + Cov(y_{t-t_1-i}^{(k)}, y_{t-t_1-i'}^{(k)})\Big) - \sum_{i=0}^{t_0-1}\sum_{i'=0}^{t_0-1} Cov(y_{t-i}^{(k)}, y_{t-t_1-i'}^{(k)})\Bigg] \\ &= 2t_0 Var(y_t^{(k)}) + 2\Bigg[\sum_{i=1}^{t_0-1}(t_0-i)\Big(2Cov(y_t^{(k)}, y_{t-i}^{(k)}) \\ &\quad - Cov(y_t^{(k)}, y_{t-t_1+i}^{(k)}) - Cov(y_t^{(k)}, y_{t-t_1-i}^{(k)})\Big) - t_1 Cov(y_t^{(k)}, y_{t-t_1}^{(k)})\Bigg] \end{aligned} \tag{4.8}$$

After a little algebraic calculation using

$$Cov(y_t^{(k)}, y_{t-t'}^{(k)}) = Var(y_t^{(k)}) - \frac{1}{2}Var(y_t^{(k)} - y_{t-t'}^{(k)})$$

(4.8) is

$$\sum_{t^*=-t_0+1}^{t_0-1}(t_0-|t^*|)Var(y_t^{(k)} - y_{t-t_1+t^*}^{(k)}) - 2\sum_{t^*=1}^{t_0-1} Var(y_t^{(k)} - y_{t-t^*}^{(k)}).$$

This completes the proof. □

5. OPTIMAL COEFFICIENTS OF l-STEP GCE

We have the four types of l-step GCE's : $y_t^{(k)}$, $y_t^{(k)} - y_{t-t_0}^{(k)}$, $S_t^{t_0}(k)$ and $S_t^{t_0}(k) - S_{t-t_1}^{t_0}(k)$ for $t_0 \geq 1$, $t_1 \geq t_0$ and each $k = 0, 1, \cdots, l-1$. Defining specific values of t_0 and t_1, we assume there are H l-step GCE's of interest. Denote them by z_{tkh} for $h = 1, 2, \cdots, H$. Note that z_{tkh} and $z_{tkh'}$, $h \neq h'$ can be the same type of l-step GCE's when they are from different characteristics. For example, $y_t^{(k)}$ for Labor Force is the same notation as $y_t^{(k)}$ for Unemployed.

To have the consistency in total among estimates or to preserve additivity of estimates, we set the object function O_k, $k = 0, 1, \cdots, l-1$ for variance : $O_k = \sum_{h=1}^{H} \delta_h Var(z_{tkh}) - \lambda_1(\mathbf{1}'\mathbf{a}_k - 1) - \lambda_2(\mathbf{1}'\mathbf{b}_k - 1)$ where λ's are Lagrange multipliers and δ_h's are the weights which represent the relative importance of the corresponding estimators. For example, H l-step GCE's are equally valuable, then $\delta_h = 1/H$ for all h. By a suitable choice of z_{tkh}'s, we obtain the one set of optimal coefficients by minimizing the object function O_k for each $k = 0, 1, \cdots, l-1$. Then we can use it commonly for different estimators and characteristics.

By Theorems 4.1 and 4.2 for the 3-way balanced $r_1^m(l) - r_2^{m-1}$ design, $Var(z_{tkh})$ can be expressed as

$$Var(z_{tkh}) = \mathbf{a}_k' C_{1kh} \mathbf{a}_k + \mathbf{b}_k' C_{2kh} \mathbf{a}_k + \mathbf{b}_k' C_{3kh} \mathbf{b}_k. \tag{5.1}$$

For example, when $z_{tkh} = y_t^{(k)}$, $\mathbf{C}_{ikh} = P_{ik}$ for $i = 1, 2, 3$. Note that in (5.1), the last $M = mr_1$ elements of $\mathbf{a}_k$ for $k = 0, 1, \cdots, l-1$ are zero and the last M elements of $\mathbf{b}_{l-1}$ are also zero by the construction of the l-step GCE. To eliminate these zero's constraint on $\mathbf{a}_k$ and $\mathbf{b}_{l-1}$, we introduce the matrix I_{k0} which is a $(k+2)mr_1 \times (k+2)mr_1$ identity matrix except the last mr_1 diagonal elements being zero where $k = 0, 1, \cdots, l-1$. Using this I_{k0} matrix, define $C_{1kh}^* = I_{k0} C_{1kh} I_{k0}'$, $C_{2kh}^* = C_{2kh} I_{k0}'$ and $C_{3kh}^* = C_{3kh}$ for $k = 0, 1, \cdots, l-2$. For $k = l-1$, let $C_{2,l-1,h}^* = I_{l-1,0} C_{2kh} I_{l-1,0}'$ and $C_{3,l-1,h}^* = I_{l-1,0} C_{3,l-1,h} I_{l-1,0}'$. Then $Var(z_{tkh})$ given in (5.1) is rewritten as

$$Var(z_{tkh}) = \mathbf{a}^{*\prime}_k C_{1kh}^* \mathbf{a}^*_k + \mathbf{b}^{*\prime}_k C_{2kh}^* \mathbf{a}^*_k + \mathbf{b}^{*\prime}_k \mathbf{C}^*_{3kh} \mathbf{b}^*_k \tag{5.2}$$

where $\mathbf{a}^*_k = I_{k0}\mathbf{a}_k$ for all $k = 0, 1, \cdots, l-1$, $\mathbf{b}^*_k = \mathbf{b}_k$ for $k = 0, 1, \cdots, l-2$, $\mathbf{b}^*_{l-1} = I_{k0}\mathbf{b}_{l-1}$.

By (5.2), the objective function O_k is $O_k^* = \sum_{h=1}^{H} \delta_h Var(z_{tkh}) - \lambda_1(\mathbf{1}'\mathbf{a}^*_k - 1) - \lambda_2(\mathbf{1}'\mathbf{b}^*_k - 1)$ where $\mathbf{1}$ is a vector of ones. By letting $C_{ik}^* = \sum_{h=1}^{H} \delta_h C_{ikh}^*$, we optimize this objective function O_k^* and obtain the following estimates of the optimal coefficients in the 3-way balanced $r_1^m(l) - r_2^{m-1}$ design. Hereafter, we call these optimal coefficients as compromise coefficients.

LEMMA 5.1. *Suppose that a multi-level rotation design is balanced in 3-ways. For given weights δ_h, $h = 1, 2, \cdots, H$, the compromise coefficients of $\mathbf{a}^*_k$ and $\mathbf{b}^*_k$ minimizing the weighted variance $\sum_{h=1}^{H} \delta_h Var(z_{tkh})$ are given by*

$$\begin{pmatrix} \widehat{\mathbf{a}^*}_k \\ \widehat{\mathbf{b}^*}_k \end{pmatrix} = \begin{pmatrix} C_{1k}^{**} & C_{2k}^* - c_1 J_k (C_{1k}^{**})^{-1} C_{2k}^{*\prime} \\ C_{2k}^* - c_3 J_k (C_{3k}^{**})^{-1} C_{2k}^{*\prime} & C_{3k}^{**} \end{pmatrix}^{-1} \begin{pmatrix} c_1 \mathbf{1}_k \\ c_3 \mathbf{1}_k \end{pmatrix}$$

where $C_{1k}^{**} = C_{1k}^* + C_{1k}^{*\prime}$, $C_{3k}^{**} = C_{3k}^* + C_{3k}^{*\prime}$, $c_1 = \mathbf{1}_k'(C_{1k}^{**})^{-1}\mathbf{1}_k$, $c_3 = \mathbf{1}_k'(C_{3k}^{**})^{-1}\mathbf{1}_k$ and $J_k = \mathbf{1}_k \mathbf{1}_k'$ where $\mathbf{1}_k$ has appropriate size depending on k.

PROOF. Derivatives of the object function O_k^* with respect to $\mathbf{a^*}_k$, $\mathbf{b^*}_k$, λ_1 and λ_2 yield

$$\frac{\partial O_k^*}{\partial \mathbf{a^*}_k} = (C_{1k}^{**})\mathbf{a^*}_k + C_{2k}^{*'}\mathbf{b^*}_k - \lambda_1 \mathbf{1}_k = 0,$$
$$\frac{\partial O_k^*}{\partial \mathbf{b^*}_k} = C_{2k}^{*}\mathbf{a^*}_k + (C_{3k}^{**})\mathbf{b^*}_k - \lambda_2 \mathbf{1}_k = 0,$$
$$\frac{\partial O_k^*}{\partial \lambda_1} = \mathbf{1}_k'\mathbf{a^*}_k - 1 = 0 \text{ and } \frac{\partial O_k^*}{\partial \lambda_2} = \mathbf{1}_k'\mathbf{b^*}_k - 1 = 0. \tag{5.3}$$

This gives

$$\lambda_1 = c_1^{-1}\big(1 + \mathbf{1}_k'(C_{1k}^{**})^{-1}C_{2k}^{*'}\mathbf{b^*}_k\big) \quad \text{and} \quad \lambda_2 = c_3^{-1}\big(1 + \mathbf{1}_k'(C_{3k}^{**})^{-1}C_{2k}^{*'}\mathbf{a^*}_k\big).$$

Substituting these λ_1 and λ_2 into (5.3), we have the desired $\mathbf{a^*}_k$ and $\mathbf{b^*}_k$ for $k = 0, 1, \cdots, l-1$. □

6. NUMERICAL EXAMPLES

To illustrate the efficiency of l-step GCEs, we use simple estimators as competitive ones. Because, in practical sense, the simple estimators are used to estimate population characteristics in multi-level rotation sampling design, we calculate the efficiency of l-step GCE's relative to simple estimators under $5^1(3) - 0^0$ design of CEX.

Through out this section, we use two types of variance. The first one is constant, $Var(x_{t,i}^{(j)}) = 100$ and the second one increases as recall time j increases, $Var(x_{t,i}^{(j)}) = (1+0.3j)^2 100$. Weaker correlation is expected when time lag is longer and the second-order correlation is usually smaller than the first order correlation. Thus, we assume $\rho_{1t} = \rho_1^t$ for the first-order correlation and $\rho_{2t} = \rho_2\rho_1^t$ for the second-order correlation. To calculate the variance of l-step GCE's, we use the compromised coefficients minimizing the variance of $0.1y_t^{(j)} + 0.1(y_t^{(j)} - y_{t-1}^{(j)}) + 0.5\overline{S}_t^{12}(j) + 0.3(\overline{S}_t^{12}(j) - \overline{S}_{t-12}^{12}(j))$ for l-step GCEs since yearly mean (*i.e.*, $\overline{S}_t^{12}(j)$) is most important and yearly mean change is next in multi-level rotation design.

Table 6.1 compares l-step GCEs with simple estimators using the CEX design under $\rho_{1t} = 0.4^t, 0.6^t, 0.8^t$ and $\rho_{2t} = 0.0, 0.3\rho_{1t}, 0.6\rho_{1t}$. The values in this table are the variance of l-step GCE divided by variance of the corresponding simple estimator. When the variance of $x_{t,i}^{(j)}$ is constant (*i.e.*, $\sigma_j^2 = 100$), then l-step GCE is slightly better than simple estimator. However, when σ_j^2 is increases as recall time j increase (*i.e.*, $\sigma_j^2 = (1+0.3j)^2 100$), l-step GCE is better than simple estimator. In particular, l-step GCE is much better than simple estimator for yearly

TABLE 6.1 *Relative efficiency*

			common variance				increasing variance			
ρ_1	ρ_2	level(j)	y_t	$y_t - y_{t-1}$	$\overline{S}_t^{12}$	$\overline{S}_t^{12}$-$\overline{S}_{t-12}^{12}$	y_t	$y_t - y_{t-1}$	$\overline{S}_t^{12}$	$\overline{S}_t^{12}$-$\overline{S}_{t-12}^{12}$
		0	1.000	1.000	0.999	0.999	1.000	1.000	0.999	0.999
0.4	0.0	1	0.967	0.920	1.050	1.048	0.936	0.898	1.003	1.001
		2	0.990	0.984	1.000	1.000	0.947	0.943	0.951	0.950
		0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.3	1	0.974	0.934	1.037	1.035	0.941	0.911	0.986	0.985
		2	0.993	0.989	1.001	1.000	0.947	0.950	0.941	0.942
		0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.4	0.6	1	0.980	0.946	1.027	1.025	0.945	0.922	0.973	0.972
		2	0.996	0.992	1.000	1.000	0.944	0.954	0.929	0.930
		0	0.999	1.000	0.995	0.994	0.999	1.000	0.995	0.994
0.6	0.0	1	0.916	0.778	1.063	1.060	0.867	0.752	0.977	0.978
		2	0.971	0.952	0.993	0.988	0.912	0.905	0.913	0.912
		0	0.999	1.000	0.997	0.997	0.999	1.000	0.997	0.997
0.6	0.3	1	0.941	0.822	1.037	1.036	0.885	0.794	0.946	0.948
		2	0.984	0.971	0.997	0.994	0.911	0.931	0.883	0.887
		0	0.999	1.000	0.999	0.998	0.999	1.000	0.999	0.998
0.6	0.6	1	0.960	0.865	1.023	1.022	0.896	0.834	0.921	0.923
		2	0.990	0.983	0.998	0.996	0.895	0.945	0.838	0.847
		0	0.990	1.001	0.968	0.944	0.990	1.001	0.968	0.944
0.8	0.0	1	0.823	0.512	1.044	1.045	0.745	0.489	0.906	0.910
		2	0.915	0.858	0.964	0.918	0.800	0.810	0.784	0.775
		0	0.994	1.001	0.985	0.977	0.994	1.001	0.985	0.977
0.8	0.3	1	0.891	0.597	1.021	1.023	0.782	0.571	0.836	0.847
		2	0.959	0.922	0.985	0.962	0.775	0.903	0.687	0.714
		0	0.996	1.001	0.993	0.990	0.996	1.001	0.993	0.990
0.8	0.6	1	0.938	0.699	1.010	1.011	0.775	0.670	0.737	0.767
		2	0.979	0.958	0.993	0.981	0.703	0.964	0.549	0.606

mean and the relative efficiency of l-step GCE is greater in final estimator (for example, $\overline{S}_t^{12}(2)$) than in preliminary estimators (for example, $\overline{S}_t^{12}(j), j = 0, 1$). This tables also shows that the effects of the first and the second-order correlations. For each fixed first-order correlation, variance of all 4 characteristics of final level increase as the second-order correlation increases under the common variance. Hence ignorance of the second-order correlation results in underestimation of variance as indicated in Park, Kim and Choi (2001). However, under the increasing variance, relative efficiency reveals completely reverse phenomena except monthly change. Since increasing variance may include effect of recall time bias, this shows that l-step GCE is effective estimator for 3-way balanced $r_1^m(l) - r_2^{m-1}$ design.

A generalized composite estimator (GCE) was introduced and improved by many researchers. But the GCE has a critical deficiency to apply in a 3-way balanced multi-level rotation design since it can not be calculated, timing with a survey month. To overcome this problem, we derived the general variance formulae of the l-step GCE in the 3-way balanced $r_1^m(l) - r_2^{m-1}$ design when the first-order and second-order correlations are presented.

As discussed in Cantwell and Caldwell (1998) and Park, Kim and Choi (2001), there should be biases from different interview times, different rotation groups and different recall times at each survey month in a multi-level rotation design. Since previous works showed that the biases overwhelms the variance of an estimator in a rotation sampling, the derivation of MSE of the l-step GCE is one of our future work. Since the two correlations depends on the rotation pattern, we need to compare the design efficiency for some selected multi-level rotation designs and to investigate the effect of the second-order correlation on the variance and MSE of the l-step GCE. These are also a part of our future work.

REFERENCES

BAILAR, B. (1975). "The effects of rotation group bias on estimates from panel survey", *Journal of the American Statistical Association*, **70**, 23–30.

BREAU, P. AND ERNST, L. (1983). "Alternative estimators to the current composite estimators", *Proceedings of the section on Survey Research Methods, American Statistical Association*, 397–402.

CANTWELL, P.J. (1990). "Variance formulae for composite estimators in rotation designs", *Survey Methodology*, **16**, 153–163.

CANTWELL, P.J. AND CALDWELL, C.V. (1998). "Examining the revisions in Monthly Retail and Wholesale Trade Surveys under a rotating panel design", *Journal of Official Statistics*, **14**, 47–59.

FULLER, W.A., ADAM, A. AND YANSANEH, I.S. (1992). "Estimators for longitudinal surveys with application to the U.S. Current Population Survey", *Statistics Canada Symposium* , 309–324.

KUMAR, S. AND LEE, H. (1983). "Evaluation of composite estimation for the Canadian Labor Force Survey", *Proceedings of the section on Survey Research Methods, American Statistical Association*, 403–408.

PARK, Y.S., KIM, K.W. AND CHOI, J. (2001). "One-level rotation design balanced on time in monthly sample and in rotation group", *Journal of American Statistical Association*, **96**, 1483-1496.

PARK, Y.S., KIM, K.W. AND KIM, N.Y. (2003). "Three-way balanced multi-level rotation sampling designs", *Journal of the Korean Statistical Society*, **32**, 245–259.

RAO, J.N.K. AND GRAHAM, J.E. (1964). "Rotation designs for sampling on repeated occasions", *Journal of the American Statistical Association*, **59**, 492–509.