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Another discovery in the technology-based classroom : Joy's Similar Quadrilaterals

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Along with the continual debate relating to the use of technology, especially since LOGO in 1980, technology has always been the issue to the society of mathematics education about what is the role of technology in teaching and learning, how it can facilitate for the better understanding of learners, especially what we can do more with it comparing to the traditional teaching and learning environments. Here I propose a way of using technology[GSP] for creative exploration, which makes it possible to extend our knowledge that leads to new discovery.

Key Words : Technology, GSP, Extension of knowledge, Mathematical discovery with exploration

I. Introduction

To find the centers of a triangle is one of the popular activities that students can actively participate in with The Geometer's Sketchpad[GSP] (Jackiw, 1991). Especially, the centroid, i.e., balancing point, of a triangle is in general first introduced to students either on the blackboard directly by the instructor or through the activity to find it using a hard board paper. In either case, GSP is expected to play an important role for the accurate construction of the centroid and confirmation of students' understanding.

After 'New Math' movement in the 1960's, people turned around to 'Back to Basics' followed by 'Problem Solving'in mathematics education. Along with the use of technology in mathematics education, people considered the creative thinking of learners as one of the most important things that we should take care of in addition to problem solving (Baranes, Perry, & Stigler, 1989; Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993). Since the appearance of LOGO (Papert, 1980), turtle geometry, the development of technology rapidly improved in quality as well as in quantity. The standard, Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989),

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put the emphasis on the use of technology in mathematics education: "The thoughtful and creative use of technology can greatly improve both the quality of the curriculum and the quality of children's learning" (p. 19).

Further we can expect technology to allow students to focus on problem solving and other important contents. Principles and Standards for School Mathematics (NCTM, 2000) stated that technology is essential in teaching and learning mathematics in the technology principle which newly showed up as an independent principle. It emphasizes the impacts that technology may bring: "Technology not only influences how mathematics is taught and learned but also affects what is taught and when a topic appears in the curriculum." (p. 26) But, it warned that technology should not replace students' need to learn basic facts, to compute mentally, or to do reasonable paper-and-pencil computation(NCTM, 1991, p. 19).

Technology, we believe, was not designed to just sit in classrooms taking some spaces to show off. But, unfortunately it does for most classrooms. We believe that technology can do something in mathematics classrooms. We would like to share an experience in the classroom where the gifted and talented students were taught using a technology from the perspective of appropriate use of technology in Korea. This paper is a case how technology can be used for further exploration of mathematical concepts that students already knew based on the real experience. Further, We would like to introduce the interesting geometrical fact which had not been discovered yet, which was found during the class using technology.

II. Classes with gifted students

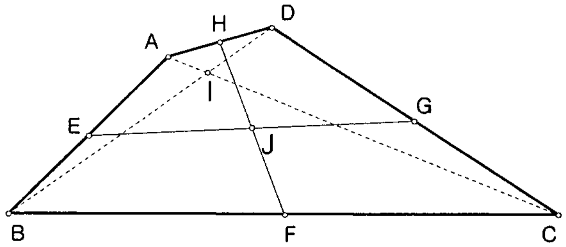
There was a special program for gifted or talented students at one major university in Korea. All of them were 8th and 9th graders and the total number was 14. The period of program was four days. One of researcher was assigned to teach Geometry with GSP for two days, four hours each day. The goals of classes were to 1) introduce a new technology, GSP, to students, 2) have them use GSP, and 3) explore geometry with it. There was nobody who heard of GSP, not to mention used it. Class began with the question to ask them to find the balancing point of the given triangle. They right away said that it is the centroid and the intersection of three medians of a triangle, which they already knew. They learn five centers of a triangle in 8th grade in Korea. Because they were aware of this very well, rather than how they could find the location of the centroid for the given triangle, most time was spent to help them to know how to use GSP, i.e., functions by clicking each menu one by one during the first day. It seemed that they were excited with functions of GSP and did not take a long time for them to get used to it. Especially, it was the function of animation that they most liked. They were able to construct five centers on a given triangle with GSP at the end of the first day although some were slow. Thus, the first two goals were achieved without any big difficulties.

The main goal of the second day was to explore geometry using GSP. Some geometrical fancy facts were introduced such as a nine point circle, a pedal triangle, a Napoleon's Theorem, etc., which they did not hear before. But, these are very closely connected with concepts of five centers, and they actually could handle these new facts with the help of GSP, but we did not prove why. Thus, they knew that they could do something using GSP that it would be very painful to do just with pencil, paper, a ruler, and a compass. Now, they were ready for the new task with their mathematical knowledge and basic skills with GSP. It was to find the balancing point of an arbitrary convex quadrilateral. They were allowed to use paper hard board and traditional tools such as a ruler and a compass, and highly encouraged to use GSP for the exploration.

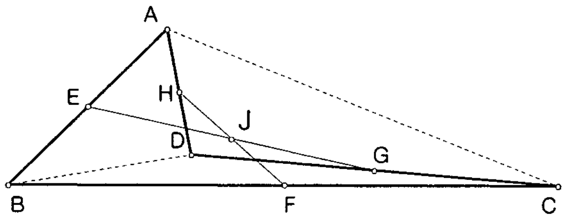
III. Students' responses with GSP for the task

Interestingly, students did not even touch any traditional tools, but directly began to use GSP. It has been only two days since a new technology was introduced. Their responses were in striking contrast compared to those of Jung (1999)'s. The participants in Jung's research were hesitant to use GSP in the beginning of the semester. It took about 2-3 weeks for them to get used to the use of GSP and after that, they relied on GSP for most tasks, however they also had a hard time with proofs from the perspectives of logical approaches using mathematical symbols. On the other hand, they liked GSP especially because GSP was very helpful in trying or testing what they were thinking in their mind and helped them to figure out geometrical facts easily. The gifted students showed similar responses except the moment when they began to use GSP.

The first response for the new task they showed as you might easily guess was to find the mid point (E, F, G, H) of each side and connect two mid points located in opposite sides, i.e., they had two bimedians ($\overline{EG}$ & $\overline{HF}$). They said that the intersection (J) of bimedians is the balancing point. Some said that the intersection of two diagonals is the balancing point (I). They were asked to test if those points are the balancing points using paper hard board. Some students tested using a board, but most did on GSP. They moved one vertex of a quadrilateral around. It was not hard for them to see these two intersections are not the balancing points (Figure 1). It seemed that it is a balancing point for familiar figures such as rectangle, rhombus, parallelograms, etc. to them, but not for irregular figures. They did not present a specific reason why it was not, but were sure of their sense or feeling that it is not for non-symmetric figures. In addition, when a quadrilateral is a concave, there even does not exist the intersection, I (Figure 2). In other words, if this method is correct, we should be able to locate a balancing point even for a concave although it might not be inside the figure.



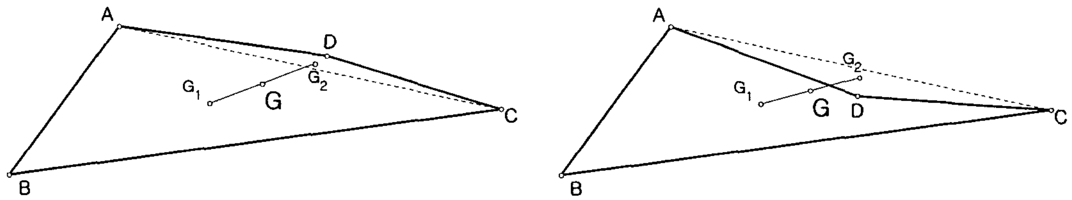
[Figure 1] Intersections of bimedians and diagonals



[Figure 2] Nonexistence of intersection of two diagonals

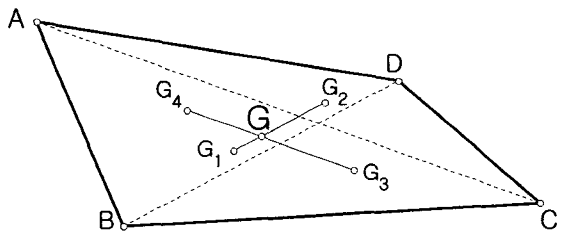
Two students suddenly suggested the very different method in finding the balancing point of a quadrilateral. It is to cut the given quadrilateral into two triangles by connecting opposite vertices of it, then find the centroid of each triangle, and find the mid point of the segment connecting two centroids. They said that it is the balancing point of a quadrilateral. In contrast to the other two cases presented above, this method was distinct in the sense that they used the concept of centroid of triangles. When this was presented to all students following the first two cases, all students seemed to agree with this method. One of them said that "Right, we have to use the centroid somewhere even for the balancing point of a quadrilateral!" But, one student came up with a counter example right away (Figure 3) by moving a vertex around on the screen. The situation was exactly the same as the first two cases. Although the mid point of the segment connecting two centroids still exist for concave and convex, it did not seem to be the balancing point for some cases. Again nobody tested this with a paper hard board by cutting and then holding it at the end of pencil. But, they were sure of their spatial senses by just looking at the figures on the screen. In figure 3, as the vertex, D, is moved to the left, so called the balancing point, G, is located outside the given figure, which might be confusing them. Here, it is suggested that they just look for the balancing point of the given convex quadrilateral to make it simple.

We spent some time over discussing if it is a balancing point or not. The conclusion was that it is not. Students could not really progress in finding the location of the balancing point, but they were happy with functions of GSP because it was really helpful in testing their conjectures. Whatever it was, they were able to construct and test what they were thinking on GSP although it turned out their method was not correct. After this, we had about ten minutes'break and came back to class for the fourth class, which is the last class period for the activity.



[Figure 3] Counter example: Mid-point of two centroids

The student who presented the third method suddenly yelled "I found it, sir. I am sure this is it."Then, he presented the correct method to find the balancing point of a quadrilateral: that is, the segment connecting two centroids of two triangles formed by a diagonal and the other segment constructed in the same way make one intersection (Figure 4). He did not prove why it is, but was confident that that is the balancing point of the given convex quadrilateral.

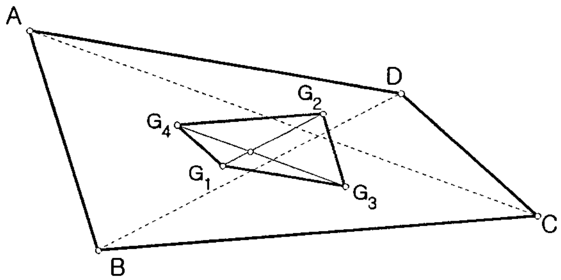


[Figure 4] The balancing point of a quadrilateral

Finally, students were able to find one method and very much pleased with the function of GSP. Especially, they were never asked to extend the idea of centroid of a triangle and did not try to think of centroid more extensively. We did not have enough time to come up with proof why this method is correct. The proof is fairly long and will not be presented here. Actually, it is too difficult for students participated in this program to prove why this is the correct method with their mathematical knowledge at this point. But, all students were invited to print out one correct construction from computer, glue it to the paper hard board, cut the paper hard board along the line, and hold the board with a pencil by locating it at the sharp end of pencil. It really did not stay balanced, but worked well when they used a little bit wider end such as the end of marker. Since each shape of a quadrilateral that they cut was distinct from student to student, they were satisfied with seeing that the method is correct for various types of quadrilaterals. We talked about what we did so far and discussed the power of technology and what we could do with technology. We believe that this type of experience was helpful to open students' eyes toward mathematics.

IV. An experience of new discovery with GSP

There were still ten minutes left. The instructor simply connected the four centroids of four triangles in a quadrilateral. At the moment, we saw something new that we had never recognized before. We could not believe what we were seeing. We were so much excited by ourselves and asked students by moving around one vertex of an original quadrilateral if they saw anything special on the screen. Two or three students said that "A similar quadrilateral to the original" after about fifteen seconds. Two quadrilaterals, the original quadrilateral and the constructed one, were similar (Figure 5), which we named "Joy's Similar Quadrilaterals."These relation holds for any type of quadrilaterals. We did not have enough time to discuss this new discovery in detail, which works for both concave and convex quadrilateral. But, it was very special moment to us as well as to students from the beginning of the program up to the moment of the end of classes. We used GSP for many years and this activity many times. But, we could not see what we saw during this class. All the instructor did was to simply connect all four centroids, which were used to locate the balancing point of a quadrilateral using two segments. He did not do so in order to check any conjecture and had nothing in his minds when connecting them. A simple try lead us to the new discovery, which is the precious fruit that technology can bring to us.



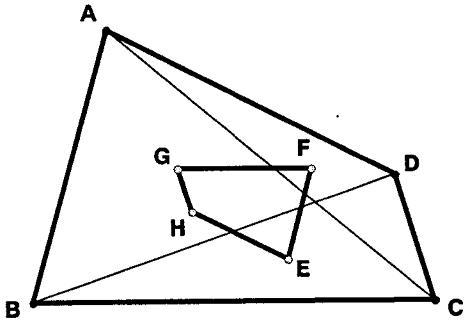
[Figure 5] Joy's Similar Quadrilaterals

V. Joy's similar quadrilaterals

For an arbitrary triangle ABC, three medians all come together in one point, called the centroid of the triangle ABC. This is the very classical concept for an arbitrary triangle. I would like to present the newly found relation for an arbitrary quadrilateral along with simple proofs based on the concept of centroid of a triangle for an arbitrary quadrilateral.

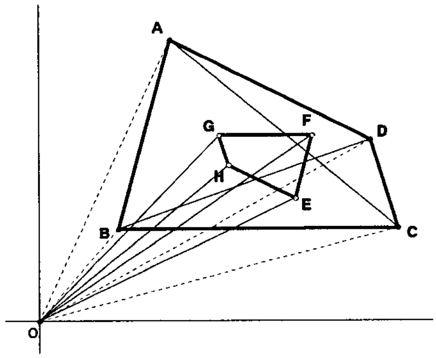
Theorem (Joy's Similar Quadrilaterals)

For any quadrilateral, □ABCD, on a plane, connect two vertices, A & C, and then find centroids, H & F, of each triangle, △ABC & △ACD, respectively. Next, connect two vertices, B & D, and then find centroids, E & G, of each triangle, △BCD & △ABD, respectively. Then, the quadrilateral, □ABCD, and the quadrilateral, □EFGH, are similar quadrilaterals (Figure 6).



[Figure 6] Construction of Joy's similar quadrilateral

Proof)
Method 1



[Figure 7] Approach to the proof with vectors

Let $\vec{A}$ denote the position vector of the vertex, A. I will name the position vectors of each vertex in this manner with respect to some origin. (It doesn't matter where the origin is.) In vector terms, the centroid of a triangle is the average of the vertices.

Thus we have $\vec{E}=\frac{1}{3}(\vec{B}+\vec{C}+\vec{D})$ $\quad \vec{F}=\frac{1}{3}(\vec{A}+\vec{C}+\vec{D})$

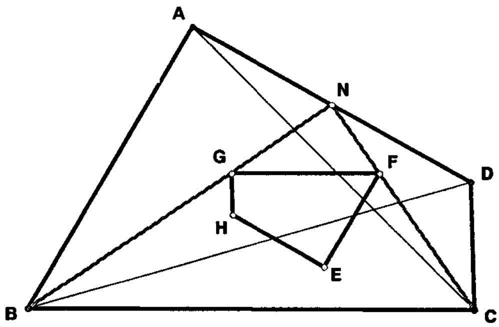
$$\vec{G}=\frac{1}{3}(\vec{A}+\vec{B}+\vec{D}) \qquad \vec{H}=\frac{1}{3}(\vec{A}+\vec{B}+\vec{C})$$

Therefore $\overrightarrow{EF}=\vec{F}-\vec{E}=\frac{1}{3}(\vec{A}-\vec{B})=\frac{1}{3}\overrightarrow{BA}$ $\qquad \overrightarrow{FG}=\vec{G}-\vec{F}=\frac{1}{3}(\vec{B}-\vec{C})=\frac{1}{3}\overrightarrow{CB}$

$$\overrightarrow{GH} = \vec{H} - \vec{G} = \frac{1}{3}(\vec{C} - \vec{D}) = \frac{1}{3}\overrightarrow{DC} \qquad \overrightarrow{HE} = \vec{E} - \vec{H} = \frac{1}{3}(\vec{D} - \vec{A}) = \frac{1}{3}\overrightarrow{AD}$$

Thus, the sides of the little quadrilateral, □EFGH,are parallel to the corresponding sides of the big quadrilateral, □ABCD, and further the ratios of corresponding sides are constant as 1:3. Therefore, □EFGH is similar to □ABCD.

Method 2



[Figure 8] Synthetic approach to the proof

Let N be the mid-point of the segment $\overline{AD}$. Since G is the centroid of △ABD, G is on $\overline{NB}$ and the ratio of $\overline{NG}$ & $\overline{NB}$ is 1 : 3. Also, since F is the centroid of △ACD, F is on $\overline{NC}$ and the ratio of $\overline{NF}$ & $\overline{NC}$ is 1 : 3. Here, let N be the mid-point of the segment $\overline{AD}$. Since G is the centroid of △ABD, G is on $\overline{NG}$ & $\overline{NB}$ is 1 : 3. Also, since F is the centroid of △ACD, F is on $\overline{NC}$ and the ratio of $\overline{NF}$ & $\overline{NC}$ is 1 :3. Hence △FNG & △CNB are similar. Thus, we know that $\overline{GF}$ & $\overline{BC}$ are parallel and its ratio is 1 : 3.

We can show that all corresponding sides are parallel and ratios are 1 : 3 respectively in the same manner. Therefore, □EFGH is similar to □ABCD.

Corollary

The ratios of corresponding sides 3 : 1 and the ratio of circumferences of ABCD and EFGH is 3 : 1 and the ratio of areas is 9 : 1.

Proof)

These are clear from the proof of the theorem of Joy's Similar Quadrilateral.

VI. Discussion

We agree with NCTM standards in many parts. Here, one of them that we would like to mention is the issue about how a technology can and should be used, but not what we have in our classrooms. Not only computers or calculators themselves, but also such a lot of information is available to us with the phenomenal growth in number of WWW servers and network (Flake, 1996). Now, it is the time that we should be wise enough to choose what we can be benefited and take full advantage of what we have because there are so many tools and much information that are there. Jung (1999) found that students liked GSP because it was helpful for accurate and easy construction. Although they did not feel very comfortable in the beginning of the semester, they easily overcame new environments and heavily relied on GSP throughout the semester not just because they had to use GSP, but rather because GSP was helpful and powerful for studying geometry.

We were able to confirm the statement given by NCTM (1989) from this experience: "The thoughtful and creative use of technology can greatly improve both the quality of the curriculum and the quality of children's learning"(p. 19). We admit that the group of students who participated in was gifted or talented with mathematics. But, they were also taught in the traditional environments and systems. We do not use calculators up to 12th grade and it is also very rare to see college students use calculators for studying mathematics in Korea. We could tell that those talented students were confident of mathematics. They clearly memorized what centroid is and several properties such as the centroid cuts the median into 2:1 and the areas cut by medians are the same. But, they seemed to be embarrassed when the instructor asked them to find the balancing point of an arbitrary quadrilateral. These were used to solve a problem using the concept of centroid of a triangle, which they simply memorized from the textbook or the mouth of the teacher.

The situation of the program was not the same as the environments of normal classrooms. In general there is a curriculum proposed by the government that we had to follow for the period of school year and each class was designed to teach 45 minutes for middle school and 50 minutes for high school. Jun & Joo (1998) found that technology has lots of potentials (Clements & Battista, 1994) for learning mathematics, but admitted that a teacher should decide either if he/she follows the flow of the current curriculum with the use of software for the limited time over the classes or uses software as much as needed by changing the whole curriculum. Although we have fine technology with us, it seemed that curricula and educational systems did not support the use of technology effectively. In addition, the class with technology requires Open-Ended Learning Environments [OELE's] (Hannafin, Hill, & Land, 1997), which provide "interactive complementary activities that enable individuals to address their unique learning interests and needs, examine content at multiple levels of complexity, and deepen understanding" (p. 94). This tells that we should consider the change of current curricula and systems so that technology can play a role as we expect it to.

We recognize the potentials of technology and new environments that technology-based classroom may bring, nevertheless, it is still hard for us to bring a technology into a classroom and have students take advantage of it for learning mathematics with understanding. We experienced mathematics that technology could help them to open their eyes and deepen their understanding. Especially, it was precious experience to see that they recognized the power of a tool so that they could use it for further exploration with what they already knew. We learned that students'viewing mathematics through technology-based environments and different approaches to mathematical concepts were changing. Due to the limited time given to us, we could not discuss what they found and we found enough. But, we consider the time that we had together is very precious. Although the direction that they came up with at first was not correct, their mathematical knowledge and spatial visualization helped them to proceed toward the correct answer. In addition, we were able to find something new with the help of technology. We might see different aspects for the other groups of students. However it is for sure that we can do something with a new tool for mathematics. We doubt that the development of curriculum and educational systems can catch that of technology. But, we do believe that we can get benefited a lot for learning mathematics with understanding by appropriately embracing technology that we have at present.

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테크놀로지 환경에서의 수학적 발견 탐구학습 : Joy의 닮은 사각형

정인철[2)]

초 록

1980년 LOGO 이후로 테크놀로지의 활용에 대하여 논의가 지속되어 왔다. 교수학습 상황에서의 테크놀로지의 역할은 무엇인지, 테크놀로지가 학습자들의 효과적인 이해를 위해서 어떤 역할을 학습자들에게 제공할 수 있는지, 그리고 특히 전통적인 교수학습 상황과는 달리 테크놀로지를 활용하여 과거에는 할 수 없었던 수학학습이라든지 또는 우리가 현재 가지고 있는 지식의 확장을 가능하게 한다는 측면에서의 논의가 수학교육계에서는 늘 있어 왔다. 본 논문은 테크놀로지를 활용하여 우리의 지식을 확장하여 탐구를 배경으로 하여 새로운 수학적 지식의 발견의 한 실례를 소개하고 탐구를 중심으로 한 수학학습에 대하여 논한다.

주요 용어 : 테크놀로지, GSP, 지식의 확장, 탐구를 통한 수학적 발견

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