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부분적으로 반복되는 프로젝트를 위한 프로젝트 내·외 학습을 이용한 프로젝트기간예측과 위험분석

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Project Duration Estimation and Risk Analysis Using Intra-and Inter-Project Learning for Partially Repetitive Projects

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■ Abstract ■

This study proposes a framework enhancing the accuracy of estimation for project duration by combining linear Bayesian updating scheme with the learning curve effect. Activities in a particular project might share resources in various forms and might be affected by risk factors such as weather. Statistical dependence stemming from such resource or risk sharing might help us learn about the duration of upcoming activities in the Bayesian model. We illustrate, using a Monte Carlo simulation, that for partially repetitive projects a higher degree of statistical dependence among activity duration results in more variation in estimating the project duration in total, although more accurate forecasting is achievable for the duration of an individual activity.

Keyword : Project Duration Estimation, Linear Bayesian Learning, Learning Curve Effect, Risk Analysis, Influence Diagram

1. Introduction

A considerable amount of effort has been devoted by project management researchers seeking to provide better problem-solving techniques. The question of how to improve the estimation of

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project duration has been one of the most important issues in project management. A large number of extensions have attempted to improve the accuracy of activity duration estimation and resource planning by using the learning curve effect [1, 2, 20, 22].

Consider a set of similar subprojects that share a group of identical activities. For example, in the Beaver Creek Resort Project in Colorado, more than 1,000 units of condominiums and townhouses are to be built. Each buildup can be regarded as a subproject and thus the total project consists of many similar or identical subprojects [3]. For partially or completely repetitive projects such as apartment building or airplane construction, incorporating the learning curve effect would be useful.

However, learning has been recognized in a narrow sense because it could be applied strictly to repetitive projects only. In other words, learning is confined to between projects only. Note that most projects are unique [18]. Thereby, we can hardly take advantage of the learning curve effect for projects in general. For instance, when two adjacent activities in a project share the same resources, we can model neither statistical dependence nor inter-learning with the learning curve. Within-project learning is not possible in the traditional learning curve model. Chatzoglou and Macaulay [4] emphasized the needs of a new approach that can re-estimate the project's variables upon the release of new information, by pointing out the existing models' inadequacy for a software development project. Learning should be feasible for both an individual project and multiple projects for a better modeling.

Research and development (R&D) projects such as software development, information system networking, or space shuttle construction, can be characterized by a high level of uncertainties. Since R&D projects are inherently novel, it is hard to estimate cost and time requirements for each activity and, as a result, the expected duration of a project is only a rough estimate [7]. Scientific and technological feasibility issues may be discovered in the middle of a project. New technologies with competitive advantages may be introduced into the market where typical advantages include cheaper cost, higher performance, and wider compatibility with other systems. Customer requirements and specifications might also change. Project managers may then be forced to modify their original courses of action.

A common weakness of most existing approaches is the assumption that activity durations in a project are statistically independent. Assume that a preceding-succeeding pair of activities is carried out by a newly formed work force. It is not easy to estimate the team's performance accurately in advance. Suppose the preceding activity took longer than expected. Then, the delay might stem from insufficient labor skill of the team, which was overestimated upon planning. This might reveal the information such that the succeeding activity is highly likely to take longer than firstly expected. Similar examples can be found in activities that share other types of resources, such as common raw materials, oil or electricity, and common equipment/facilities. Another major source of dependence among activity durations in a project might be common risk factors. For example, weather or financial risk might influence some of or all activity durations. Several attempts have been made to model dependence structure among activity durations in a project [6, 7, 14, 23]. More recently, Virto et al. [24] and Diaz et al. [8] suggested a sequential project estimation using Markov Chain Monte Carlo simulation technique. A Bayesian approach was adopted by Cho and Covaliu [5] for sequential resource allocation problem in projects.

This study proposes a hybrid model to estimate activity duration as well as project duration using inter-project learning and intra-project learning in the Bayesian scheme. We illustrate, through modeling processes and risk analysis, that our model is more practical and realistic than other existing approaches currently in use.

2. Review of Learning Curve Effect : Inter-Project Learning

It is essential to apply the learning curve effect for a set of projects that are somewhat repetitive. As a set of similar projects are serially accomplished, we might recognize a pattern about the duration of identical activities between projects. Hence, we define it as inter-project learning. We can apply the learning curve effect into the context of project management in the following way. As an activity is repeatedly carried out, activity duration, on average, decreases according to a certain pattern.

$$T_i^{(X)} = T_i^{(1)} X^l \qquad (1)$$

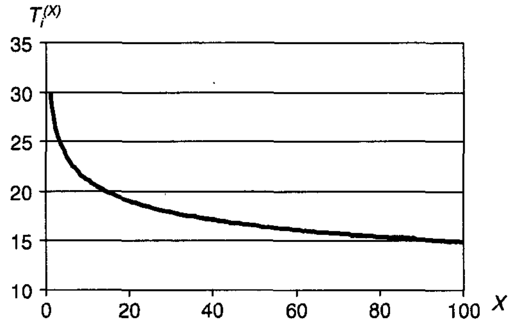
$T_i^{(X)}$: cumulative average duration of Activity i for Xth repetition

$T_i^{(1)}$: duration of Activity i for the first repetition

X : Xth repetition

l : learning curve exponent [$l = ln$ (% of learning) / ln 2]

At a learning rate of 90%, e.g., the cumulative average time of Activity i would decrease as Activity i is repeatedly pursued. Note that l is - 0.1520 given a 90% learning rate. For example, suppose Activity i takes 30 days for its first repetition. For the second repetition, the expected duration of Activity i would be 27 days. For the fourth repetition, it would be 24.3 days. Similarly, it decreases in an exponential way (see [Figure 1]).



[Figure 1] Activity duration as a function of project repetition

3. Bayesian Model for Intra-Project Learning

As discussed in Section 1, we might learn about dependence structure among activities, activity duration in particular, as long as activities share resources or lie under the influence of common risk factors. Compared to inter-project learning in the previous section, we define intra-project learning as learning that can possibly be occurred between activities within a particular project. We propose a framework for intra-project learning using a Bayesian scheme.

3.1 Graphical Model

For the sake of simplicity, consider a simple two-activity in-serial project. Conventionally, in

PERT-type network representation we have been using precedence diagrams to illustrate precedence relationships between activities. [Figure 2(a)] depicts that Activity I must be completed before the onset of Activity J. This study newly employs influence diagram as a graphical modeling tool in order to model a probabilistic dependence between activity durations (see [21] for a review of influence diagram). We model the dependence structure using influence diagram, in [Figure 2(b)], such that the duration of Activity J (T_j) might depend on that of Activity I (T_i).

3.2 Linear Bayesian Model

In the full Bayesian scheme, we update the probability distribution using entire probability mass or density function, which often results in heavy computational work and difficulty in obtaining analytic solutions. A few studies introduced linear Bayesian method in a way to relieve computational complexity inevitably coming along with the full Bayesian approach [9, 10]. We formulate Bayesian updating scheme using the Hartigan's linear Bayes' theorem [12] where the first two moments of a probability distribution are directly obtained. For random variables T_i and T_j, we set the linear equation for the approximate expectation of T_i given T_j such that $\hat{E}[T_i \mid T_j] = cT_j + d$ where c and d are chosen to minimize the variance of the equation. Then the conditional variance of T_j given T_i, $V[T_j \mid T_i]$ can be obtained from the combination of marginal variance and present data variance.

$$\frac{1}{V[T_j \mid T_i]} = \frac{1}{V[T_j]} + \frac{c^2}{V[T_i \mid T_j]} \quad (2)$$

Next, we can also obtain the estimation of the conditional mean of T_j given T_i by averaging its marginal mean and present data by the weight of variances in Eq. (2) such that :

$$\hat{E}[T_j \mid T_i] = \hat{E}[T_j] \frac{1/V[T_j]}{1/V[T_j \mid T_i]} + \left(\frac{T_i - d}{c}\right) \frac{c^2/V[T_i \mid T_j]}{1/V[T_j \mid T_i]} \quad (3)$$

3.3 Example of Bayesian Updating

Let us specify the distribution of activity duration such that it follows Normal distribution with the following means and variances (unit : days) and additionally assume the correlation coefficient of T_i and T_j as follows :

$$T_i \sim N[20, 5^2],\ T_j \sim N[30, 6^2],\ \rho_{ij} = 0.5. \quad (4)$$

By using Eqs. (2) and (3), we can update the distribution of T_j given T_i as follows :

$$T_j \mid T_i \sim N[18 + 0.6T_i, 5.20^2].$$

For example, if Activity I takes 20 days as expected, the duration of Activity J remains unchanged as before, 30 days. If Activity I takes

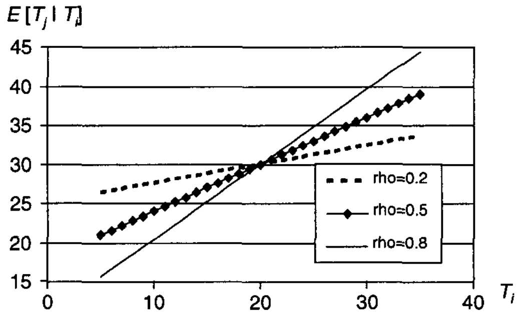
(a) Sequence of activities (b) Dependence of activity durations

[Figure 2] Precedence diagram and Influence diagram for the two-activity in-serial project more time than expected, e.g., 25 days, Activity J is expected to take 33 days, which is greater than the marginal expected duration 30 days. If Activity I takes less time than expected, e.g., 15 days, Activity J is expected to take 27 days, less than the marginal expected duration.

Let us further examine the effect of correlation coefficient on the estimation of the duration of remaining activities given that of completed activities. Consider ± 0.3 bounds from the correlation coefficient of 0.5. For a low correlation coefficient (0.2) and a high correlation coefficient (0.8), the conditional distribution of the duration of Activity J given the duration of Activity I would be :

for $\rho_{ij}=0.2, T_j \mid T_i \sim N[25.2+0.24T_i, 5.88^2]$.

for $\rho_{ij}=0.8, T_j \mid T_i \sim N[10.8+0.96T_i, 3.60^2]$.



[Figure 3] The conditional expected duration of Activity J given the duration of Activity I as a function of correlation coefficients

[Figure 3] depicts how the conditional expected duration of Activity J given the duration of Activity I changes according to correlation coefficients. As the correlation coefficient gets higher, we can identify a stronger impact on the expected duration of Activity J by the observed duration of Activity I. For example, if Activity I takes less time than expected, Activity J tends to take far less time than marginally expected as the correlation increases. On the other case when Activity I takes more time than expected, Activity J tends to take even more time as the correlation increases.

4. Hybrid Model with Intra- and Inter- Project Learning

Now we combine the linear Bayesian model with the learning curve. Upon observing the duration of Activity I for the first project, we can update the expected duration of Activity I for the Xth project as follows :

$$E[T_i^{(X)}] = T_i^{(1)} X^I \quad (5)$$

Next, we can obtain, for Activity J of the Xth project, the conditional variance of T_j given T_i using Eq. (2) and then estimate the conditional expectation of T_j given T_i using the following Equation :

$$\hat{E}[T_j^{(X)} \mid T_i^{(X)}] = \hat{E}[T_j^{(X)}] \frac{1/V[T_j]}{1/V[T_j \mid T_i]} + \left(\frac{T_i^{(X)}-d}{c}\right) \frac{c^2/V[T_i \mid T_j]}{1/V[T_j \mid T_i]} \quad (6)$$

4.1 Simulation Example

To illustrate the modeling processes and evaluate the effect of learning curve effect and Bayesian updating, consider a project that consists of ten subprojects each of which has two in-serial activities and subprojects will be carried out in a serial manner. Note that if both activities are common for all subprojects, they are not projects any longer, but a routine work. So it is reasonable to assume only a part of subproject is identical. With the above reasoning, only preceding Activity A is common for all ten subprojects. The learning rate for Activity A is assumed to be 90%. Activities for each subproject and marginal distribution of activity duration are described in <Table 1>. For a clear evaluation of learning curve effect and Bayesian updating, the marginal distribution of duration of succeeding activities of subprojects are set to equal to each other, and so do the correlation coefficients as 0.5.

We generated the posterior samples of project duration using a Monte Carlo simulation under the following four estimation models. After 50,000 iterations we compared the differences in the expected duration and its variation. The four estimation models are as follows.

- Naïve model

Neither learning curve effect nor Bayesian updating are reflected in the Naïve model. Thus the duration of all activities is generated from the marginal probability distribution described in <Table 1>.

- Learning curve model

This model allows learning for identical activities of subprojects. The marginal distribution of Activity A for all subprojects is generated using Eq. (5) for the mean and 5 for the standard deviation. The duration of succeeding activity is generated using the marginal distribution in <Table 1>.

- Bayesian model

In the Bayesian model, the conditional distribution of succeeding activity is generated using Eq. (6), in that the mean duration of succeeding activity for each subproject is conditioned on the observed duration of preceding activity duration and the standard deviation is fixed to 5.20. For the duration of preceding activity, the marginal distribution is used as in <Table 1>.

- Hybrid model

This model allows learning curve effect as well as Bayesian updating. For preceding activ-

<Table 1> Data assumed for the ten subprojects

Subproject	Preceding activity	Succeeding activity	Duration of preceding activity	Duration of succeeding activity	Correlation coefficient
1	A	B	$T_A \sim N(20, 5^2)$	$T_B \sim N(30, 6^2)$	$\rho_{AB} = 0.5$
2	A	C	$T_A \sim N(20, 5^2)$	$T_C \sim N(30, 6^2)$	$\rho_{AC} = 0.5$
3	A	D	$T_A \sim N(20, 5^2)$	$T_D \sim N(30, 6^2)$	$\rho_{AD} = 0.5$
4	A	E	$T_A \sim N(20, 5^2)$	$T_E \sim N(30, 6^2)$	$\rho_{AE} = 0.5$
5	A	F	$T_A \sim N(20, 5^2)$	$T_F \sim N(30, 6^2)$	$\rho_{AF} = 0.5$
6	A	G	$T_A \sim N(20, 5^2)$	$T_G \sim N(30, 6^2)$	$\rho_{AG} = 0.5$
7	A	H	$T_A \sim N(20, 5^2)$	$T_H \sim N(30, 6^2)$	$\rho_{AH} = 0.5$
8	A	I	$T_A \sim N(20, 5^2)$	$T_I \sim N(30, 6^2)$	$\rho_{AI} = 0.5$
9	A	J	$T_A \sim N(20, 5^2)$	$T_J \sim N(30, 6^2)$	$\rho_{AJ} = 0.5$
10	A	K	$T_A \sim N(20, 5^2)$	$T_K \sim N(30, 6^2)$	$\rho_{AK} = 0.5$

<Table 2> Simulated statistics of the four estimation models

Duration		Naïve model		Learning curve model		Bayesian model		Hybrid model	
		Mean	std. dev.	mean	std. dev.	mean	std. dev.	mean	std. dev.
Total project	T	500.03	24.77	459.78	24.71	500.54	22.74	459.59	22.89
Subproject	T_1	49.98	7.83	50.06	7.83	51.06	7.22	52.18	7.20
	T_2	50.04	7.82	48.03	7.79	47.97	7.19	48.00	7.20
	T_3	49.99	7.82	46.86	7.81	50.29	7.20	46.92	7.19
	T_4	50.00	7.78	46.19	7.80	50.86	7.18	45.69	7.21
	T_5	50.00	7.78	45.63	7.81	49.17	7.18	43.43	7.22
	T_6	50.01	7.80	45.19	7.80	49.23	7.20	45.09	7.20
	T_7	49.96	7.81	44.83	7.85	50.40	7.24	45.88	7.19
	T_8	50.01	7.81	44.57	7.82	50.75	7.21	43.22	7.17
	T_9	50.00	7.85	44.30	7.81	49.37	7.18	45.16	7.23
	T_{10}	50.04	7.84	44.13	7.82	51.35	7.21	44.03	7.17

ities, we generate the duration using Eq. (5) for the mean and 5 for the standard deviation. Conditioned on the generated duration of preceding activity, the duration of succeeding activity is generated using Eq. (6) for the mean and 5.20 for the standard deviation.

The simulated results are displayed in <Table 2>. There are a few findings from the experiment. First, the two models without learning curve effect, i.e., the Naïve model and the Bayesian model, have, on average, around 500 project duration days. On the other hand, the average duration is reduced to about 460 days for the other two models, the Learning curve model and the Hybrid model, that allow learning rate of 90%. It is obvious for the two models with learning curve effect to have shorter project duration as subprojects continue to proceed.

Secondly, it is worth to pay attention to the risks associated with the estimation of project duration. In the Bayesian scheme, we update not only the expected duration of succeeding activity but also its variation. Its conditional variance gets smaller than the marginal variance. It is apparently shown in Eq. (2). Since the conditional precision (precision is 1 over variance) is the sum of the marginal precision and the present data precision, the conditional precision is always greater than the marginal precision. It is important to note that the Bayesian updating gives more precise forecasting about individual activity duration. Accordingly, the Bayesian model and the Hybrid model yield a smaller standard deviation in estimating project duration, as in individual subprojects and projects in total, than the other two models without Bayesian component. Conclusively, it is helpful to adopt Bayesian modeling in reducing the amount of uncertainty associated with estimating project duration.

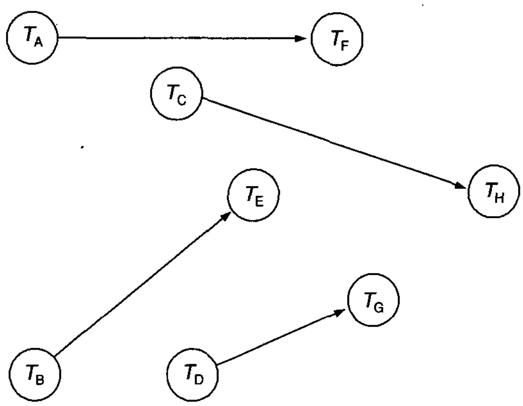
5. Example Project

Further investigation on the effect of learning by the Hybrid model is done using the air pollution control equipment project in Heizer and Render [13]. Activity description and immediate precedence relationships were adopted and for the sake of simulation we assumed the first two moments of the distribution of activity duration as in <Table 3> (duration unit : days). Among the eight activities in the project, Activities A, B, C, and D are assumed to be repeatedly carried out project after project, and other four activities are treated as changing into different entities that, although, have the same marginal distribution.

We assumed that resource sharing among activities is planned between Activities A and F, between Activities B and E, between Activities C and H, and between Activities D and G. The effect of resource sharing plan is reflected in the influence diagram, in that statistical dependence exists between activity duration as illustrated in [Figure 4]. It is well known that the project duration depends on the duration of the critical path of the networks, which can be easily identified in precedence diagrams. In our case, the project duration is computed by the longest duration path such that :

$$T = \max\{T_A + T_C + T_F + T_H,\ T_A + T_C + T_E + T_G + T_H,\ T_B + T_D + T_G + T_H\}$$

We assumed that the project is repeatedly carried out five times with the four identical activities as mentioned above. After 50,000 Monte Carlo simulations, sample statistics about the project duration are obtained.



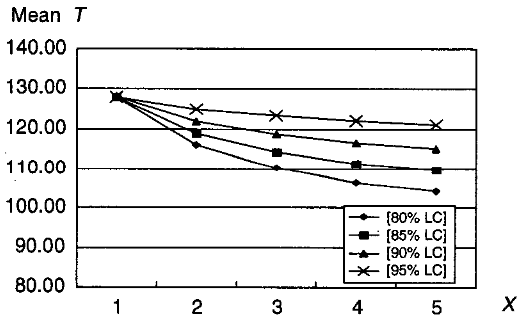
[Figure 4] Influence diagram for the air pollution control equipment project

Given controlling correlation coefficients, 0.6 in our experiment, for all statistical dependence relations described in the influence diagram, themean project duration for each of five com-

<Table 3> Activity description about the air pollution control equipment project

Activity	Description	Immediate predecessors	μ_i	σ_i^2
A	Build internal components	-	40	8^2
B	Modify roof and floor	-	30	5^2
C	Construct collection stack	A	15	4^2
D	Pour concrete and install frame	B	40	6^2
E	Build high-temperature burner	C	20	4^2
F	Install control system	C	35	4^2
G	Install air pollution device	D, E	30	8^2
H	Inspection and testing	F, G	20	3^2

pletions of the projects is computed and illustrated in [Figure 5]. It is natural for the effect of learning curve not to be happened for the first completion of project. For X = 1, the mean project duration approximately equals to each other regardless of learning rate. For the subsequent completion of project, the mean project duration continuously goes down owing to the accumulation of learning effect. This tendency becomes stronger as the learning rate decreases from 95% to 80%.



[Figure 5] Mean project duration under various learning rates

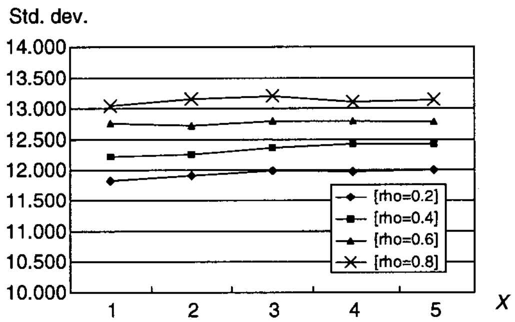
Whereas the learning curve effect of the Hybrid model can be identified in terms of the expected project duration, the effect of the Bayesian updating can effectively be shown on the perspective of uncertainty involved with estimating work. Given the learning rate of 90%, we compute the standard deviation of simulated project duration by substituting several correlation coefficients.

A sample correlation coefficient can be easily calculated for the case where historical data were accumulated. However, such statistical data accumulation is practically impossible and meaningless in project management. This is the reason why time estimation in projects has been entirely depended on the subjective judgment. Assessing a correlation coefficient directly is quite difficult even for professionals in the area of stochastic modeling. Like the derivation of utility or risk attitude, the correlation coefficient can be indirectly assessed. There were some studies dealing with the problem of assessing subjective expectations and probabilities [15, 16].

A good reference of deriving a correlation coefficient would be a study by Gokhale and Press [11]. By assuming a bivariate normal density for a pair of activities, we can obtain the concordance probability. The concordance probability for two random variables is defined by the degree of tendency of shifting in the same direction. For example, for two activities A and B, the project manager and field experts should assess the fraction of the cases that the duration of activity B will be longer than its expected duration, given that the duration of activity A is longer than its expected duration. As a next step, we can inversely estimate the correlation coefficient from the given concordance probability using the following equation, in that a one-to-one relationship exists between the correlation coefficient and the concordance probability :

$$\text{Concordance probability} = \frac{1}{2} + \frac{1}{\pi}\arcsin\rho$$

The four correlation coefficients that can be identified in the influence diagram are to be changed simultaneously from 0.2 to 0.8 by an increment of 0.2 for examining the impact of the degree of statistical dependence on estimating project duration. As the degree of statistical dependence increases, we can identify an increased level of risks associated with estimation (see [Figure 6]). This phenomenon might stem from the following reasoning. As the correlation coefficient increases, the conditional distribution of the duration of upcoming activities becomes less variable. In other words, we end up with a more precise probability distribution. The longer the preceding activity took, the greater the succeeding activity tends to take longer. The shorter the former, the greater the latter tends to take shorter.



[Figure 6] Standard deviation of project duration under various correlation coefficients

For better understanding, let us explain the impact of correlation coefficient on the individual activity duration and the total project duration, using the following example. As shown in [Figure 4], Activities D and G are correlated with each other. Consider the four cases where the correlation coefficient is either 0.2 or 0.5 and the duration of Activity D is observed as either 30 days or 50 days. The conditional duration of Activity G can be obtained using Eqs. (2) and (3), which is as follows :

Given $\rho_{DG} = 0.2$:

If $T_D = 30$, then

$$(T_G \mid T_D = 30) \sim N(27.30, 7.84^2) \quad (7)$$

If $T_D = 50$, then

$$(T_G \mid T_D = 50) \sim N(32.67, 7.84^2) \quad (8)$$

Given $\rho_{DG} = 0.5$:

If $T_D = 30$, then

$$(T_G \mid T_D = 30) \sim N(23.33, 6.93^2) \quad (9)$$

If $T_D = 50$, then

$$(T_G \mid T_D = 50) \sim N(36.67, 6.93^2) \quad (10)$$

First, let us identify the impact on the individual activity duration. By comparing (7) to (9) or alternatively (8) to (10), we can see the impact of correlation coefficient on the variation of the duration of upcoming activity. As correlation coefficient increases, the conditional variance for the upcoming activity decreases from 7.84^2 to 6.93^2. This implies that for increasing correlation, forecasting on the level of an individual activity becomes more precise.

Second, let us see the impact on the total project duration by comparing the total duration of Activities D and G. If Activity D took 30 days, the total expected duration of Activities D and G would be, from (7) and (9) :

Given $\rho_{DG} = 0.2$: $T_D + E[T_G \mid T_D = 30] = 57.30$ days.

Given $\rho_{DG} = 0.5$: $T_D + E[T_G \mid T_D = 30] = 53.33$ days.

If Activity D took 50 days, the total expected duration of Activities D and G would be, from (8) and (10) :

Given $\rho_{DG} = 0.2$: $T_D + E[T_G \mid T_D = 50] = 82.67$ days.

Given $\rho_{DG} = 0.5$: $T_D + E[T_G \mid T_D = 50] = 86.67$ days.

Depending on whether Activity D took 30 days or 50 days, the range of the total duration would be 25.37 days (82.67-57.30) for the case of

$\rho_{DG}=0.2$, whereas the range would be 33.34 days (86.67-53.33) for the case of $\rho_{DG}=0.5$. This implies that it is more likely for the succeeding activity to end up with a different duration compared to its marginal expected duration for increasing correlation. Therefore, the variation of the total project duration increases as the correlation coefficient increases.

In summary, higher statistical dependence among activity duration results in more variation in estimating the project duration in total, although we can attain a more precise forecasting for the duration of an individual activity.

6. Conclusion

Today projects are often planned and managed in an inefficient way, resulting in higher costs. In the future, undoubtedly even more uncertainty will be involved in the management of projects because of many reasons such as rapidly-changing technology and the increased complexity of the projects. Clearly, the question of how to deal with uncertainty is increasing in significance. It is essential to develop a more realistic and coherent methodology than those currently available for scheduling and controlling projects, so we can apply it to projects in the real world.

This study contributes to problem solving with respect to estimating project duration in the area of project management. There have been two avenues with respect to how to deal with project duration. One cohort of studies has mainly used the learning curve effect. In this case, there exists a theoretical flaw, a deterministic approach to activity duration, which is manifestly unrealistic. More seriously, this approach is applicable only to a set of completely repetitive projects. The other cohort has used a stochastic approach. They used MCMC simulation techniques or full Bayesian approaches, which inevitably encompass computational complexity. These studies were purely academic since it requires highly professional knowledge and skills in terms of stochastic modeling.

On the other hand, the Hybrid model proposed in this study firstly combines two kinds of learning, inter-project learning and intra-project learning, in the area of project management, and overcomes the weaknesses of the above two approaches. In particular, intra-project learning is modeled on the basis of a linear Bayesian scheme. Due to the substantial reduction in computational load, the linear Bayesian updating model can be used to field managers with a minimal level of understanding of the formula. For those pairs of activities that share resources or risks, by combining old information with the newly available information, we can update the duration of upcoming activities. The old information implies the marginal activity duration prior to the onset of project initiation, whereas the newly available information means the observed duration of completed activities. The degree of association between a pair of activities is adequately captured by a correlation coefficient, which might be derived from a concordance probability. These updating processes continue repeatedly from the onset of a project throughout the entire project period. This study explicitly models these processes, which is really happening in the real world projects. In conclusion, this study contributes to the field of project management by introducing an estimation model in a more practical and realistic way.

It might be worthwhile to announce the limitation of this study. We separately treat the learning curve effect from the Bayesian learning. The former is applied to recurrent identical activities between projects. The latter is applicable for non-recurring activities that share resources or risks with preceding activities within a particular project. Future study might seek a way to apply both learning on the same activities that are recurrent and resource-sharing with other recurrent activities. Another possible extension of study would be to develop a framework for assessing the degree of dependence, which, we asserted in our study, usually exists among activities through resource sharing or being influenced by common environmental factors. Research on such topics may contribute to more efficient management of projects and conservation of resources than has been possible with techniques currently in use.

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