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# 가격보호 정책, 반품 정책과 물량할인 정책을 사용한 3단계 공급사슬의 협력방안

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# Coordination Under Price Protection, Mid/End Life Returns, and Quantity Discount for a Three-Level Supply Chain

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■ Abstract ■

The coordination of a three-level supply chain consisting of a supplier, a retailer, and a discount outlet (DCO) is studied here. We assume that the product is sold in two consecutive periods : a Normal Sales Period (NSP) and a subsequent Clearance Salvage Period (CSP). A benchmark case is studied initially in which the supply chain is coordinated by a single agent. Thus, the supplier, the retailer, and the discount outlet design a common system that allows centralized decision making about stocking quantities, markdown time schedules, and policies on disposing of leftovers to deliver the greatest possible expected supply chain profit. Next, we consider a decentralized supply chain. Here, decisions are made without coordination. The objective is to maximize an individual party's expected profits. The focus of the study is on the following questions: what factors make the coordination an effective approach for the supply chain? How do we coordinate the supply chain so as to maximize the supply chain joint expected profit? These and other related study issues are explored in this paper.

Keyword : Supply Chain Coordination, Newsvendor Inventory Control Model

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## 1. Introduction

The importance of coordination in the supply chain has recently been discussed in a considerable body of literature. According to the main argument, while the importance of achieving integration in the supply chain is generally well recognized, designing a sophisticated integrated system for application in the real world is an arduous task. A more realistic route is to design a coordination mechanism (including contractual forms, compensation schemes, and side payments) that aligns the self-interests of individuals with the integrated interests of the supply chain. For example, Jeuland and Shugan [6] suggested using the quantity discount as a mechanism for coordinating a bilateral monopoly channel. Monahan [20] developed a model for establishing an optimal discount schedule from the vendor's perspective, and showed a price discount schedule with a single break point, achieving the desired outcome for the vendor.

Monahan's work has been advanced by Lee and Rosenblatt [16], Kohli and Park [10], Weng [35], and Corbett and Groote [2], among others. Parlar and Weng [22] described coordination between a firm's manufacturing and supply departments. Weng [34] assumed demand to be stochastic and analyzed channel coordination in wholesale and retail prices. More recently, Lee, *et al.* [17] discussed coordination by Wholesale and Retail Price Protection. Wang and Gerchak [32] analyzed a shelf-space dependent demand inventory model, and proposed using a holding cost subsidy as a coordination mechanism. Taylor [28] studied coordination strategies using target-level rebate policies in which a manufacturer gives a performance- based rebate for a retailer when sales exceed a target level.

The seminal work of Pasternack [23] showed that a manufacturer's returns policy can not only induce a larger order through the risk sharing, but also coordinate a supply chain to eliminate the phenomenon of double marginalization and generate the greatest joint profits for the supply chain. Since then, extensions of Pasterneck's basic models have been attempted in many directions. These include extending the model to consider price-sensitive demands [4, 8, 14, 19] and comparing the returns policy with other coordinating mechanisms (a markdown allowance in Tsay [30] and a two-part tariff price only contract in Lariviere [11]), among many others. More recent work has attempted to include the agents' individual agendas and their attitudes toward risk in the basic model [13, 31, 33]. Others have studied the model under a non-Newsboy framework, and showed that a returns policy could stimulate retail competition to benefit the manufacturer [21]. New approaches are also frequently reported. For example, Tsay [29] studied a Quantity Flexibility Contract in which a manufacturer fully rebates a portion of the leftovers (up to the order quantity). Donohue [3] discussed a coordinating returns policy under the assumption that the manufacturer can produce a second production lot after the forecast update. Taylor [27] analyzed a two-period model in which a wholesale price protection policy is employed with mid-life and end-life returns to coordinate the supply chain. This two-returns model was also formulated in Lee [15] to study the returns policy in a Manufacturer-Retailer-Discount Outlet setting.

In this study, along a similar vein, we explore coordination effects in a three-level supply chain in which a supplier, acting as a channel captain coordinates a retailer and a discount outlet. Assume that a retailer sells a "short life-cycle good" (e.g., fashion items) to possible consumers. If the product cannot be sold after the first Normal Sale Period (NSP), the retailer has two available options : (1) she can wholesale the ownership of the leftovers to a downstream discount outlet (DCO), which in turn will try to clean out leftovers in a secondary Markdown Sale Market at the following Clearance Salvage Period (CSP), or (2) the retailer can return the leftovers to the upstream supplier, who will then salvage the buyback leftovers.

Under this problem setting we consider two supply chain models-a centralized supply chain and a decentralized supply chain. In the centralized supply chain the supplier-retailer-discount outlet are coordinated to form an integrated system, and jointly design an integrated ordering and leftovers salvage policy to deliver the greatest possible expected supply chain profit. Thus, the centralized supply chain acts as if the system is managed by a single entity, and designs the most beneficial policy for maximizing the supply chain profit. Any policy different from the one made by the centralized supply chain, while potentially preferable to one party or the other, will most likely lead to system inefficiency. In the decentralized supply chain, ordering and leftovers salvage decisions are made without coordination. The purpose is to address the following managerial questions : (1) what factors, if any, make coordination an effective approach? (2) What are the coordination strategies and policies that maximize the supply chain joint expected profit? The approach used in this paper-comparing and analyzing centralized-coordinated and decentralized-uncoordinated supply chains-is widely applied in supply chain literatures (see, for example, [15, 22, 29]). By comparing and analyzing centralized and decentralized systems, we identify how is the significance of the improvement delivered by centralizing the supply chain, when supply chain efficiency can be improved by centralization, and how this can be achieved. Note that although achieving centralization in the supply chain is generally well recognized, in real-world applications doing so will very likely require enormous managerial efforts. The tradeoff is therefore between the cost of maintaining a complicated centralized system and the costly consequence of operating a simple decentralized system. Hence, when is it worthwhile to centralize the supply chain, and how should this be proceed? In this work, we try to provide answers to these questions.

This paper is structured as follows. In section 2, the problem description, assumptions, and notations are presented. In section 3, we examine the integrated model and design an optimal policy for the benchmark case. In section 4, we consider a decentralized model. In section 5, we study coordination strategies for decentralized systems. A brief discussion in section 6 completes the paper.

## 2. The Modeling Issues

The selling period consists of two sequential and non-overlapping periods - a Normal Sales Period (NSP) managed by the retailer, and a Clearance Salvage Period (CSP) managed by a markdown specialist. The chronology of events for the model is described below :

(1) First, the retailer decides order quantity based on a forecast of the expected demand. The expected demand is closely related to the planned lengths of the NSP and CSP (operation time schedule); thus, to choose an optimal order size the retailer needs to simultaneously plan the time schedule. This scenario is similar to that in Bartmann and Beckmann [1], in which a newsvendor simultaneously chooses an optimal order quantity and a selling period. Notice that the estimated time schedule (the lengths of the NSP and CSP) is information that is private to the retailer, and will only be used by the retailer as an internal aid to decisions on the quantity of the order. It is not a firm commitment to the discount outlet, and can be changed in the later phase of the selling season. We assume that the inventory is allowed to be replenished only once at the beginning of the NSP. When the random demand at any point in time exceeds availability, selling opportunities are lost.

(2) At the end of NSP, the leftovers, if any, are divided between the two agents - the Discount Outlet (DCO) and the supplier. A portion of the leftovers are returned to and salvaged by the supplier (through reuse or remanufacture), and the rest of the leftovers are put into a markdown sale in the CSP by the DCO. We assume that the supplier operates a full returns policy [4, 8, 19, 23] so that the returned quantity is only bounded by the ordered quantity. We also assume that the salvage capacity (markdown sale demands) in the DCO is stochastic and that the supplier's practice of reusing or remanufacturing is nearly unlimited and deterministic. This assumption is an extension of the literature in which the salvage capacity is often assumed to be deterministic and unlimited [30]. In reality, salvage operations frequently involve the use of a mixture of clearance salvage paths. For example, retailers tend to liquidate leftovers in specialized discount outlets (e.g., TJ Maxx) and/or through online business to consumer (B2C) sites such as eBay or Amazon. com, or to entrust leftover merchandise to a third party logistic companies. Each of these paths has a different consumer base, demand characteristics, and distributions ; thus, salvage capacities are more likely to be limited and in many cases probabilistic in nature. On the other hand, supplier/manufacturers commonly use a wide spectrum of technologies to recover various raw materials, to remanufacture, and to repair and reuse from leftovers [7, 26] ; thus, salvage processes are likely to be deterministic and to possess ample capabilities.

The following assumptions and notations are used for modeling purposes. Demand in the NSP and CSP is probabilistic depending on the duration of the sales period, and is assumed to be comprised of two components. The first component, representing the expected demand or the location parameter of the random demand, is influenced by the duration of the sales period. The second component, representing the probabilistic scaling parameter of the random demand, is independent of the duration of the sales period. We define $\tau$ as the exogenously determined total life cycle. We formulate length of the NSP (CSP) as a fraction $0 \le \alpha \le 1$ of $\tau$, i.e., $\alpha\tau$ $((1-\alpha)\tau)$. Let $D_N(\alpha)$ and $D_C(\alpha)$ denote expected demands during the NSP and CSP, respectively. <Table 1> summarizes the additive and multiplicative two components random demands formulated in our model.

〈Table 1〉 Random Demand

| | | |
|---|---|---|
| $D_N(a) = k_N a$ | expected random demand of the retailer in the NSP | |
| $D_C(a) = k_C(1-a)$ | expected random demand of the DCO in the CSP | |
| $k_N$ and $k_C$ | are constant demand rates | |
| $Y$ and $X$ | are probabilistic scaling parameter of the random demands | |
| | Additive | Multiplicative |
| NSP | $y + D_N(a), E(Y) = 0$ | $yD_N(a), E(Y) = 1$ |
| CSP | $x + D_C(a), E(X) = 0$ | $xD_C(a), E(X) = 1$ |

Because of its simplicity and flexibility, this two-component approach has been used in various studies to formulate random demands (see [4, 12, 18, 24] for the price-dependent random demand model, and [5] for random yield models). <Table 2> lists the notations used in this paper.

〈Table 2〉 List of Notations

| | |
|---|---|
| $*P$ | the NSP retail price |
| $*M$ | the supplier's unit manufacturing cost |
| $*C$ | the wholesale price paid by the retailer to the supplier |
| $*W$ | the wholesale price of leftovers paid by the DCO to the retailer at the end of the NSP |
| $*V_S$ | the supplier's salvage values |
| $*V_D$ | the DCO's markdown sale price |
| $Q$ | the retailer's order quantity |
| $a$ | the fraction of the retailer's normal sales period ($\overline{a} = 1 - a$) to the total life cycle |
| $f(y)$, and $g(x)$ | the probability densities of $Y$, $X$ |
| $F(y)$ and $G(x)$ | the cumulative distribution functions ($\overline{F}(y)$ and $\overline{G}(x)$ complementary CDF) |
| $\wedge$ | subtraction (-) and division (/) in additive and multiplicative model |
| $\vee$ | summation (+) and production (×) in additive and multiplicative model |
| $I(Q, a) = \max[Q - (y \vee D_N(a)), 0]$ | the retailer's leftovers at the end of the NSP |
| $\overline{\theta} \in [0, 1]$ | the retained portion of the leftovers ($\theta := 1 - \overline{\theta}$ returned leftovers) |
| $q$ | DCO's optimum(maximum) markdown sales quantity of leftovers |

We also use the following notations for modeling purposes :
$\xi := Q \wedge D_N(a)$, $\delta := (Q - q) \wedge D_N(a)$, and
$\overline{\delta}_i = \{\overline{\delta}_1 := \delta, \overline{\delta}_2 := 0\}$.
$\phi := k_N - k_C$, and $\phi(a) := -ak_C + yk_N$.
$\zeta := \{retained\ quantity = \overline{\theta} I\} \wedge D_N(a)$ with
$\zeta_q := q \wedge D_C(a)$ and $\zeta_I := I \wedge D_C(a)$.

Note) * We assume that (1) $P \geq C \geq M$, (2) $V_D \geq W$, and (3) $V_S \leq V_D$.

## 3. Centralized Supply Chain Model

To provide an efficient benchmark, we consider an integrated system in which the supplier-retailer-DCO form a common system, and jointly design an integrated ordering and leftovers $(Q, \theta)$ salvage policy to deliver the greatest possible expected system profits. As in most previous studies (e.g., [23]), we formulate the problem as a Newsvendor inventory control model (see Porteus [25] for a review) with the objective of maximizing expected profit. Denote $\Omega_R(Q, a) := Q\overline{F}(\xi) + \int_0^{\xi} \{y \vee D_N(a)\} dF$ and $\Omega_D(\theta | I(Q, a), a) := \overline{\theta} I \overline{G}(\zeta) + \int_0^{\zeta} \{x \vee D_C(a)\} dG$ as the expected sales units for the NSP and CSP. The joint objective function of the supply chain is given as follows :

$$\max_{Q, a} \prod(Q, a) = P\Omega_R(Q, a) - MQ + \int_0^{\xi} \max_{0 \leq \theta \leq 1} \{V_S \theta I + V_D \Omega_D(\theta | I(Q, a), a)\} dF. \quad (1)$$

Note that in Pasternack's [23] model, the leftovers have no value to the retailer so she will always want to return as many of them as possible to the supplier; thus, there is no need to decide how much to return. If, however, as in our model, the DCO can receive some salvage value for the leftovers, and if the quantity that the DCO can successfully salvage is stochastic, then the DCO and the retailer clearly have another decision to make - namely, how much to return to the supplier and how much to keep and salvage themselves. Furthermore, with this assumed setup it is more likely that the decision made by the retailer and DCO will be based on the leftovers they have and not what they originally ordered. The objective function in (1) reveals that the retailer and DCO need to base their stocking decision on expected sales, which is a function of how the transfer time ($\alpha$) will be determined. Thus, an optimal policy must solve for an optimal integrated triplet ( $Q^I, \alpha^I, \theta^I$) (the superscript "I" denotes an integrated system). Proposition 1 provides a summary of the optimal policies.

***Proposition* 1.** *(See Appendix 1 for the proof for Proposition 1)*

**1.1.** *$\Pi$ is jointly concave with respect to $(Q, \theta, \alpha)$. Denote $I_1 = Q_1 - (y \vee D_N(\alpha)$. The optimal policies are given as follows :*

$$Cases = \left\{ \text{when } Q_1 < q^I \text{ choose } P_2 = (\alpha_2, \theta_2, Q_2) \right.$$

$$\text{when } Q_1 \geq q^I \text{ choose } P_1 = \begin{cases} I_1 \leq q & P_{1N} = (\alpha_1, \theta_{1N}, Q_1) \\ I_1 > q & P_{1P} = (\alpha_1, \theta_{1P}, Q_1) \end{cases}$$

Where $P_{ij}$ denote optimal policies in which subscripts i = 1:a flexible policy in which the quantity of returns is a function of the quantity of leftovers, and i = 2:a "fixed" no returns policy. For policy type 1, subscripts j=P: partial returns ($0 < \theta \leq 1$), and j = N: no returns.

*The optimal triplet ($Q^I, \alpha^I, \theta^I$) is given as follows :*
*(i) Retained leftovers (markdown quantity) satisfy $\overline{I\theta}^I = q^I = D_C \vee G^{-1}(1 - V_s / V_D)$, and $\theta_{IN}, \theta_2 = 0$.*
*(ii) As in <Table 2>, denote $\xi := Q \wedge D_N(\alpha)$, $\delta := (Q-q) \wedge D_N(\alpha)$, and $\overline{\delta}_i = \{\overline{\delta}_i := \delta, \overline{\delta}_2 := 0\}$. $\phi := k_N - k_C$, and $\phi(\alpha) := -\alpha k_C + y k_N$.*
*Order quantities $Q^I = Q_i$ $i = 1,2$ satisfy $P - M = PF(\xi) - \omega_i$ with $\omega_i = V_s F(\overline{\delta}_i) + \int_{\overline{\delta}_i}^{\xi} V_D \overline{G}(\zeta_I) dF$.*
*(iii) The length of the NSP $\alpha_i$ $i = 1,2$ satisfy*
*Additive :*

$$Pk_N F(\xi) - V_S \phi F(\overline{\delta}_i) - V_D k_C F(\overline{\delta}_i) \{ G(\zeta_q) + \overline{G}(\zeta_q) \} - V_D \int_{\overline{\delta}_i}^{\xi} \{ k_C G(\zeta_I) + k_N \overline{G}(\zeta_I) \} dF = 0$$

*Multiplicative :*

$$Pk_N \int_0^{\xi} y dF - V_s \int_0^{\overline{\delta}_i} \phi(\zeta_q) dF - V_D k_C \int_0^{\overline{\delta}_i} \left\{ \int_0^{\zeta_q} x dG + \zeta_q \overline{G}(\zeta_q) \right\} dF - V_D \int_{\overline{\delta}_i}^{\xi} \left\{ k_C \int_0^{\zeta_I} x dG + k_N y \overline{G}(\zeta_I) \right\} dF = 0$$

**1.2** *Ceteris paribus, the optimal solution : (i) $q^I$ increases as $\overline{\alpha}$ or $V_D$ increases, and $V_s$ decreases. (ii) $Q_1$ increases as $V_D$, $V_S$, or $P$ increases, and M decreases. For the additive model $Q_1$ increases as $\alpha$ increases if $k_N > k_C$. (iii) $\alpha_1$ increases as P increases, and $V_D$ decreases. For the additive model $\alpha_1$ increases as $V_S$ decreases if $k_N > k_C$. For the multiplicative model $\alpha_1$ increases as $V_s$ decreases if $k_N(Q-q) > k_C q$.* □

To provide an efficient benchmark, we assume that a single agent coordinates the supply chain. The optimal policy proposed in Proposition 1.1

delivers the greatest possible expected supply chain profit. Any policy different from the one described in Proposition 1.1, while possibly preferable to one party or the other, will lead to a lower supply chain profit. It reveals that the optimal retain policy is a *retain up to policy*. When $I > q$, that is, when the leftovers are more than the targeted markdown sale quantity, the leftovers are shared between the two parties with retained quantity $q$ satisfying $V_s = V_D \overline{G}(\zeta_q)$ (since $q = D_C \vee G^{-1}(1 - V_s / V_D)$). This involves (1) the likelihood that the agent will clean out the allocated leftovers ($\overline{G}(\_)$) and (2) their salvage values, $V_D$ and $V_S$. At the optimal solution, the expected marginal revenue of retaining one unit of leftovers (= $V_D \overline{G}(\zeta_q)$) must be identical to the marginal cost of giving up the salvaging of one unit of leftovers (= $V_s$). In other words, it satisfies the classical optimality condition of marginal revenue=marginal cost.

Proposition 1.2 (ii) shows that $Q$ increases as $\alpha$ increases if $k_N > k_C$. That is, if the NSP has a higher demand rate ($k_N > k_C$), $Q$ increases as the length of the NSP increases. On the other hand, if CSP has a higher demand rate ($k_N > k_C$), then it is possible that $Q$ will decrease in order to generate fewer leftovers as the markdown sale period ($\overline{\alpha}$) decreases. These results help a manager to determine a simultaneously optimal stock level and selling period by explaining the relationship between these two decision variables. We also see that the retained leftover quantity (markdown quantity) $q$ increases as the markdown period ($\overline{\alpha}$) increases; thus, the longer the planned markdown period the more leftover items will be stocked to meet the markdown clearance demand. Finally, we see that the return quantity ($I - q$) increases as the supplier's salvage value $V_s$ increases.

The findings discussed in this section enhance understanding of the coordination mechanism in which salvage operations are stochastic. Previous studies (for example, Tsay [30]) have proposed an optimal strategy prescribing that surplus merchandise be liquidated exclusively by the party that can earn a higher salvage value at the stage of clearance. This conclusion relies on a crucial assumption: any amount of surplus can be liquidated deterministically. In practice, this assumption is questionable. For example, in the retail industry the salvage operation frequently involves markdown clearance operations where demand is very likely a stochastic process. We have shown that with the stochastic salvage capacity assumption, a flexible returns policy in which agents' salvage values weighted by their probabilities of cleaning out of leftovers are made identical gives a greater system profit. This result is particularly intuitive when more and more independent outlets (discount stores, e-based stores, various reversed logistics paths) have recently become available for liquidating channel overstocks. According to our findings, the management not only needs to plan for an adequate stock level at the beginning of the primary selling period, but also needs to design an optimal mix of salvaging paths and responsibility sharing for system overstocks.

# 4. Decentralized Optimal Policies

In this section we consider a decentralized system. We assume that the retailer determines order quantity and time schedule ($Q^{UC}, \alpha^{UC}$) (the superscript "UC" denotes an uncoordinated and decentralized system), and that the DCO determines the optimal markdown sales quantity $q^{UC}$, in order to individually maximize their objective functions. The DCO's objective function after the end of the NSP, for a given $(I(Q,\alpha),\alpha)$ is $\Pi_D(q|I,\alpha)=V_D\Omega_D(q|I,\alpha)-wq$. Denote $\Omega_S:=W\bar{\theta}I$ where $\bar{\theta}I=\{q,I\}$ as the retailer's actualized wholesale revenue from leftovers. In the decentralized supply chain, the retailer and the DCO will add/subtract the wholesale revenue (cost) to/from their individual objective functions. The retailer's problem at the beginning of the NSP is given as follows.

$$\max_{Q,\alpha}\Pi_R(Q,\alpha|q)=P\Omega_R(Q,\alpha|q)-CQ + \int_0^{\xi}\max_W \Omega_S(W|I(Q,\alpha),\alpha,q)dF.$$

Proposition 2 states the individually optimal policies.

***Proposition* 2.** *(See Appendix 2) Assume that $g(x)$ has an increasing or constant failure rate (ICFR) failure rate.*

**2.1** *$\Pi_R(\Pi_D)$ is concave in $(Q,\alpha,W)$ $(\theta)$. Denote $I_1=\max[Q_1-(y\vee D_N(\alpha)),0]$. The optimal policies are given as follows :*

$$Cases=\left\{\begin{array}{l}\text{when } Q_1<q \text{ choose } P_2=(\alpha_2,\theta_2,Q_2)\\ \text{when } Q_1\geq q \text{ choose } P_1=\left\{\begin{array}{ll} I_1\leq q & P_{1N}=(\alpha_1,\theta_{1N},Q_1)\\ I_1>q & P_{1P}=(\alpha_1,\theta_{1P},Q_1)\end{array}\right.\end{array}\right.$$

*(i) DCO's markdown quantity is $q^{UC}:=D_C\vee G^{-1}(1-W_1/V_D)$.*

*(ii) Denote $\tilde{\omega}_q:=\lfloor q,\zeta_q\rfloor$ for [additive, multiplicative] and $\tilde{\omega}_I:=\lfloor I,\zeta_I\rfloor$ for [additive, multiplicative] models. The wholesale price of the leftovers is a form of "Inverse Demand Price Function : $W=V_D\bar{G}(\zeta)$".*

$$W=\left\{\begin{array}{ll} W_1 \text{ statisfies simultaneously} & \text{if } q(W_1)\leq I\\ \left\{\begin{array}{l}\text{First Order Condition } W_1=V_Dg(\zeta_q)\tilde{\omega}_q\\ \text{Inverse Demand Price } W_1=V_D\bar{G}(\zeta_q)\end{array}\right. & \\ W_2=V_D\bar{G}(\zeta_I) & \text{if } q(W_1)>I\end{array}\right.$$

*(iii) Let $\rho:=g/\bar{G}$. $Q^{UC}=Q_i$ $i=1,2$ satisfieswith $P-C=PF(\xi)-\omega_i$ with*

$$\omega_i=V_D\int_{\bar{\delta}_i}^{\xi}\bar{G}(\zeta_I)(1-\rho(\zeta_I)\tilde{\omega}_I)dF.$$

*(iv) The length of the NSP $\alpha_i$ $i=1,2$ satisiesy*

*Additive :*

$$Pk_NF(\xi)+V_D\int_{\bar{\delta}_i}^{\xi}g(\zeta_I)\phi IdF-\left\{k_C\int_0^{\bar{\delta}_i}W_1dF+k_N\int_{\bar{\delta}_i}^{\xi}W_2dF\right\}=0.$$

*Multiplicative :*

$$Pk_N\int_0^{\xi}ydF-V_D\int_{\bar{\delta}_i}^{\xi}g(\zeta_I)\phi(\zeta_I)\zeta_IdF-\left\{k_C\int_0^{\bar{\delta}_i}W_1\zeta_qdF+k_N\int_{\bar{\delta}_i}^{\xi}W_2dF\right\}=0.$$

**2.2** *Ceteris paribus, the optimal solution : (i) $q^{UC}$ increases as $\bar{\alpha}$ or $V_D$ increases, and $W_1$ decreases. (ii) $Q_1$ increases as $V_D$, $W_2$, or $P$ increases, and $C$ decreases. For the additive model $Q_1$ increases as $\alpha$ increases if $k_N>k_C$. (iii) $\alpha_1$ increases as $P$ increases and $(W_1,W_2)$ decreases.*

**2.3** *System Distortion : Denote $\Delta\Pi(\alpha):=\partial\Pi/\partial\alpha-\partial\Pi_R/\partial\alpha$. Assume that the remaining decision variables of the triplet $(Q,\alpha,\theta)$, other than the specified one, are identical between the centralized and decentralized systems.*

*(1) Additive model : $Q^{UC}\leq Q^I$, $\theta^I(<)\geq\theta^{UC}$ if $V_s<(\geq)W_1$, and $\alpha^{UC}\geq(<)\alpha^I$ if $0\geq(<)\Delta\Pi(\alpha)=-V_s\phi F(\bar{\delta}_i)-V_D\int_{\delta}^{\xi}g(\zeta_I)\phi IdF-k_C\left\{\int_0^{\bar{\delta}_i}(V_D-W_1)dF+\int_{\bar{\delta}_i}^{\xi}V_DG(\zeta_I)dF\right\}$. Here, $k_N-k_C\geq0\Rightarrow0\geq$*

$\Delta\Pi(\alpha)$ ; *thus, the decentralized system maintains a longer normal sales period.*

*(2) Multiplicative model* : $Q^{UC} \leq Q^{I}$, $\theta^{I}(<) \geq \theta^{UC}$ *if* $V_s < (\geq) W_1$, *and* $\alpha^{UC} \geq (<) \alpha^{I}$ *if* $0 \geq (<) \Delta\Pi(\alpha) =$

$$V_s \int_0^{\bar{\delta}_i} \phi(\zeta_q) dF - V_D \int_{\bar{\delta}_i}^{\xi} \zeta_I g(\zeta_I) \phi(\zeta_I) dF - k_C V_D \left\{ \int_0^{\bar{\delta}_i} \int_0^{\zeta_q} x\, dG\, dF + \int_{\bar{\delta}_i}^{\xi} \int_0^{\zeta_I} x\, dG\, dF \right\}$$

□

In proposition 2, we assume a supply chain composed of an independent retailer and a DCO that are managed to selfish, rather than system-wide, objectives. Proposition 2.1 reveals that the objective function is concave if the density function $g(x)$ has an *increasing or constant failure rate* ICFR. While the proposition limits the distribution, the ICFR class is broad enough to include most of the distribution one would choose to employ. For example, the normal and the exponential are both relatively widely used densities that are quite probable for formulating random demands. Proposition 2.3 reveals two sources of system distortions that lead to a sub-optimal supply chain profit. We have discussed in Proposition 1 that optimal supply chain performance requires execution of a precise set of strategies. Unfortunately, those actions are not always in the best interest of individual members of the supply chain. The supply chain members are primarily concerned with optimizing their own objectives, and this self-serving focus leads to system distortions. Let us now discuss these distortions in a more detailed fashion.

***Distortion* 1** : The retailer's local sale margin $(P-C)$ is less than the full system margin $(P-M)$ ; thus, this drives the retailer to order less than provided for under the centralized policy $(Q^{UC} \leq Q^{I})$.

***Distortion* 2** : The system's expected salvage revenues $V_D \int_0^{\xi} \Omega_D dF$ (salvage revenue in DCO) and $V_s \int_0^{\delta} (1-q)\, dF$ (salvage revenue of supplier) versus the retailer's local salvage revenue $W \int_0^{\xi} \Omega_S dF$ (wholesale revenue from leftovers) : (i) The underage cost of the retained leftovers (markdown sale quantity) in the centralized system $V_D - V_S$ is different from $V_D - W_1$ in the decentralized system ; thus, $\theta^{I} \neq \theta^{UC}$. (ii) The time schedule in the decentralized system is made based on analyzing the trade-offs regarding the retailer's normal sales revenue and the wholesale revenue from leftovers ( $W \int_0^{\xi} \Omega_S dF$) ; thus, the result is confined to optimizing the individual benefit to the retailer. In the centralized system, the trade-off analysis focuses on optimizing system-wide profits. It considers the retailer's normal sale revenue, and the true system salvage revenue of both the DCO ( $V_D \int_0^{\xi} \Omega_D dF$) and the supplier ( $V_S \int_0^{\delta} (1-q)\, dF$).

Having analyzed system distortions, a manager should then design a set of supply chain coordinating contracts such that each firm's objective becomes aligned with the coordinated supply chain objective. We will discuss these coordinating contracts in the next section. Before we proceed to the next section, however, we will furnish a numerical experiment to provide a better understanding of the system distortions discussed in this section.

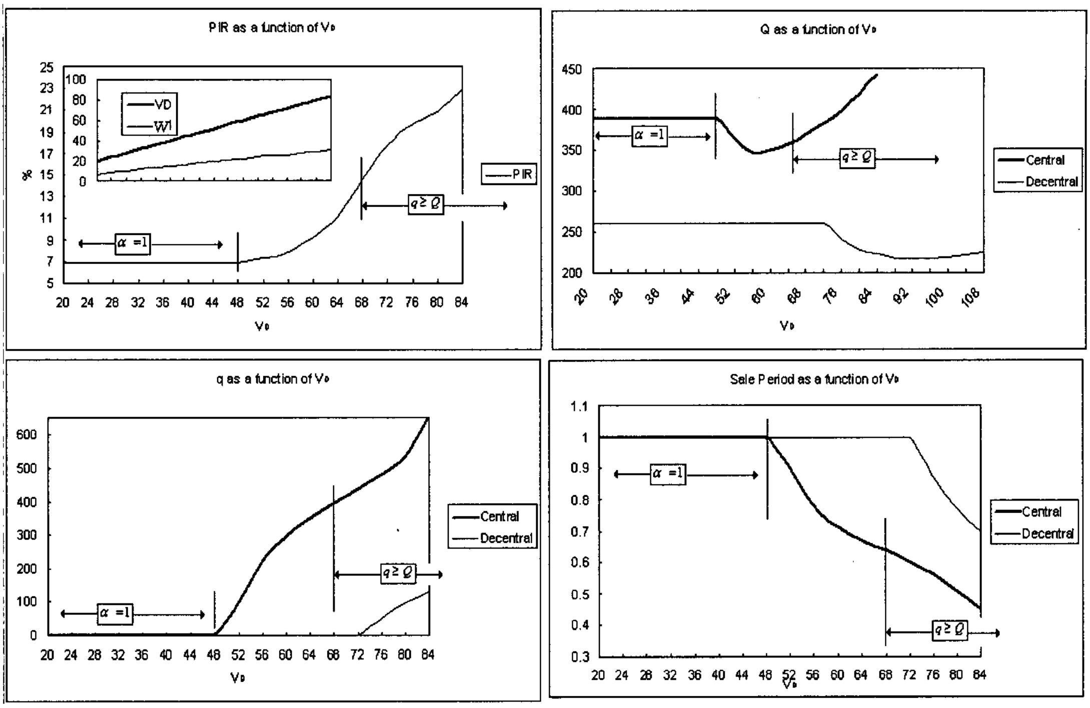
Let us now furnish a numerical example to explore and compare centralized and decentralized policies as discussed in Propositions 1 and 2.

The following assumptions are used in the numerical computation : (a) The demand is multiplicative with density functions $g(x)$ and $f(y)$ exponentially distributed with $f(y)=e^{-y}$ and $g(x)=e^{-x}$. (b) The base parameters take the following values : $V_D=78$, $V_S=14$, $P=110$, $M=20$, $C=30$, $k_N=200$, and $k_C=420$.

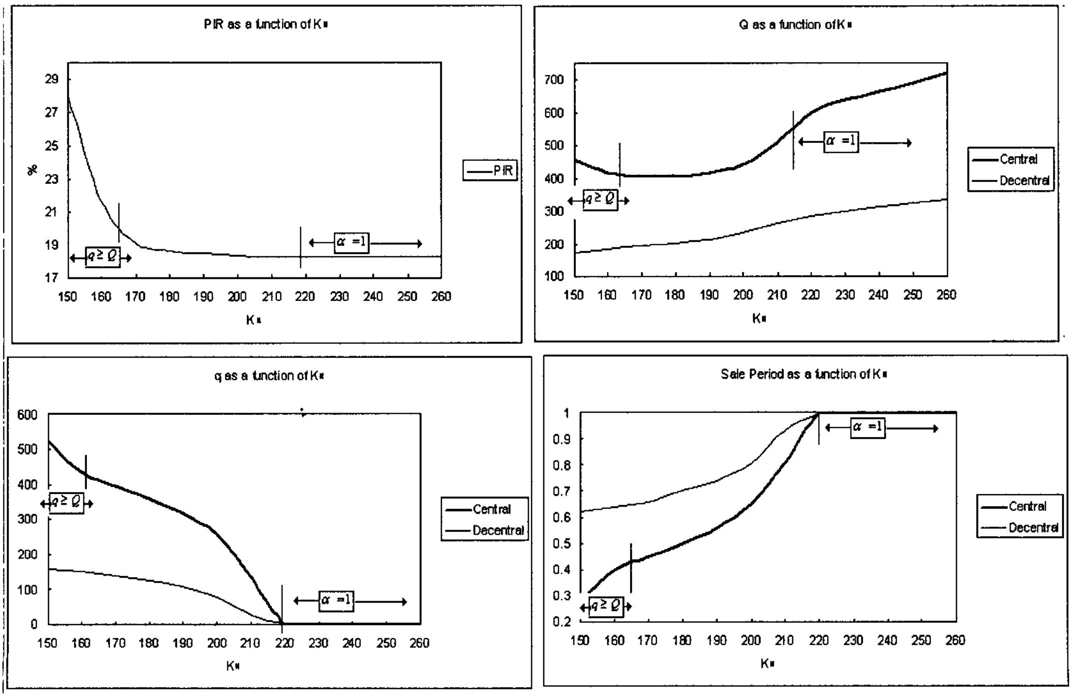
**$V_D$ Variation.** We first investigate the effects of centralization on the expected profits by varying the markdown sale price $V_D$. We keep the other parameters constant and change $V_D$ from $20 to $84 in increments of $4. Each scenario (P = 20, 24, ⋯, 84) is solved, and the Percentage Increase Ratio, PIR= $[\Pi_{Central}/\Pi_{Decentral}-1]\times 100\%$, is plotted as a function of $V_D$. For the centralized policy, our parameter range covers three regions : $\alpha^I=1$ when $V_D \leq 48$ (the normal sale period covers the entire life cycle), $\alpha^I < 1$ and $q^I < Q^I$ when $48 < V_D \leq 68$ (policy $P_1$), and $q^I \geq Q^I$ when $V_D > 68$ (policy $P_2$). [Figure 1] shows that the centralized policy outperforms the decentralized policy in all cases ; however, the benefits of integration increase as $V_D$ increases. It illustrates that PIR increases as $V_D - W_1$ and $W_1 - V_S$ ( $V_S$ remains unchanged) increase. It is seen that $V_D - W_1$ and $W_1 - V_S$ represent the differences in local (the retailer's) and system salvage revenues (Distortion 2). [Figure 1] also shows that, in general, the two systems show similar decision-making trends ; however, the trends in the decentralized system are less sensitive compare to those in the centralized system. For example, both order quantities show constant-decreasing-increasing trends but the variation in the centralized system is much greater than that in the decentralized system. We also see that $q^I \geq q^{UC}$ and $\alpha^I \leq \alpha^{UC}$ in all cases, and that both $q^I$ and $q^{UC}$ increase as $\alpha^I$ and $\alpha^{UC}$ decrease. Clearly, the trends show that a retailer in the centralized system maintains a shorter sales period so that the leftovers can be moved to the DCO in a more timely fashion in order to take advantage of a more time-elastic market. We also identify that the order quantity in the centralized system is significantly larger than that in the decentralized system due to Distortions 1 and 2.

**$k_N$ Variation** Here, the retailer's demand rate varies from 150 to 260, in increments of 10, while other parameters are kept constant. [Figure 2] illustrates again that the two policies show similar decision-making patterns ; however, the trends revealed by the decentralized system are less sensitive compared to those in the centralized system. We see that PIR gradually dies down as the differences in time schedule and markdown quantity between the centralized and the decentralized system diminish. In fact, the optimal $(q, \alpha)$ policies in the two systems are identical, and the only difference between them is limited to order quantity $Q^{UC} \leq Q^I$ after $k_N \geq 220$.

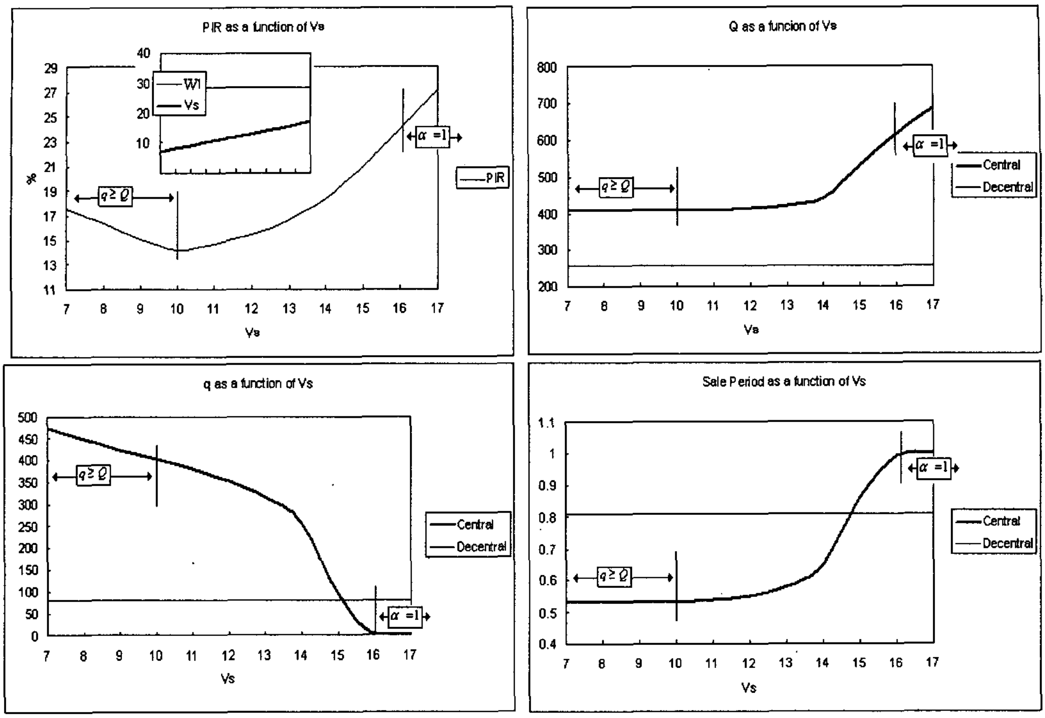
**$V_s$ and C Variation.** [Figure 3] illustrates how the decentralized system's ignorance of the supplier's expected salvage revenue (Distortion 2) can affect the system. We see that ( $V_S \leq 10$) $Q^I$ initially remains unchanged as $V_S$ increases, since at this $V_S$ range the DCO takes all of the leftovers ( $q \geq Q$). However, as $V_S$ further increases, the order quantity in the centralized system sharply increases as the system salvage revenue increases, whereas the decentralized system remains unchanged. We see that this has caused the PIR to sharply increase. We also see that the normal sales period for the centralized



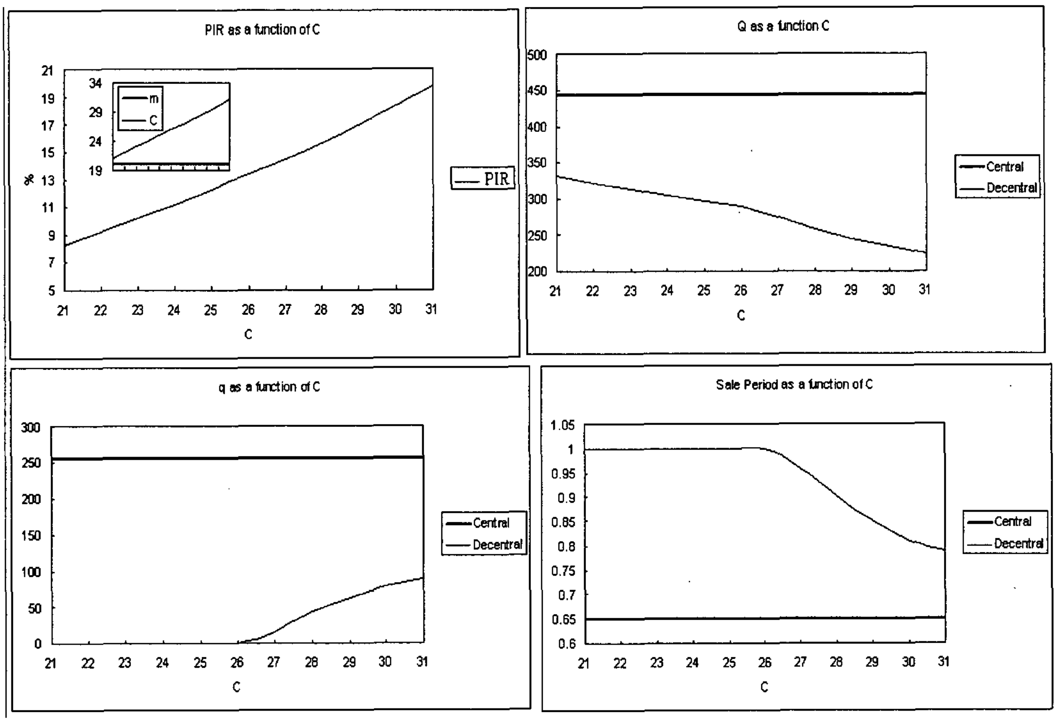

[Figure 1] Centralized and Decentralized Policies as a function of $V_D$




[Figure 2] Centralized and Decentralized Policies as a function of $K_N$




[Figure 3] Centralized and Decentralized Policies as a function of Vs




[Figure 4] Centralized and Decentralized Policies as a function of C system can be longer than that for the decentralized system when the supplier's salvage revenue is very high. That is, the system no longer feels that it is profitable to keep the leftovers in the DCO when the supplier's salvage revenue becomes very lucrative.

Different results can be observed from the $C$ variation. Here, regardless of the fact that the decentralized system reacts sensitively to the $C$ variation, the centralized system completely ignores the variation in the parameter. This phenomenon is widely discussed in the literature as the double marginalization effect. [Figure 1] through [Figure 3] reveal that centralized system designs $\alpha$ are based on a system-wide viewpoint; thus, $\alpha$ decreases as $V_D$ increases [Figure 1], $k_N$ decreases, or $q$ increases ( $q$ increases as $V_S$ decreases). [Figure 4] reveals that in the decentralized system, the markdown quantity ( $q$) increases as $Q$ decreases $\Rightarrow I$ decreases. Clearly, these two trends reveal inconsistencies.

# 5. System Coordination Strategies

In this section, we consider coordination strategies that can align the individual policies with the jointly optimal policies of supply chain. Our scenario is given as follows. First, a decentralized policy ( $Q^{UC}, \alpha^{UC}, q^{UC}$) is designed. Coordination strategies using several channel policies as means are then designed to modify the terms of trade so as to drive the individual system to adopt the centralized policy ( $Q^I, \alpha^I, q^I$). The channel leader uses three types of well-known channel policies for coordination. These are (1) incremental quantity discount (QD), (2) rebate for the retained leftovers (RR), e.g., Markdown Allowance [30] or Price Protection [17], and (3) rebate for the returned leftovers, e.g., end-life returns (ER) and mid-life returns (MR). In addition to those in <Table 2>, <Table 3> lists the notations used in the following sections.

〈Table 3〉 List of Additional Notations

| | |
|---|---|
| $B$ | minimum order quantity to qualify for a quantity discount |
| $C_d < C$ | unit discount wholesale price for all units $Q > B$ |
| $\beta$ | per unit compensation for the retained leftovers given by the supplier. For example, in Price Protection $\beta = \eta(C - W)$, $0 \le \eta \le 1$[17], and in Markdown Allowance $\beta = \eta C$ and $0 \le \eta \le 1$ (Tsay [30]). |
| $r$ and $r_e$ | mid-life (MR) and end-life (ER) per unit buyback price of the leftovers (returns value). We assume $r_e \le V_D$ to ensure that the retailer will operate a clearance salvage operation. |
| $W = (W_S, W_R)$ | $W_S$ and $W_R$ are wholesale prices of leftovers paid by the DCO to supplier and retailer |
| $V_D\overline{G}(\zeta) = \{V_D\overline{G}(\zeta_q), V_D\overline{G}(\zeta_I)\}$ | "Inverse Demand Price". $V_D\overline{G}(\zeta_q) = V_S$ when $q = q^I$. |
| We also use the following notations : | |
| $\tilde{x} := x + D_C(\alpha)$ and $\tilde{y} := y + D_N(\alpha)$. | |
| $E_y\{\Omega_s(V)\} = \int_0^{\xi} V\overline{\theta} I\, dF$ | wholesale revenue in which $(Q, \alpha, q) = (Q^I, \alpha^I, q^I)$ and wholesale price= $V$ |

In what follows, we present two coordination strategies.

(1) *The Centralized Coordination Strategy* (CCS) : In CCS, the supplier employs three supply chain coordinating channel policies--QD, MR, and ER. QD and MR are used to eliminate distortions in $(Q, \alpha)$, and ER is used to eliminate distortion in $q$. The coordination strategy works as follows. At the end of the NSP, the supplier buys back (using mid-life returns) all mid-life leftovers ( $I$). The buy-

back value ($r$) is designed to be no less than the retailer's wholesale price for the leftovers. The supplier sells a portion of the buyback leftovers ($q$) to the DCO, charging a resale value $W_S$ (the subscript "S" denotes the supplier) per unit, and salvages the rest of the leftovers ($I-q$). At the end of the CSP, the supplier again buys back any unsold end-life leftovers from the DCO, paying $r_e$ per unit.

(2) *The Decentralized Coordination Strategy* (DCS) : In DCS, the supplier and the retailer work together to coordinate the supply chain. The supplier employs QD, MR, and RR to eliminate distortions in $(Q, \alpha)$, and the retailer use ER to eliminate distortions in $q$. At the end of the NSP, the retailer sells a portion of the leftovers $q$ to the DCO, charging $W_R$ (the subscript "R" denotes the retailer) per unit ; and the supplier buys back the rest of the leftovers $I-q$ paying $r$ per unit. The supplier also pays the retailer compensation $\beta$ per unit for the retained leftovers ($q$). We assume that $W_R+\beta \geq r$. At the end of the CSP, the retailer purchases back any unsold end-life leftovers from the DCO, paying $r_e$ per unit.

The main difference between the *Centralized Coordination Strategy* (CCS) and the *Decentralized Coordination Strategy* (DCS) resides in the supplier's willingness to control the wholesale price of leftovers. In CCS, the supplier centralizes the wholesale price-setting decision. Here, the supplier interferes with the transaction between the retailer and the DCO by buying back all of the leftovers and setting the wholesale price ($W=W_S$) by herself. In DCS, the supplier decentralizes the price-setting decision and allows the retailer to determine the wholesale price ($W=W_R$).

Let $\upsilon$ be less than and approximately equal to $\zeta_I$ so that $\varepsilon(\upsilon) := V_D\int_{\upsilon}^{\zeta_I}\tilde{x}\,dG \cong 0$. Denote $R_{S-R}$ ($R_{S-D}$) as compensation packages given by the supplier to the retailer (DCO). We formulate the objective function of the supplier with coordination strategies CCS and DCS as follows. Notice that in CCS, the retailer earns {*expected wholesale revenue*} + $\int_{\delta}^{\xi}\varepsilon(\upsilon)dF$ for the returned leftovers where $\int_{\delta}^{\xi}\varepsilon(\upsilon)dF$ is an additional incentive.

$$\Pi_S(C_d, \beta, r, r_e, W_S) = (C-M)Q - R_{S-R} + \int_0^{\delta} V_S(1-q)\,dF - R_{S-D} + \begin{cases} CCS & E_y\{\Omega_S(W_S)\} \\ DCS & 0 \end{cases},$$

where $R_{S-D} = \begin{cases} CCS & \int_0^{\xi} r_e \int_0^{\zeta}(\bar{\theta}I - \tilde{x})\,dG\,dF \\ DCS & 0 \end{cases}$, and

$$R_{S-R} = (C-C_d)(Q-B) + \begin{cases} CCS: & \int_0^{\delta}\{r(I-q) + V_D\bar{G}(\zeta_q)q\}dF + \int_{\delta}^{\xi} V_D\bar{G}(\zeta_I)I\,dF + \int_{\delta}^{\xi}\varepsilon(\upsilon)\,dF \\ DCS & \int_0^{\delta}\{r(I-q) + \beta q\}dF + \int_{\delta}^{\xi}\beta I\,dF \end{cases}$$

The DCO's and the retailer's expected profits under the two coordination strategies are given as follows.

$$\Pi_D := V_D\Omega_D + R_{S-D} + R_{R-D} - \begin{cases} CCS & E_y\{\Omega_S(W_S)\} \\ DCS & E_y\{\Omega_S(W_R)\}, \end{cases}$$

$$\Pi_R := P\Omega_R - CQ + (C-C_d)(Q-B) + R_{S-R} - R_{R-D} + \begin{cases} CCS & 0 \\ DCS & E_y\{\Omega_S(W_R)\}, \end{cases} \text{ and}$$

$$R_{R-D} = \begin{cases} DCS & \int_0^{\xi} r_e \int_0^{\zeta}(\bar{\theta}I - \tilde{x})\,dG\,dF \\ CCS & 0 \end{cases}$$

We see that in CCS the supplier interferes with the transaction between the retailer and the DCO by buying back all of the leftovers and setting the wholesale price ( $W_s$) by herself ; whereas in DCS, the supplier lets the retailer determine $W_R$ Coordination strategies for CCS and DCS are given in Proposition 3. We will focus on presenting coordination strategy for the additive model.

***Proposition* 3** *(See Appendix 3 for the proof.) : For a pair of arbitrarily decided wholesale prices* $M \le C = P - \omega_1$ *and* $V_D\overline{G}(\zeta) \le W = V_D - \omega_2$, *the supply chain can be coordinated by offering*

**3.1 *Coordinating Strategies*** : *(i) an end-life return rebate*

$$0 \le r_e = \begin{cases} r_{e1} = \dfrac{W - V_D\overline{G}(\zeta_q)}{G(\zeta_q)} & q(r_{e1}) \le I \\ r_{e2} = \dfrac{W - V_D\overline{G}(\zeta_I)}{G(\zeta_I)} & q(r_{e1}) > I \end{cases} \text{ where}$$

$$q(r_{e1}) = D_C + G^{-1}\left\{\frac{V_D - W}{V_D - r_{e1}}\right\} = D_C + G^{-1}\left\{1 - \frac{V_S}{V_D}\right\},$$

*(ii) A channel policy : Denote*

$$\Delta\Pi(a) := k_C V_D\left\{\int_0^{\xi} G(\zeta_q)dF + \int_{\delta}^{\xi} G(\zeta_I)dF\right\}.$$

$$\text{CCS: } P^{CCS}(r, C_d) := \begin{cases} r = V_S + \dfrac{\Delta\Pi(a)}{\phi F(\delta)} \le C \\ C_d = M + \dfrac{\Delta\Pi(a)}{\phi} \le C. \end{cases}$$

$$\text{DCS: } P^{DCS}(r, C_d, \beta) := \begin{cases} r = \beta = \dfrac{V_S\phi F(\delta) + \Delta\Pi(a)}{k_N F(\xi)} \\ C_d = M - \dfrac{V_S F(\delta)k_c}{k_N} + \dfrac{\Delta\Pi(a)}{k_N} \le C. \end{cases}$$

**3.2 *Expected Profits*** : *Denote* $0 \le \zeta_1 := (P - C)/(P - M) \le 1, 0 \le \zeta_2 = \{V_D - W\}/\{V_D - V_D\overline{G}(\zeta)\} \le 1$ *(since* $V_D\overline{G}(\zeta) \le W$*), and* $E_y\{\Omega_S\} := E_y\{\Omega_S(V_D\overline{G}(\zeta))\}$. *Let* $\Pi_{SR}$ *and* $E_y\{\Pi_{LD}\}$ *be the joint profits for the Supplier-Retailer channel and the Leader (CCS : Supplier, DCS : Retailer)-DCO channel respectively. We see that* $\Pi_{SR} = (P - M)Q - P\int_0^{\xi} F(y)dy + V_S\int_0^{\delta} F(y)dy + E_y\{\Omega_s\}$, *and* $E_y\{\Pi_{LD}\} = \int_0^{\xi} V_D\left\{G(\zeta)\overline{\theta}I - \int_0^{\zeta} G(x)dx\right\}dF$. *Let* $\mu \in [0, 1]$ *be a mutually agreeable share of the profise between the retailer and the supplier. Designing* $0 \le B = Q - \dfrac{(P - M)(\mu - \zeta_1) + \Lambda(\mu)}{C - C_d} \le Q$, *where*

$$\Lambda(\mu) := \begin{cases} CCS : P(1 - \mu)\int_0^{\xi} F(y)dy - (r - V_S\mu)\int_0^{\delta} F(y)dy \\ DCS : (P(1 - \mu) - \beta)\int_0^{\xi} F(y)dy - \mu V_s\int_0^{\delta} F(y)dy \end{cases}$$

*leads to the expected profits :*

$$\begin{cases} Retailer: \Pi_R = \mu\Pi_{SR} + (1 - \mu)E_y\{\Omega_s\} + \begin{cases} CCS: 0 \\ DCS: E_y\{(1 - \zeta_2)\Pi_{LD}\} \end{cases} \\ DCO: E_y\{\Pi_D\} = E_y\{\zeta_2\Pi_{LD}\} \\ Supplier: \Pi_S = (1 - \mu)\{\Pi_{SR} - E_y\{\Omega_s\}\} + \begin{cases} CCS: E_y\{(1 - \zeta_2)\Pi_{LD}\} \\ DCS: 0 \end{cases} \end{cases}$$

$$\text{Denotes CCS: } \pi = \frac{P\int_0^{\xi} F(y)dy - r\int_0^{\delta} F(y)dy}{P\int_0^{\xi} F(y)dy - V_S\int_0^{\delta} F(y)dy}$$

$$\text{and DCS: } \pi = \frac{(P - \beta)\int_0^{\xi} F(y)dy}{P\int_0^{\xi} F(y)dy - V_S\int_0^{\delta} F(y)dy}.$$

*It is seen that* $\Lambda(\mu) \ge 0\ \forall\mu \in [0, \pi]$ *and* $\Lambda(\mu) < 0\ \forall\mu \in [\pi, 1]$ ; *thus, the supplier designs*

*(i)* $\mu \in [0, \pi]$: $\zeta_1 \le \mu \Rightarrow 0 \le B \le Q$, *and*

*(ii)* $\mu \in [\pi, 1]$: $\zeta_1 \le \mu + \Lambda(\mu)/(P - M) \Rightarrow 0 \le B \le Q$

□

Proposition 3 reveals that the coordination strategies are not unique, and shows that a continuum of solution exists. Possible profit-sharing arrangements differ in their divisions of the channel profit. With coordinating channel policies, the supplier (in CCS or supplier-retailer in DCS) can manipulate the channel followers. Obviously, the channel leader will set $\zeta$ as low as possible and capture as high a percentage as possible of the channel profits. In another interesting property, depending on which strategy is used the channel leader's profit ( $E_y\{(1-\zeta_2)\Pi_{LD}\}$) in the Leader-DCO channel can go to either the supplier (CCS) or the retailer (DCS). Thus, who will play what role is also an important issue in the multi-lateral supply chain. Note, however, that we have not taken into consideration the extra costs (e.g., transportation costs) that might be incurred by the supplier in handling leftovers in CCS. As noted by Hal Upbin, CEO of Kellwood Co, the suppliers are sometimes reluctant to take anything back since "the cost of handling would be absurd" [29]. It is seen that $1-\zeta_2 = r_e/V_D$ ; thus, the profit share $1-\zeta_2$ of the channel leader (the retailer in DCS or supplier in CCS) increases as $r_e$ of the increases, and at the expense of the DCO's profit share. Similarly, $C-C_d$ increases as $C-M$ increases, and $\zeta_1$ decreases as $C-M$ increases; thus, the channel leader's profit share increases at the expense of the follower's as $C-M(C-C_d)$ increases.

## 6. Discussion and Conclusion

This article further enhances understanding of the importance of supply chain coordination by examining a stochastic salvage capacity newsvendor inventory model. Previous research development relies on a crucial assumption : any amount of surplus can be liquidated deterministically. In practice, this assumption is questionable. For example, in the retail industry the salvage operation frequently involves markdown clearance operations where demand is very likely a stochastic process. We have shown that with the stochastic salvage capacity assumption, a partial returns policy in which agents' salvage values, weighted by their probabilities of the cleanout of leftovers, are made identical results in a greater system profit.

Our second research topic has focused on system distortions in the given problem setting. We have shown that, in general, the distortions stem from decisions made by agents based on the concept of decentralized revenue or cost structures. As a result, the decentralized system usually (1) orders less, (2) keeps a longer normal sales period, and (3) allocates more leftovers to the supplier.

We have furnished a numerical experiment to enhance understanding of the research findings. We first investigate the effects of centralization on the expected profits by varying the markdown sale price $V_D$. We show that the centralized system outperforms the decentralized system in all cases; however, the benefits of integration increase as $V_D$ increases. We also show that in general, the two systems show similar decision-making trends ; however, the trends in the decentralized system are less sensitive compare to those in the centralized system. For example, our numerical experiment reveals that the centralized system maintains a shorter sales period so that the leftovers can be moved to the DCO in a more timely fashion in order to take advantage of a more time-elastic market. A same phenomenon can also be observed from varying the retailer's demand rate $k_N$.

Our numerical experiment also reveals that when the supply chain is coordinated, the system operates as if the supply chain is operating a common system. Internal transactions that do not contribute to increasing the supply chain joint profit are eliminated completely ; therefore, the internal parameter ( $C$ ) has no impact on the determination of jointly optimal policies. The centralized supply chain completely ignores the variation in $C$. This phenomenon is widely discussed in the literature as the double marginalization effect. Finally, our numerical experiment reveals that the decentralized supply chain frequently shows inconsistency in decision making. For example, in the decentralized system, we frequently observed that the markdown quantity increases as the order $Q$ decreases. Clearly, these two trends reveal inconsistencies.

Our third research topic studies how to cope with system distortions in decentralized policies, we have considered two coordination strategies for aligning the individually optimal policies. In the Centralized Coordination Strategy (CCS), the supplier buys back all leftovers from the retailer, and sells a portion of them to the DCO. The supplier uses quantity discount, mid-life returns and end-life returns to eliminate distortions. In the Decentralized Coordination Strategy (DCS), members in supply chain need mechanisms to work together to secure better joint performance. Here, quantity discount, mid-life returns, and rebates for retained leftovers (e.g., a markdown allowance or wholesale price protection) are used as coordination mechanisms to eliminate distortions. The retailer also uses an end-life returns policy to eliminate distortions involving the policy for disposing of leftovers. We have shown that the profit share of channel leader increases at the expense of those of followers when the value of the channel rebates increases.

In the present study, we set out to analyze the possibility of designing a contract to coordinate a three-level supply chain. In our view, the analysis has some limitations. First, our research is done in a much simplified setting by considering a supplier with a sole retailer/DCO case. In discussing the topic, the inclusion of multiple-heterogeneous retailers/DCOs with different demand distributions might provide more meaningful results. Recent work (for example, Webster and Weng [33]) has shown that using a returns policy to coordinate a supply chain frequently results in a higher returns cost ; thus, when demand is lower than expected, coordination often leads to a manufacturer's resulting realized profit being lower than that in the no returns case. Future research should examine the agent's attitude toward risk and its consequences. Finally, our research does not allow us to study the possibility of a situation involving multiple markdown periods. Generally, in a real-world application, a markdown operation may consist of more than one discount period. The multiple markdown period problem has been studied by Khouja [9] on a single company level. Future work on a progressive multiperiod markdown model could certainly shed further light on the topic. These limitations indicate some of the possible extensions to a future study.

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*Appendix 1 Proof for Proposition 1 :*

Let $\Pi'_{\mathbf{x}} := d\Pi(x)/dx$ (e.g., $\Pi'_{\boldsymbol{\alpha}} = \partial\Pi/\partial\alpha$ ), $\varpi_q := \lfloor q, \varsigma_q \rfloor$, and $\varpi_I := [I, \varsigma_I]$ for [additive, multiplicative] model.

**Proof for Proposition 1.1 :** *(i) Sufficiency condition: We will show the proof for policy* $\mathrm{P}_1$. $\Pi_{\boldsymbol{\theta\theta}} = -V_D g(I\bar{\theta} \wedge D_C)\varpi_I < 0 \Rightarrow \Pi$ is concave in $\theta$. Let $\Pi := \Pi(Q, \alpha, \theta^*)$ where $I\bar{\theta}^* = D_C \vee G^{-1}(1 - V_S/V_D)$. For [additive, multiplicative] models denote

(1) [ $\lambda_C := -V_D g(\varsigma_I) < 0$ , $\lambda_C := -V_D g(\varsigma_I)/D_C < 0$ ],

(2) [ $\phi_N := k_N > 0$ , $\phi_N := \xi k_N > 0$ ],

(3) [ $\lambda_N := -(P - V_D)f(\xi) < 0$ , $\lambda_N := -(P - V_D)f(\xi)/D_N < 0$ ], and

(4) [ $\phi := k_N - k_C$ , $\phi := -\varsigma_I k_C + yk_N \le 0$ ].

{Proof : Denote $\varsigma := I/D_C$ and $\bar{\varsigma} := yk_N/k_C \ge 0$. $\phi \le 0$ if $\varsigma \ge \bar{\varsigma}$, and $\varsigma'_{\boldsymbol{\alpha}} = -D_C^{-1}\phi$ ; hence, $\varsigma'_{\boldsymbol{\alpha}}(\varsigma > \bar{\varsigma}) \ge 0$, $\varsigma'_{\boldsymbol{\alpha}}(\varsigma < \bar{\varsigma}) \le 0$, and $\varsigma'_{\boldsymbol{\alpha}}(\bar{\varsigma}) = 0$. $\varsigma(\alpha > 0) \ge \varsigma(\alpha = 0)$ if $\varsigma(\alpha = 0) \ge \bar{\varsigma}$ since $\varsigma'_{\boldsymbol{\alpha}}(\varsigma > \bar{\varsigma}) \ge 0$, and $\varsigma(\alpha > 0) \le \varsigma(\alpha = 0)$ if $\varsigma(\alpha = 0) \le \bar{\varsigma}$ since $\varsigma'_{\boldsymbol{\alpha}}(\varsigma < \bar{\varsigma}) \le 0$ ; thus, only one of $\varsigma \ge \bar{\varsigma}$ ($\varsigma'_{\boldsymbol{\alpha}} \ge 0$) or $\varsigma \le \bar{\varsigma}$ ($\varsigma'_{\boldsymbol{\alpha}} \le 0$) can apply. However, $\varsigma \to \infty$ as $\alpha \to 1$; hence, it must be that $\varsigma \ge \bar{\varsigma}$ and $\phi \le 0$ $\forall \alpha \in [0,1]$.}

We see that $\Pi''_{\boldsymbol{\alpha\alpha}} = \lambda_N k_N{}^2 + \int_\delta^\xi \lambda_C \phi^2 dF \le 0$, $\Pi''_{\mathbf{Q\alpha}} = -\lambda_N \phi_N - \int_\delta^\xi \lambda_C \phi dF$ , and $\Pi''_{\mathbf{QQ}} = \lambda_N + \int_\delta^\xi \lambda_C dF \le 0$.

Thus, $\Pi''_{\boldsymbol{\alpha\alpha}}\Pi''_{\mathbf{QQ}} - (\Pi''_{\boldsymbol{\alpha}\mathbf{Q}})^2 = a + b > 0$, where $b = \int_\delta^\xi \lambda_C dF \times \int_\delta^\xi \lambda_C \phi^2 dF - \left(\int_\delta^\xi \lambda_C \phi dF\right)^2 \ge 0$ (by Cauchy-Shwarz's inequality), and $a = \int_0^\xi \lambda_N \lambda_C (\phi_N - \phi)^2 dF > 0$, and the objective function $\Pi := \Pi(Q, \alpha, \theta^*)$is concave in $(Q, \alpha)$

*(ii) Optimal Policies : We will show the derivation for the additive model. The multiplicative model can be shown similarly.* {*Lemma 1* : $Q_1 < q \Rightarrow Q_{2N} < q$ and $Q_1 \ge q \Rightarrow Q_{2N} \ge q$ : Define $\Gamma_{ij} := PF(\xi) - \omega_{ij}$. Proposition 1.1 states that $Q_{ij}$ satisfies $P - M = \Gamma_{ij}$. $\Gamma_{ij}$ s are the strictly increasing functions of $Q$, and $P - M$ is a constant ; thus, $\Gamma_a > \Gamma_b \Rightarrow Q_a < Q_b$. For an arbitrary $\theta \ge 0$ and $Q$, $V_D\bar{G}(\bar{\theta}I \wedge D_C) \ge V_D\bar{G}(I \wedge D_C) \Rightarrow \Gamma_{2N} \ge \Gamma_1$ ; thus, (R1): $Q_1 \ge Q_{2N}$, and $Q_1 < q \Rightarrow Q_{2N} < q$. Assume otherwise that $Q_1 \ge q$ then, via (R1) one of the following two cases can apply : (Case 1) : $Q_1 \ge q \ge Q_{2N}$ or (Case 2) : $Q_1 \ge Q_{2N} \ge q$. Assume first that (Case 1) applies. Since $V_D\bar{G}(\bar{\theta}I_1 \wedge D_C) = V_S$ $\forall I_1 > q$ and $Q_{2N} \le q \Rightarrow V_S \le V_D\bar{G}(Q_{2N} \wedge D_C)$, $V_D\bar{G}(\bar{\theta}I_1 \wedge D_C) < V_D\bar{G}(Q_{2N} \wedge D_C) \le V_D\bar{G}(I_{2N} \wedge D_C) \Rightarrow \Gamma_1 > \Gamma_{2N} \Rightarrow Q_1 < Q_{2N}$, which

contradicts (R1). Thus, we conclude (Case 2).} The first derivative of the objective function reveals $\partial\Pi/\partial\theta = 0 \Rightarrow \; V_S - V_D\bar{G}(\bar{\theta}I \wedge D_C) = 0$. The optimal policy for disposing of leftovers is derived from the following four observed cases. Situation S1 : $Q_1 \geq q \Rightarrow Q_{2N} \geq q$ (see Lemma 1). S1 consists of two sub-cases. (S1-1): $I \leq q \Rightarrow V_S - V_D\bar{G}(I \wedge D_C) \leq 0 \Rightarrow \; \partial\Pi/\partial\theta \leq 0 \;\; \forall\theta \in [0,1]$. Together, $\partial^2\Pi/\partial\theta^2 = -V_D g(I\bar{\theta} \wedge D_C)\varpi_I < 0$ and $\partial\Pi/\partial\theta \leq 0$ imply that $\Pi$ is decreasing and concave; thus, optimal $\theta^* = 0$. (S1-2): $I > q$ then $\theta^* \in [0,1]$ satisfies $\partial\Pi/\partial\theta = 0$. Situation S2 : $Q < q \Rightarrow V_S < V_D\bar{G}(Q \wedge D_C) \Rightarrow$ $\partial\Pi/\partial\theta < 0$ regardless of leftovers; thus $\theta^* = \theta_{2N} = 0$. The optimal policies for $(Q,\alpha)$ are derived for cases S1 and S2 by substituting the optimal $\theta^*$ to the necessary conditions $\partial\Pi/\partial\alpha = 0$ and $\partial\Pi/\partial Q = 0$. □

**Proof for Proposition 1.2 :**

In the [multiplicative, additive] models :

(a) [$\partial q/\partial\alpha = -k_C\varsigma_q < 0$, $\partial q/\partial\alpha = -k_C < 0$],

(b) [$\partial q/\partial V_S = -D_C/V_D g(\varsigma_q) < 0$, $\partial q/\partial V_S = -1/V_D g(\varsigma_q) < 0$], and

(c) [$\partial q/\partial V_D = D_C V_S/V_D{}^2 g(\varsigma_q) > 0$, $\partial q/\partial V_D = V_S/V_D{}^2 g(\varsigma_q) > 0$]. Thus, $q'$ increases as $\bar{\alpha}$ or $V_D$ increases, and $V_S$ decreases. The other cases in (ii) and (iii) can be derived in a similar manner.□

## *Appendix 2. Proof for Proposition 2.*

**Proof for Proposition 2.1 :** *The optimal policies can be derived in a similar manner to that in Proposition 1.1. We will show the proof of sufficiency condition. The policy* $\mathrm{P}_1$.

$\Pi_{D\theta\theta} = -V_D g(I\bar{\theta} \wedge D_C)\varpi_I < 0 \Rightarrow \Pi_D$ is concave in $\theta$. Define $\rho := g/\bar{G}$ as the failure rate., and for the [additive, multiplicative] model let

(1) [$\lambda_N := -(P - V_D)f(\xi) < 0$, $\lambda_N := -(P - V_D)f(\xi)/D_N < 0$],

(2) [$\lambda_C := -V_D(g_I'(\varsigma_I)I + 2g)$, $\lambda_C := -V_D(g_\varsigma'(\varsigma_I)\varsigma_I + 2g)/D_C$]. Here, $(g_I'(\varsigma_I)I + 2g)$ $= -\{\bar{G}((\varsigma_I))(1-\rho(\varsigma_I)I)\}_I' \geq 0$ and $(g_\varsigma'(\varsigma_I)\varsigma_I + 2g) = -\{\bar{G}(\varsigma_I)(1-\rho(\varsigma_I)\varsigma_I)\}_\varsigma' \geq 0$, if $g(x)$ has an increasing failure rate (IFR).

We see that $\Pi''_{R\mathbf{QQ}} = \lambda_N + \int_\delta^\xi C dF$, $\Pi''_{R\mathbf{Q\alpha}} = -\lambda_N\phi_N - \int_\delta^\xi \lambda_C\phi dF$, and $\Pi''_{R\boldsymbol{\alpha\alpha}} = \lambda_N\phi_N{}^2 + \int_\delta^\xi \lambda_C\phi^2 dF \leq 0$.

Therefore, $\Pi''_{R\boldsymbol{\alpha\alpha}}\Pi''_{R\mathbf{QQ}} - (\Pi''_{R\boldsymbol{\alpha}\mathbf{Q}})^2 = a + b \geq 0$, where $b = \int_\delta^\xi \lambda_C dF \times \int_\delta^\xi \lambda_C\phi^2 dF - \left(\int_\delta^\xi \lambda_C\phi dF\right)^2 \geq 0$ (by Cauchy-Swarz's inequality) and $a = \int_\delta^\xi \lambda_N\lambda_C(\phi_N - \phi)^2 dF > 0$ ; thus, the objective function $\Pi_R$ is concave in $(Q,\alpha)$ □

**Proof for Proposition 2.2 :** (i) In the [multiplicative, additive] models :

(a) $[\partial q/\partial\alpha = -k_C\varsigma_q < 0, \partial q/\partial\alpha = -k_C < 0]$,

(b) $[\partial q/\partial W_1 = -D_C/V_D g(\varsigma_q) < 0, \partial q/\partial W_1 = -1/V_D g(\varsigma_q) < 0]$,

(c) and $[\partial q/\partial V_D = D_C W_1/V_D^2 g(\varsigma_q) > 0,\ \partial q/\partial V_D = W_1/V_D^2 g(\varsigma_q) > 0]$ thus, $q'$ increases as $\overline{\alpha}$ or $V_D$ increases, and $V_S$ decreases.

(ii) $\partial Q_1/\partial V_D = -\int_\delta^\xi \overline{G}(\varsigma_I)(1-\rho(\varsigma_I)\varpi_I)dF \big/ (\partial^2\Pi/\partial Q^2)$. Proposition 2.1 reveals that

$$\begin{cases} W_1 = V_D g(\varsigma_q)\varpi_q \\ q = D_C \vee G^{-1}(1-W_1/V_D) \Rightarrow W_1 = V_D\overline{G}(\varsigma_q) \end{cases} \Rightarrow 1-\rho(\varsigma_q)\varpi_q = 0.$$

$0 = (1-\rho(\varsigma_q)\varpi_q) \le (1-\rho(\varsigma_I)\varpi_I)$, since $q > I$ and $g(x)$ has an IFR or $\rho' > 0\ \ \forall y \in [\delta, \xi]$; thus, $\partial Q_1/\partial V_D \ge 0$. The results for other cases in (ii) and (iii) can be demonstrated in a similar manner.

□

**Proof for Proposition 2.3 :** The results are obtained by analyzing $\partial\Pi/\partial\alpha - \partial\Pi_R/\partial\alpha$, $\partial\Pi/\partial\theta - \partial\Pi_D/\partial\theta$, and $\partial\Pi/\partial Q - \partial\Pi_R/\partial Q$. □

*Appendix 3 Proof for Proposition 3 :*

**Proof for Proposition CCS :** *(i)Coordinating* $q$ : Since $\partial\Pi/\partial\theta - \partial\Pi_D/\partial\theta = W_S - V_S - r_e G(\varsigma_q)$, when $q(r_{e1}, W_S) \le I$, designing $r_{e1} = (W_S - V_S)/G(\varsigma_q)$ coordinates the supply chain. On the other hand, when $q(r_{e1}, W_S) > I$ the supply chain can be coordinated by equating $q(r_e, W_S) = I \Rightarrow r_{e2} = (W_S - V_D\overline{G}(\varsigma_I))$ $/G(\varsigma_I)$.

*(ii) Coordinating* $Q$ : Upon substituting $\varepsilon(\upsilon) = V_D \int_\upsilon^{\varsigma_I} \tilde{x}\, dG \cong 0\ \ \forall\ y \in [\delta, \xi]$, $\partial\Pi/\partial Q - \partial\Pi_R/\partial Q = C_d - M - (r - V_S)F(\delta)$ ; thus, $C_d = M + (r - V_S)F(\delta)$ coordinates the distortion in order quantity.

*(iii) Coordinating* $\alpha$ : $\partial\Pi/\partial\alpha - \partial\Pi_R/\partial\alpha = -\Delta\Pi(\alpha) + (r - V_S)\phi F(\delta)$ ; thus, $r = V_S + \Delta\Pi(\alpha)/\phi F(\delta)$ coordinates the distortion in $\alpha$. (iv) The profit expected by the supplier, retailer, and the DCO upon substituting the coordinating channel policies, is given as follows.

*(1) Supplier-DCO* : The supplier incurs buyback cost $E_y\{\Omega_S\}$; thus, the expected profit for the supplier-DCO channel is $E_y\{\Pi_{SD}\} = \int_0^\xi V_D\{\theta I - \int_0^\varsigma G(x)dx\}dF - E_y\{\Omega_S\}$. This implies that

$$E_y\{\Pi_{SD}\} = \int_0^\xi V_D\{(1-\overline{G}(\varsigma))\theta I - \int_0^\varsigma G(x)dx\}dF = \int_0^\xi V_D\{G(\varsigma)\theta I - \int_0^\varsigma G(x)dx\}dF$$

The DCO's individual expected profit upon substituting the coordination strategy is given by

$$E_y\{\Pi_D\} = \int_0^{\xi}\left\{(V_D - W_S)\bar{\theta}I - (V_D - r_e)\int_0^{\varsigma} G(x)dx\right\}dF$$

$$= \int_0^{\xi}\left\{\omega_2\bar{\theta}I - \left(V_D - \frac{W_S - V_D\bar{G}(\varsigma)}{G(\varsigma)}\right)\int_0^{\varsigma} G(x)dx\right\}dF$$

$$= \int_0^{\xi}\zeta_2\left\{V_D G(\varsigma)\bar{\theta}I - V_D\int_0^{\varsigma} G(x)dx\right\}dF = E_y\{\zeta_2\Pi_{SD}\}, \text{ and } E_y\{\Pi_S\} = E_y\{[1-\zeta_2]\Pi_{SD}\}.$$

(2) Denote $\tilde{\Pi}_R := \Pi_R - E_y\{\Omega_S\} - (C - C_d)(Q - B)$. $\tilde{\Pi}_R = (P - C)Q - P\int_0^{\xi} F(y)dy + r\int_0^{\delta} F(y)dy$, and $\Pi_{SR} - E_y\{\Omega_S\} = (P - M)Q - P\int_0^{\xi} F(y)dy + V_S\int_0^{\delta} F(y)dy$.

(i) $\mu \le \pi \Rightarrow \Lambda(\mu) \ge 0 \Rightarrow (\mu = \zeta_1) \times \{\Pi_{SR} - E_y\{\Omega_S\}\} \ge \tilde{\Pi}_R$; thus, equating $\zeta_1 = \mu \Rightarrow B = Q - \Lambda(\mu)/(C - C_d) < Q$ leads to the retailer's share of the joint profit $\Pi_R - E_y\{\Omega_S\} = \zeta_1\{\Pi_{SR} - E_y\{\Omega_S\}\}$.

(ii) $\mu > \pi \Rightarrow \Lambda(\mu) < 0 \Rightarrow (\mu = \zeta_1) \times \{\Pi_{SR} - E_y\{\Omega_S\}\} < \tilde{\Pi}_R$. We see that $\mu\{\Pi_{SR} - E_y\{\Omega_S\}\} - \tilde{\Pi}_R = (P - M)(\mu - \zeta_1) + \Lambda(\mu)$. Here, $\zeta_1 \le \mu + \Lambda(\mu)/(P - M) \Rightarrow (P - M)(\mu - \zeta_1) + \Lambda(\mu) \ge 0 \Rightarrow B \le Q$, *and* $\zeta_1 \ge \mu + \{\Lambda(\mu) - Q(C - C_d)\}/(P - M) \Rightarrow (P - M)(\mu - \zeta_1) + \Lambda(\mu) \le Q(C - C_d) \Rightarrow 0 \le B$.

**Proof for Proposition DCS** : The proof for the coordination strategies is similar to that in CCS.