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Conversion of the Sonic Conductance C and the Critical Pressure Ratio b into the Airflow Coefficient μ

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In a case of computer simulation used for the verification of pneumatic system performance one of the main problems is that various parameters can be used to describe flow characteristics of the system components. The Standard ISO 6358 offers two parameters : the sonic conductance C and the critical static pressure ratio b, but the parameters can not be directly utilised in an analysis of a pneumatic system. In the standard analysis there is applied the airflow coefficient μ, but it is not presented in the vendors' catalogues. In the paper the numerical algorithm for calculation of the airflow coefficient μ (which is required for computer simulation) as a function of sonic conductance C and a critical pressure ratio b (recommended by the standard) is presented. Additionally, because of the iterative character of the described algorithm, an artificial neural network approach to solve the problem is proposed.

Key Words : Pneumatics, ANN, CAD, Flow Properties

1. Introduction

In a case of computer simulation for the verification of pneumatic system performances one of the main problems is that various parameters can be used to describe flow characteristics of elements (Grymek and Kiczkowiak, 2000).

During calculation and selection of the pneumatic elements from a catalogue there are preferred parameters which can be easily found in the vendors' catalogues (Grymek and Kiczkowiak, 1998) :

- control valve nominal diameter d (m)
- volumetric airflow rate in normal conditions[1] Q_N (m^3/s)
- flow coefficient[2] K_v (according to VDI/VDE 21732 standard)

There are many models of an air-flow rate (Kaminski, 2003 ; Baek and Kwon, 2003). For a computer simulation, where the differential model by E.W. Gerc (1969, 1985) is utilised, the flow properties are described by an airflow coefficient μ. For these models a mass flow rate as function of μ (Gerc, 1969, 1985 ; Iwaszko, 1999) is given by Saint Venant-Wantzel formula :

$$\dot{m}=\mu\cdot f\cdot p\cdot\sqrt{\frac{2\cdot\kappa}{\kappa-1}}\cdot\sqrt{\frac{1}{R\cdot T}}\cdot\varphi(Y) \qquad (1)$$

where :

μ : Airflow coefficient defined as a proportion of the real flow rate and the theoretic (isentropic) one (Iwaszko, 1999)

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[1] for the absolute pressure 0.7 MPa (for some vendors 0.73 MPa) and the pressure drop 0.1 MPa

[2] VDI/VDE 2173 *Richtlinien. Stromungstechnische Kenngrößen von Stellventilen und deren Bestimmung*

f : Inlet area (m^2), $f=\frac{\pi\cdot d^2}{4}$
p, T : Accumulated input parameters[3] of gas: pressure (Pa) and temperature (K)
κ : Adiabatic exponent, $\kappa=1.4$
R : Individual gas constant for air, $R=287$ J/(kg·K)
$\varphi(Y)$: Expansion function defined as

$$\varphi(Y)=\begin{cases}\sqrt{Y^{2/\kappa}-Y^{(\kappa+1)/\kappa}} & \text{for } 1\geq Y<0.5282\\ 0.2588 & \text{for } 0<Y\leq 0.5282\end{cases} \tag{2}$$

Y : Accumulated pressures ratio

The mass flow rate can also be expressed as function of the Mach number (Iwaszko 1999) :

$$\dot{m}=f\cdot\frac{p}{\sqrt{T}}\cdot\sqrt{\frac{\kappa}{R}}\cdot M\cdot\left(1+\frac{\kappa-1}{2}\cdot M^2\right)^{\frac{\kappa+1}{2(1-\kappa)}} \tag{3}$$

Methods of experimental determination of the airflow coefficient μ assume that its value is constant. But for real pneumatic devices it is not constant and depends on the accumulated pressures ratio Y (see Figure 1).

A mass flow rate according to ISO 6358 is given by:

$$\dot{m}=\frac{p_1}{T_0}\cdot\rho_{\mathrm{ANR}}\cdot\sqrt{T_{\mathrm{ANR}}}\cdot Y_{\mathrm{st}} \tag{4}$$

$$Y_{\mathrm{st}}=\begin{cases}C & \text{for } 0\leq p_2/p_1\leq b\\ C\cdot\sqrt{1-\left(\frac{p_2/p_1-b}{1-b}\right)^2} & \text{for } b<p_2/p_1\leq 1\end{cases} \tag{5}$$

where:

C : Sonic conductance ($s\cdot m^4/kg$)
p_1 : Absolute static inlet pressure[4] (Pa)
p_2 : Absolute static outlet pressure (Pa)
T_0 : Absolute inlet temperature (K)
ρ_{ANR} : Gas density for standard conditions ANR[5] (kg/m^3)
b : Critical pressures ratio

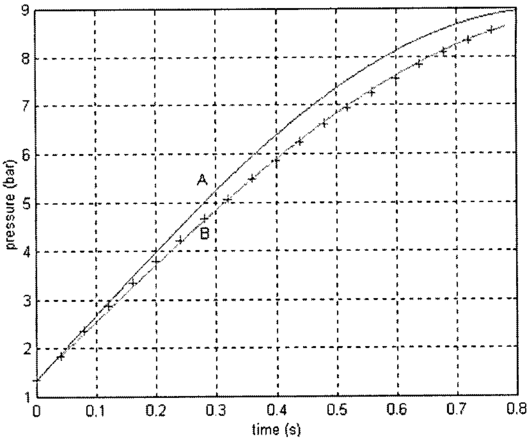

Fig. 1 Pressure changes during the filling of a chamber, '+' experimental results, computer simulation with the use of (A) ISO 6358 and (B) Saint Venant-Wantzel with changeable μ

The mass flow rate can also be expressed as function of the Mach number (Iwaszko 1999) :

$$\dot{m}=f\cdot\frac{b_1}{\sqrt{T_0}}\cdot\sqrt{\frac{\kappa}{R}}\cdot M\cdot\sqrt{1+0.2\cdot M^2} \tag{6}$$

In a computer simulation, a replacement of the formula Eq. (1) by the formula Eq. (4) is not possible despite their similarity. The first one, Eq. (1), utilises the accumulated pressures while the second one, Eq. (4), — the static pressures. The replacement, which can be found in some papers, always leads to divergence between the experimental results and the simulation. The exemplary results are presented in Fig. 1. An analysis of the problem was described by Iwaszko (Iwaszko, 1999).

It is why there is a need of conversion from C, b, d and Y into μ.

2. Requirements

The procedure needed should convert the sonic conductance C, the critical pressures ratio b, the pipe diameter d and the accumulated pressure ratio Y into the airflow coefficient μ (see Figure 2).

In the context of utilisation in a CAD system it should additionally (Kiczkowiak et al., 2002):

- be numerically stable; give precise result in

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[3] accumulated parameters are the local parameters of gas when it is isentropicly compressed because of coming to stop (Szumowski, 1989)

[4] measurement according to ISO 6358 standard

[5] $p_{\mathrm{ANR}}=0.1$ MPa, $T_{\mathrm{ANR}}=293.15$ K

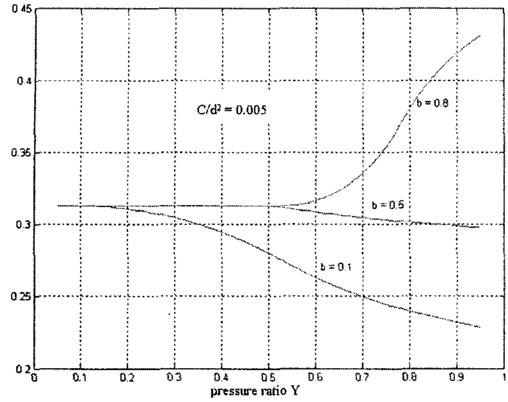

Fig. 2 Examples[6] of the airflow coefficient μ characteristics as a function of pressures ratio Y

the whole range of input data,

- allow to apply the standard fast numerical procedures,
- allow for low cost calculations.

There can be found in literature (Iwaszko, 1999) a solution of the task. Its main disadvantage is the lack of direct solution. It allows to calculate a set of points $\mu(C, b, d, Y)$, for a given pneumatic ele-ment (C=const, b=const, d=const). From these points, by approximation, the characteristic $\mu = f(Y)$ can be obtained. Such approach (Kiczkowiak et al., 2002):

- needs a determination of the whole characteristic $\mu = f(Y)$ in order to obtain a single value of the airflow coefficient,
- is very inaccurate because of the approximation,
- does not generate a uniform set of points (in relation to Y) — the approximation is more difficult and its errors are (very often) higher.

The above mentioned disadvantages result in the need for a new method of direct calculation of the airflow coefficient μ.

3. Proposed Algorithm

The algorithm proposed below is a significant modification of the one presented in (Iwaszko, 1999). There are input parameters: C/d^2, b, Y. A single value of μ is obtained as the result.

Comparing the Eq. (1) with the Eq. (3) the formula is obtained:

$$\mu = \frac{M \cdot \left(1 + \frac{\kappa - 1}{2} \cdot M^2\right)^{\frac{\kappa + 1}{2(1-\kappa)}}}{\sqrt{\frac{2}{\kappa - 1}} \cdot \varphi(Y)} \tag{7}$$

where only the value of the Mach number is unknown.

Comparing the Eq. (4) with the Eq. (6) and assuming the supersonic flow ($Y_{st} = C$), the maximum value of the Mach number can be calculated from:

$$M_{max} = \sqrt{\frac{\sqrt{1 + \frac{32 \cdot (\kappa - 1) \cdot \left(\frac{C \cdot p_{ANR}}{d^2 \cdot \pi}\right)^2}{\kappa \cdot R \cdot T_{ANR}}} - 1}{\kappa - 1}} \tag{8}$$

The static pressure ratio S as function of the Mach number can be expressed as (Iwaszko, 1999):

$$S = b + (1 - b) \cdot \sqrt{1 - \left(\frac{\pi \cdot M \cdot \sqrt{\kappa \cdot R \cdot T_{ANR}}}{4 \cdot \rho_{ANR} \cdot \frac{C}{d^2}}\right)^2 \cdot \left(1 + \frac{\kappa - 1}{2} \cdot M^2\right)} \tag{9}$$

and the pressure ratio Y as (Iwaszko, 1999):

$$Y = S \cdot \left(1 + \frac{\kappa - 1}{2} \cdot M^2\right)^{\frac{k}{1-\kappa}} \tag{10}$$

Assuming the sonic flow ($S = b$ and $M = M_{max}$) the border value of the pressure ratio can be calculated from:

$$Y_{gr} = b \cdot \left(1 + \frac{\kappa - 1}{2} \cdot M_{max}^2\right)^{\frac{\kappa}{1-k}} \tag{11}$$

Inserting Eq. 9 into Eq. 10 a relation between the Mach number M and the pressure ratio Y is obtained:

$$Y = \left(b + (1 - b) \cdot \sqrt{1 - \left(\frac{\pi \cdot M \cdot \sqrt{\kappa \cdot R \cdot T_{ANR}}}{4 \cdot p_{ANR} \cdot \frac{C}{d^2}}\right)^2 \cdot \left(1 + \frac{\kappa - 1}{2} \cdot M^2\right)}\right) \cdot \left(1 + \frac{\kappa - 1}{2} \cdot M^2\right)^{\frac{k}{1-k}} \tag{12}$$

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[6] The graph presented is the result of ANN evaluation. It is similar to the Figure 30a from (Iwaszko, 1999) because of the same input data

The proposed algorithm is as follows:

(1) The maximum value of the Mach number $M_{\max}$ is calculated from the Eq. (8).

(2) The border value of the pressure ratio Y_{gr} is calculated from the Eq. (11).

(3) If $Y \leq Y_{gr}$ then $M = M_{\max}$.

(4) If $Y > Y_{gr}$ then a value of the Mach number is calculated by solving the Eq. (12).

(5) Value of the airflow coefficient is calculated form the Eq. (7).

The only disadvantage of the algorithm, for CAD system, is the need of iterative calculations in order to solve the equation (12).

4. Utilisation of Artificial Neural Network

Because of the disadvantages mentioned above it was decided to verify the feasibility of an artificial neural network (ANN) for an approximation of the relation $\mu = f(C/d^2, b, Y)$. Exemplary courses of the relation are presented in the Figure 1. Nowadays ANN technology is quite often applied in pneumatics (KyoungKwan AHN and TU Diep Cong Thanh, 2004) as well as in other technical fields (Cheol Kim, 2002).

Utilising the proposed algorithm the learning set of data was generated which consisted of 13440 vectors. The nearly uniform set covered the entire range of the μ changes. The ranges of the input parameters were as follows: $C/d^2 = 1 \cdot 10^{-4} - 25 \cdot 10^{-4}$ with the step of $2 \cdot 10^{-4}$, $b = 0 - 0.95$ with the step of 0.05 and $Y = 0.05 - 0.99$ with the step of 0.02. In order to improve calculations for small values of μ vectors with $C/d^2 = 2 \cdot 10^{-4}$ were added to the set.

Assuming computer simulation of artificial neural network, after design work, a topology of the ANN was proposed. It was a feed — forward ANN with 3 inputs (for C/d^2, b and Y values), 1 output (for μ value) and 2 hidden layers. The first hidden layer consisted of 81 neurones, the second one of 9. All layers had biases. The output neurone had a linear activation function but the hidden layers had hyperbolic tangents. The network was trained with the back — propagation algorithm utilising the Levenberg — Marquard optimisation technique and after 1000 epochs the mean square error less than $8.18 \cdot 10^{-9}$ was obtained. The maximal relative error (on the learning set) was less than $37.6 \cdot 10^{-4}$ (see figure 2). Additionally the test set of 54 vectors was generated and the maximal relative error obtained was less than $7.7 \cdot 10^{-4}$.

5. Conclusions

The modified numerical algorithm for calculation of an airflow coefficient μ for the known value of a sonic conductance C and a critical pressure ratio b is proposed. The algorithm is numerically stable, computationally less expensive (than the original one (Iwaszko, 1999)) and more accurate (does not need an approximation procedure). Its utilisation results in more accurate computer simulation of a pneumatic system than the usage of the flow rate model from ISO 6358 standard (see Fig. 1).

Utilising the algorithm the learning data set for artificial neural network was produced. The obtained ANN (compared to the numerical algorithm) gives very small relative error (less than 0.4% — see Fig. 3) and can be practically utilised in computer aided calculations of a pneumatic system. But the obtained increase of calculation speed (comparing to the numerical algorithm) is

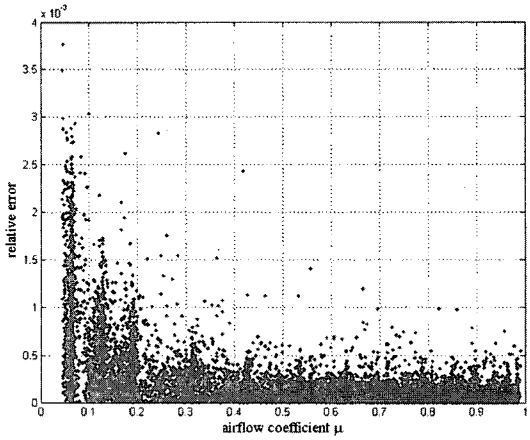

Fig. 3 Relative error for airflow coefficient μ calculated by ANN

not as large (several percent) as it was expected. It can be improved by a hardware implementation of the ANN. In such case it would be better to redesign the ANN to one with a single hidden layer.

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