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RINGS WHOSE PRIME RADICALS ARE COMPLETELY PRIME

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ABSTRACT. We study in this note rings whose prime radicals are completely prime. We obtain equivalent conditions to the complete 2-primal-ness and observe properties of completely 2-primal rings, finding examples and counterexamples to the situations that occur naturally in the process.

Throughout all rings are associative with identity. Given a ring R, the prime radical and the set of all nilpotent elements are denoted by $P(R)$ and $N(R)$, respectively. In this note we concern some conditions which are generalizations of domains. A ring is called *reduced* if it has no nonzero nilpotent elements; and a ring R is called *2-primal* if $P(R) = N(R)$, due to Birkenmeier-Heatherly-Lee[3]. It is obvious that a ring R is 2-primal if and only if $R/P(R)$ is a reduced ring. Hirano[7] used the term *N-ring* for what is called a 2-primal ring, showing that an N-ring R is strongly π-regular if and only if the n by n full matrix ring over R is strongly π-regular, where n is a positive integer. The class of 2-primal rings contains commutative rings and reduced rings.

Given a ring R, $r_R(-)$ ($l_R(-)$) is used for the right (left) annihilator over R. An element c of a ring R is called *right regular* if $r_R(c) = 0$, *left regular* if $l_R(c) = 0$, and *regular* if $r_R(c) = 0 = l_R(c)$. A ring is called a *domain* if every nonzero element is regular. It is well-known that a square matrix over a division ring is regular if and only if it is invertible. An ideal I of a ring R is called *completely prime* if R/I is a domain. We shall call a ring R *completely 2-primal* if $R/P(R)$ is a domain. It is obvious that domains are completely 2-primal and completely

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2-primal rings are 2-primal; but commutative rings and reduced rings need not be completely 2-primal. We use $P_c(R)$ for the intersection of all completely prime ideals of a ring R, and define $C(P(R)) = \{r \in R \mid r + P(R) \text{ is regular in } R/P(R)\}$. Shin proved that R is 2-primal if and only if every minimal prime ideal of R is completely prime [14, Proposition 1.11]; while Birkenmeier-Heatherly-Lee proved that a ring R is 2-primal if and only if $P(R) = P_c(R)$ [3, Proposition 2.1].

The following concepts are due to Shin[14]:

$$N(P) = \{a \in R \mid aRb \subseteq P(R) \text{ for some } b \in R\backslash P\};$$
$$N_P = \{a \in R \mid ab \in P(R) \text{ for some } b \in R\backslash P\};$$
$$\overline{N}_P = \{a \in R \mid a^m \in N_P \text{ for some positive integer } m\},$$

where P is a prime ideal of a ring R. Note that $N(P) \subseteq P$, $N(R) \subseteq \overline{N}_P$, and $N(P) \subseteq N_P \subseteq \overline{N}_P$.

PROPOSITION 1. *Given a ring R the following conditions are equivalent:*

(1) *R is completely 2-primal;*
(2) *R is 2-primal and $P_c(R)$ is completely prime;*
(3) *Every minimal prime ideal of R and $P_c(R)$ are both completely prime;*
(4) *$C(P(R)) = R\backslash P(R)$;*
(5) *$N(P) = N_P = \overline{N}_P = P(R)$ for any minimal prime ideal P of R;*
(6) *$N(P) = N_P = \overline{N}_P = P(R)$ for any minimal completely prime ideal P of R.*

PROOF. The proofs of (1)⇔(2) and (2)⇔(3) are obtained from [3, Proposition 2.1] and [14, Proposition 1.11]. (1)⇔(4) is a restatement of the definition.

(1)⇒(5): Let P be a minimal prime ideal of R. Clearly $N(P) \subseteq N_P \subseteq \overline{N}_P$. Let $a \in \overline{N}_P$, then $a^n b \in P(R)$ for some $b \in R\backslash P$ and positive integer n. Since $R/P(R)$ is a domain by the condition, $aRb \subseteq P(R)$ and so $a \in N(P)$; hence we have $N(P) = N_P = \overline{N}_P$. By [14, Corollary 1.9], $N(P) = \cap\{Q \mid Q \text{ is a prime ideal of } R \text{ with } Q \subseteq P\}$. But P is a minimal prime ideal of R and so $N(P) = P$. From the condition that $R/P(R)$ is a domain, $P(R)$ is the unique minimal prime ideal of R and thus $N(P) = N_P = \overline{N}_P = P(R)$.

(5)⇒(6): Given $a \in N(R)$ with $a^n = 0$, we have $a \in \overline{N}_P = N_P = N(P) = P(R)$ for any minimal prime ideal P of R by the condition, verifying that $N(R) = P(R)$ (i.e., R is 2-primal). So every minimal prime ideal of R is completely prime by [14, Proposition 1.11]; hence an ideal of R is a minimal prime ideal if and only if it is a minimal completely prime ideal.

(6)$\Rightarrow$(1): Let P be a minimal completely prime ideal of R. Note that R is 2-primal by the condition that $\overline{N}_P = P(R)$, and so any minimal completely prime ideal is a minimal prime ideal by [14, Proposition 1.11]. So P is a minimal prime ideal of R and $N(P) = P$ by [14, Corollary 1.10]; hence we have $P = N(P) = N_P = \overline{N}_P = P(R)$, proving that R is completely 2-primal. □

The *index* (*of nilpotency*) of a nilpotent element x in a ring R is the least positive integer n such that $x^n = 0$. The *index* of a subset I of R is the supremum of the indices of nilpotency of all nilpotent elements in I. If such a supremum is finite, then I is said to be *of bounded index.*

Lemma 2. *Let R be a ring of bounded index. If I is a nonzero nil one-sided ideal of R then I contains a nonzero nilpotent ideal.*

Proof. By Levitzki's Lemma [6, Lemma 1.1] or [11, Lemma 5]. □

A ring R is called *strongly prime* if R is prime with no nonzero nil ideals, and an ideal P of R is called *strongly prime* if R/P is strongly prime. We use $N_r(R)$ to denote the upper nilradical of a ring R, i.e., the unique maximal nil ideal of R by [13, Proposition 2.6.2].

Corollary 3. *Suppose that a ring R is of bounded index. Then the following conditions are equivalent:*

(1) *R is completely 2-primal;*
(2) *$P_c(R)$ is completely prime with $N(R) = N_r(R) = P_c(R) = P(R)$;*
(3) *Every minimal strongly prime ideal of R and $P_c(R)$ are both completely prime;*
(4) *$N(P) = N_P = \overline{N}_P = P(R)$ for any minimal strongly prime ideal P of R.*

Proof. Since R is of bounded index by hypothesis it follows that $R/P(R)$ is also of bounded index because $P(R)$ is nil; thus we have $P(R) = N_r(R)$ by Lemma 2; hence R is 2-primal if and only if $N(R) = P(R) = N_r(R)$. Note that $N(R) = N_r(R)$ if and only if every minimal strongly prime ideal of R is completely prime [8, Corollary 13]. Thus if R is a 2-primal ring of bounded index, then we have that an ideal I of R

is a minimal prime ideal if and only if I is a minimal completely prime ideal if and only if I is a minimal strongly prime ideal, with the help of [14, Proposition 1.11]. So we obtain the equivalences by Proposition 1. □

The hypothesis and conditions in Proposition 1 and Corollary 3 are not superfluous by the following.

EXAMPLE 4. (1) The hypothesis in Corollary 3 is not superfluous. Let $\mathbb{V}$ be a vector space over F, the field of integers modulo 2, such that $\mathbb{V}$ is countably infinite dimensional with a basis $\{v(0), v(1), v(-1), \ldots, v(i), v(-i), \ldots\}$. Note that there is a unique endomorphism $f(i)$ of $\mathbb{V}$ for $i = 1, 2, \ldots$ such that $f(i)(v(j)) = 0$ if $j \equiv 0 (\text{mod } 2^i)$ and $f(i)(v(j)) = v(j-1)$ if $j \not\equiv 0 (\text{mod } 2^i)$. Let S be the ring, without identity, of endomorphisms of $\mathbb{V}$ generated by the endomorphisms $f(1), f(2), \ldots$; next let R be the ring obtained from S by adjoining the identity map of $\mathbb{V}$. Then R is semiprime and $N_r(R) = S$ by [2, p.540]. Since $R/S \cong F$ and $N(R) = S \neq 0$, $N_r(R) = N(R)$ is the only strongly prime ideal of R. Next since every completely prime ideal of R must contain S, we have $S = P_c(R) = N_r(R) = N(R)$. Consequently $P_c(R)$ is completely prime. However R is not of bounded index and R is not completely 2-primal since R is semiprime.

(2) The conditions in Proposition 1 and Corollary 3 are not superfluous. Let D be a domain and $R = \begin{pmatrix} D & D \\ 0 & D \end{pmatrix}$, the 2 by 2 upper triangular matrix ring over D. Since $P(R) = \begin{pmatrix} 0 & D \\ 0 & 0 \end{pmatrix}$ and $R/P(R) \cong D \oplus D$, R is 2-primal but not completely 2-primal. $P_c(R) = P(R) = N(R) = N_r(R) = \begin{pmatrix} D & D \\ 0 & 0 \end{pmatrix} \cap \begin{pmatrix} 0 & D \\ 0 & D \end{pmatrix} = \begin{pmatrix} 0 & D \\ 0 & 0 \end{pmatrix}$, so $P_c(R)$ is not completely prime.

For a commutative case, let R be the ring $\mathbb{Z}_6$ of integers modulo 6. Since $P_c(R) = P(R) = N(R) = N_r(R) = 2R \cap 3R = 0$, $P_c(R)$ is not completely prime and R is not completely 2-primal; but R is commutative (so 2-primal).

Notice that each prime ideal in the previously mentioned rings is minimal prime, minimal strongly prime, and completely prime. □

A ring is called *abelian* if every idempotent in it is central.

PROPOSITION 5. *Let R be a completely 2-primal ring. Then the identity is the only nonzero idempotent* (so R is abelian).

PROOF. Let $0 \neq e^2 = e \in R$, then $e \notin P(R)$. Since $R/P(R)$ is a domain, $1 + P(R) = e + P(R)$ and $(1-e)^2 = 1 - e \in P(R)$; hence $1 - e = 0$ and $e = 1$. □

The converse of Proposition 5 is not true in general by Example 4(1). For, the Jacobson radical of R is $N_r(R)$ and R is a local ring with $R/N_r(R) \cong F$; hence the identity is the only nonzero idempotent.

PROPOSITION 6. *The class of completely 2-primal rings is closed under subrings.*

PROOF. Let R be a completely 2-primal ring and S be a subring of R. Then R is clearly 2-primal, and so the subring S is also 2-primal by [3, Proposition 2.2]. Take $a \in S$ with $a \notin P(S)$, then a is not nilpotent since $P(S) = N(S)$, forcing $a \notin P(R)$ since $P(R) = N(R)$. Here assume $a \notin C(P(S))$ with $C(P(S)) = \{r \in S \mid r + S \text{ is regular in } S/P(S)\}$. Then there is $b \in S$ such that $b \notin P(S)$ and ab (so ba) is contained in $P(S) = N(S) \subseteq N(R) = P(R)$. Since $R/P(R)$ is a domain and $a \notin P(R)$, we have $b \in P(R)$. Notice that $P(S) = P(R) \cap S$ because $P(S) = N(S)$ and $P(R) = N(R)$. So we get $b \in P(S)$, a contradiction; consequently $a \in C(P(S))$. Thus $C(P(S)) = S \backslash P(S)$ and S is completely 2-primal.□

The class of 2-primal rings is closed under direct sums [3, Proposition 2.2]; however this result does not hold for completely 2-primal rings by $D \oplus D$ with D a domain. Closely related to Proposition 6, one may consider affirmative situations for factor rings of completely 2-primal rings. However this argument needs not hold by the following.

EXAMPLE 7. Let $\mathbb{Z}$ be the ring of integers and $\mathbb{Z}[x]$ be the polynomial ring with an indeterminate x over $\mathbb{Z}$. Then clearly $\mathbb{Z}[x]$ is completely 2-primal, but $\frac{\mathbb{Z}[x]}{(a+x)(b+x)\mathbb{Z}[x]}$ is not completely 2-primal for all $a, b \in \mathbb{Z}$ with $a \neq b$. In fact, $((a+x)+I)((b+x)+I) = 0$ for $(a+x)+I \neq 0$ and $(b+x)+I \neq 0$, but $P(S) = 0$, where $S = \frac{\mathbb{Z}[x]}{(a+x)(b+x)\mathbb{Z}[x]}$ and $I = (a+x)(b+x)\mathbb{Z}[x]$. □

Let R be a ring and $UTM_n(R)$ be the n by n upper triangular matrix ring over R, where n is a positive integer. If R is 2-primal then so is $UTM_n(R)$ by [3, Proposition 2.5]; however this result does not hold for completely 2-primal rings by Proposition 5 since $UTM_n(R)$ is nonabelian when $n \geq 2$. But some kind of subrings of $UTM_n(R)$ are completely 2-primal as in the following.

PROPOSITION 8. *Following* [13, Example 2.7.38], *let S be a ring and T be the n by n upper triangular matrix ring over S, where n may* *be infinite. Let* $N = \{(a_{ij}) \in T \mid a_{ii} = 0$ *for all* i *and there is* k *such that* $a_{ij} = 0$ *for* $i \geq k\}$ *and set* R *be the subring of* T *generated by the ide-ntity and* N. *Then* S *is completely 2-primal if and only if so is* R.

PROOF. It suffices to show the necessity by Proposition 6. Notice that every prime ideal of R is of the form $\{r \in R \mid$ the set of all diagonal entries of r is $P\} \supseteq N$, where P is any prime ideal of S. So we obtain $\frac{R}{P(R)} \cong \frac{S}{P(S)}$; hence R is completely 2-primal since S is completely 2-primal by the condition. □

The following is proved by Propositions 6 and 8.

COROLLARY 9. *Let* S *be a ring and define*

$$R = \left\{ \begin{pmatrix} a & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_{(n-1)n} \\ 0 & 0 & 0 & \cdots & a \end{pmatrix} \mid a, a_{ij} \in S \right\},$$

where n *is a positive integer. Then* R *is completely 2-primal if and only if so is* S.

A ring R is called *strongly* π*-regular* if for every a in R there exist a positive integer n, depending on a, and an element b in R satisfying $a^n = a^{n+1}b$. It is obvious that a ring R is strongly π-regular if and only if R satisfies the descending chain condition on principal right ideals of the form $aR \supseteq a^2R \supseteq \cdots$, for every a in R. Dischinger[4] showed that the strongly π-regularity is left-right symmetric. A ring R is called π*-regular* if for each $a \in R$ there exist a positive integer n, depending on a, and $b \in R$ such that $a^n = a^n b a^n$. Strongly π-regular rings are π-regular by Azumaya[1], and it is easy to show that the Jacobson radicals of π-regular rings are nil. $J(R)$ denotes the Jacobson radical of given a ring R. The following result is similar to [7, Theorem 1].

PROPOSITION 10. *Suppose that a ring* R *is completely 2-primal. Then the following conditions are equivalent:*

(1) R *is* π*-regular;*
(2) R *is a local ring with* $P(R) = J(R)$*;*
(3) $R/P(R)$ *is strongly* π*-regular and* $P(R)$ *is the unique prime ideal in* R*;*
(4) $R/P(R)$ *is* π*-regular and* $P(R)$ *is the unique prime ideal in* R*;*

(5) *$R/P(R)$ is π-regular and $P(R)$ is the unique primitive ideal in R;*
(6) *$R/P(R)$ is π-regular and $P(R)$ is the unique maximal ideal in R;*
(7) *R is strongly π-regular.*

PROOF. (1)$\Rightarrow$(2): $J(R)$ is nil since R is π-regular, and so R being completely 2-primal implies $J(R) = P(R)$. Take $a \notin P(R)$. Since R is π-regualr and completely 2-primal, $a^n b$ and ba^n are nonzero idempotents in R for some $b \in R \backslash P(R)$ and positive integer n; hence $a^n b = 1 = ba^n$ by Proposition 5 and so a is invertible.

(2)$\Rightarrow$(3), (3)$\Rightarrow$(4), (4)$\Rightarrow$(5), (5)$\Rightarrow$(6), (6)$\Rightarrow$(4), and (7)$\Rightarrow$(1) are obvious.

(3)$\Rightarrow$(7): By [5, Theorem 2.1].

(4)$\Rightarrow$(2): For $a \notin P(R)$, $a^n + P(R) = (a^n + P(R))(b + P(R))(a^n + P(R))$ for some $b \in R$ and positive integer n because $R/P(R)$ is π-regular. Since R is completely 2-primal by hypothesis, $a^n b + P(R)$ and $ba^n + P(R)$ are both nonzero idempotents. But $P(R)$ is idempotent-lifting, and hence there are nonzero idempotents $e, f \in R$ such that $e + P(R) = a^n b + P(R)$ and $f + P(R) = ba^n + P(R)$. By Proposition 5, e and f are the identity of R; hence $1 - a^n b, 1 - ba^n \in P(R)$ and it follows that a is invertible. □

Given a ring R, the polynomial ring and the formal power series ring over R are denoted by $R[x]$ and $R[[x]]$ with x the indeterminate, respectively; also $R[X]$ and $R[[X]]$ denotes the polynomial ring and the formal power series ring over R with X a set of commuting indeterminates over R, respectively. Birkenmeier-Heatherly-Lee proved that a ring R is 2-primal if and only if so is $R[X]$ [3, Proposition 2.6]. In the following we have the same result for completely 2-primal rings.

PROPOSITION 11. *A ring R is completely 2-primal if and only if so is $R[X]$.*

PROOF. It is well-known that $P(R[X]) = P(R)[X]$, so we have

$$\frac{R}{P(R)}[X] \cong \frac{R[X]}{P(R)[X]} = \frac{R[X]}{P(R[X])}.$$

Thus $R[X]$ is completely 2-primal if so is R, and the converse follows by Proposition 6. □

However this result does not hold for formal power series rings by the following.

EXAMPLE 12. Let S be a division ring and T be the n by n upper triangular matrix ring over S, where n is infinite. Let $N = \{(a_{ij}) \in T \mid a_{ii} = 0$ for all i and there is k such that $a_{ij} = 0$ for $i \geq k\}$ and set R be the subring of T generated by the identity and N. Then R is completely 2-primal by Proposition 8. Next we use a method of [12, Example 1.1]. Let e_{ij} be the infinite matrix over S with (i,j)-entry 1 and elsewhere 0, and take

$$f(x) = e_{12}x + (e_{34} + e_{56})x^2 + \cdots + (\Sigma_{i=0}^{2^{n-1}-1} e_{(2^n+2i-1)(2^n+2i)})x^n + \cdots,$$

$$g(x) = e_{23}x + (e_{45} + e_{67})x^2 + \cdots + (\Sigma_{i=0}^{2^{n-1}-1} e_{(2^n+2i)(2^n+2i+1)})x^n + \cdots$$

in $R[[x]]$. Then $f(x)$ and $g(x)$ are nilpotent of index 2 but $f(x) + g(x)$ is non-nilpotent. Hence $f(x) \notin P(R[[x]])$ or $g(x) \notin P(R[[x]])$, implying that $R[[x]]$ is not completely 2-primal. □

Note that the ring in the preceding example is not of bounded index, but we obtain an affirmative situation when given rings are of bounded index as follows.

PROPOSITION 13. *Suppose that R is a ring of bounded index. Then R is completely 2-primal if and only if so is $R[[X]]$.*

PROOF. It suffices to show the necessity by Proposition 6. $P(R[[X]]) \subseteq P(R)[[X]]$ by [10, Corollary 1.2]. $P(R)$ is nil of bounded index by hypothesis, so $P(R)[[X]]$ is also nil of bounded index by [9, Theorem 2.4]; hence we have $P(R)[[X]]) = P(R[[X]])$ by Lemma 2, considering the nil ideal $\frac{P(R)[[X]]}{P(R[[X]])}$ of $\frac{R[[X]]}{P(R[[X]])}$ that is of bounded index. Consequently

$$\frac{R[[X]]}{P(R[[X]])} = \frac{R[[X]]}{P(R)[[X]]} \cong \frac{R}{P(R)}[[X]]$$

and therefore$R[[X]]$ is completely 2-primal if so is R. □

Given a ring R it is obvious that $C(0)$ is contained in $C(P(R))$; hence one may conjecture that the converse also holds for completely 2-primal rings, based on the definition. However it needs not be true by the following.

EXAMPLE 14. Let $\mathbb{Z}$ be the ring of integers and $\mathbb{Z}_n$ be the ring of integers module n, say $n \geq 2$ and $\mathbb{Z}_n = \{\bar{0}, \bar{1}, \ldots, \overline{n-1}\}$. Set $R =$

$\left\{\begin{pmatrix} m & \bar{a} \\ 0 & m \end{pmatrix} \mid m \in \mathbb{Z} \text{ and } \bar{a} \in \mathbb{Z}_n\right\}$ with matrix operations and module operations. Then R is completely 2-primal because $P(R) = \begin{pmatrix} 0 & \mathbb{Z}_n \\ 0 & 0 \end{pmatrix}$ and $\frac{R}{P(R)} \cong \mathbb{Z}$. But we have

$$C(0) = \left\{\begin{pmatrix} m & \bar{a} \\ 0 & m \end{pmatrix} \in R \mid \bar{m} \neq \bar{0} \text{ is invertible in } \mathbb{Z}_n\right\}$$

and

$$C(P(R)) = R \backslash P(R) = \left\{\begin{pmatrix} s & \bar{a} \\ 0 & s \end{pmatrix} \in R \mid s \neq 0\right\},$$

showing $C(0) \subsetneqq C(P(R))$. □

If completely 2-primal rings are π-regular then $C(0)$ coincides with $C(P(R))$ by Proposition 10.

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