신장트리 기반 유전자 알고리즘에 의한 비선형 fcTP 해법

(Solving Nonlinear Fixed Charge Transportation Problem by Spanning Tree-based Genetic Algorithm)

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요 약 수송문제는 산업공학 및 OR 그리고 전자계산학 분야에서 중요한 문제 중의 하나로 인식된다. 수송 문제가 시설을 수립하거나 고객들의 요구를 이행하기 위한 추가적인 고정 비용과 연관될 때, fcTP(fixed charge Transportation Problem)라 한다. fcTP는 이전의 고전적인 방법으로 해결하기 어려운 NP-hard 문제들 중의 하나이다. 본 논문에서는 비선형 ftTP를 해결하기 위한 신장트리 기반 유전자알고리즘을 제안한다. 특히, 염색체(chromosome)에 대한 feasibility criteria와 repairing procedure를 포함하는 GA 염색체 표현에 대해 새로운 아이디어를 제안한다. 또한, 본 논문에서 제안하는 방법의 효율성을 입증하기 위한 여러 가지 수치 실험 결과를 기술한다.

키워드 : 비선형 fixed charge 문제, 수송문제, 신장트리 기반 유전자 알고리즘

Abstract The transportation problem (TP) is known as one of the important problems in Industrial Engineering and Operational Research (IE/OR) and computer science. When the problem is associated with additional fixed cost for establishing the facilities or fulfilling the demand of customers, then it is called fixed charge transportation problem (fcTP). This problem is one of NP-hard problems which is difficult to solve it by traditional methods. This paper aims to show the application of spanning-tree based Genetic Algorithm (GA)approach for solving nonlinear fixed charge transportation problem. Our new idea lies on the GA representation that includes the feasibility criteria and repairing procedure for the chromosome. Several numerical experimental results are presented to show the effectiveness of the proposed method.

Key words : nonlinear fixed charge problem, transportation problem, spanning tree-based genetic algorithm

1. Introduction

In this several decades, there have been many researchers reported new models or methods to determine the transportation or the logistic activities that can give the least cost [1,2]. A general definition of logistics is: *the quality of a flow of materials*, such as frequency of departures (number per time unit), transportation time, adherence to the transportation time schedule, and so on [3]. Products can be manufactured and sent to distribution centers, retailers or plants.

The classical transportation problem (TP) refers to a special class of linear programming. It is well known as a basic network problem. The first formulation and discussion of a planar transportation model was introduced by Hitchcock [4]. The objective is to find a way to transport homogeneous products from several sources to several destinations so that the total cost can be minimized. In the past several decades a variety of deterministic and/or stochastic models have been developed. In more real world applications, it is

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often that the problem is also associated with the fixed cost for establishing the facilities or fulfilling the demand of customers from a certain source center. It has been shown that this fixed cost transportation problem (fcTP) is NP-hard problem and more practical transportation problem (TP) in production-distribution sector [5]. Many practical transportation and distribution problems such as the minimum cost network flow (transshipment) problem with fixed charge in logistics can be formulated as fixed charge transportation problems. This problem is often formulated and solved as a mixed integer network-programming problem. Any general mixed integer programming solution method can be used to solve the fcTP such as a branch-and-bound method and a cutting plane method. Because they do not take advantage of the special network structure of the fcTP, however, these methods are generally inefficient and computationally expensive [6]. Several heuristic methods was proposed for solving linear fixed cost transportation problem [7,8]. Another variant for the fcTP for a single product is given by Chien [9]. Here, transportation costs are modeled with a fixed charge per truck and no variable cost.

Since it was introduced by Holland [10], Genetic Algorithm (GA) has taken a great attention of researchers. It has been used to solve many difficult combinatorial optimization problems (see for example [1,2,11]). When using a GA, one of the important factors is the method for representing the chromosome. It has been found to give great effect on the effectiveness and the efficiency of the GA. As transportation problem (TP) as one of network design problems, it has a special data structure in solution characterized as a spanning tree. Thus, a spanning tree-based representation would be appropriate for this problem and recently applied it to a telecommunication network design problem [12] and an allocation-location problem of customers to warehouses [13].

In this paper, we consider the fixed charge transportation problem which has nonlinear term in objective function. We attempt to use the spanning tree-based genetic algorithm to find the best heuristic solution to the problem. Our proposed GA adopts the Prüfer number encoding which is the one of the popular tree encoding [14,15]. Our innovation lies in the representation used (Prufer numbers), especially in developing a new feasibility criteria for Prufer number, adopting non-standard genetic operations and the repairing procedure to ensure that Prufer numbers always generate feasible spanning tree. These make our proposed method can be applied for relatively large size problems.

The rest of this paper is organized as follows: In the next Section 2, the Mathematical formulation of this problem is given. We describe the design of our algorithm in Section 3. In Section 4, numerical experimental results are presented to demonstrate the efficiency of the proposed method. Finally, some concluding remarks are given in Section 5.

2. Problem Description

In the fcTP, two types of costs are considered simultaneously when the best course of action is selected, a variable cost proportional to the activity level and a fixed cost. The fcTP seeks the determination of a minimum cost transportation plan for a homogeneous commodity from a number of plants to a number of consumers. It requires the specification of the level of supply at each plant, the amount of demand at each consumer, and the transportation cost and fixed cost from each plant to each consumer. The goal is to allocate the supply available at each plant so as to optimize a criterion while satisfying the demand at each consumer. The usual objective function is to minimize the total variable cost and fixed costs from the allocation. It is one of the simplest combinatorial problems involving constraints. In the some cases, the variable cost be associated with quadratic variables in which the cost function will be nonlinear. This transportation problem with given m plants and n consumers can be formulated as follows:

$$\min \quad f(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij}^2 + d_{ij}g_{ij}(x))$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j, \quad j = 1,2,\ldots,n$$

$$x_{ij} \ge 0, \qquad \forall i,j$$

$$\text{with} \quad g_{ij}(x) = \begin{cases} 0, & \text{if } x_{ij} \le 0 \\ 1, & \text{if } x_{ij} > 0 \end{cases}$$

where $x=[x_{ij}]$ is the unknown quantity to be transported on the route (i,j) that from plant i to consumer j, c_{ij} is the shipping cost per unit from plant i to consumer j. The total shipping cost is depended on quadratic of the shipping units shown as a nonlinear term. And d_{ij} is the fixed cost associated with route (i,j). a_{ij} is the number of units available at plant i, and b_j is the number of units demanded at consumer j.

In this paper, we assume a balanced transportation problem, because the unbalanced transportation problem can be converted to a balanced transportation problem by introducing a dummy plant or a dummy consumer.

3. Spanning Tree-based Genetic Algorithm Design

Nowadays, as the use of computers is rapidly increasing, many evolutionary computation methods for solving optimization problems have been introduced such as Genetic Programming, Evolutionary Strategies or Evolutionary Programming, Tabu Search, Simulated Annealing and so on. Among them, Genetic Algorithm (GA) has been one of popular methods. GA has been shown as a robust optimization technique to solve many real world problems [1,2]. Michalewicz used a GA for solving linear and nonlinear transportation problems [16]. In the method, each chromosome is represented by using a $m \times n$ matrix.

3.1 Spanning Tree-based Genetic Algorithm

As told before, TP is a network problem which its feasible solution has spanning tree topology. Thus, spanning tree-based representation would also appropriate for this problem. The use of the Prufer number representation for solving various network problems was introduced by Gen and Cheng [1]. They utilized the Prufer number [2] as

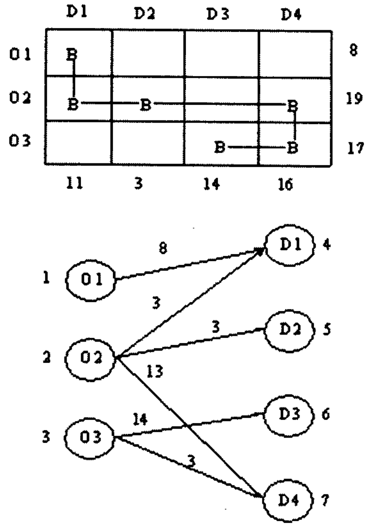

Figure 1 Illustration of a basis on the transportation tableau and the transportation graph

it is capable of equally and uniquely representing all possible trees in a network graph. They noted that the use of the Prufer number is more suitable for encoding a spanning tree, especially in some research fields like some extended transportation problems [17], production/distribution problem [11, 18], minimum spanning problems, and so on [2,11].

Transportation graph in Figure 1 can be represented as a spanning tree such as in Figure 2 with its Prüfer number.

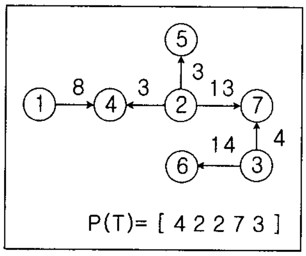

Figure 2 A spanning tree and its Prüfer number

3.2 Initialization

The initialization of a chromosome (a Prüfer number) is performed from randomly generated $m+n-2$ digits in range $[1, m+n]$. When generating the Prufer number, there will be a possibility that it cannot be adapted into the transportation network graph. Due to this reason, the feasibility should be checked before decoding the Prufer number into the spanning tree. Gen and Cheng developed the feasibility criteria for the Prufer number; however, it seems that their criterion is a little bit complex [2]. Moreover, they did not introduce any procedure for handling the infeasible chromosome. In their technique, they randomly generate the Prufer number. This means that the probability for each node to appear in the Prufer number is the same. We found that this technique cannot generate the feasible chromosome when the difference between the number of source nodes and the number of demand nodes is very big. It only works when the number of source nodes and demand nodes are almost the same. Here, we developed a new and simple feasibility criterion for checking the feasibility of the Prufer number to be decoded into a spanning tree and also the procedure for repairing infeasible chromosomes.

Feasibility Criterion

Let L_i, $i \in \{1,2,\cdots,m+n\}$ be the appearance number of node i in Prufer number $P(T)$, the Prufer number $P(T)$ is said to be feasible If

$$\sum_{i=1}^{m} L_i + 1 = \sum_{i=m+1}^{m+n} L_i + 1$$

Procedure: Repairing

```
switch = 0;
do
 ts = 0, td = 0;
 Calculate the appearance of each node
 for(i = 1; i <= m; i++)
   ts = L_i + 1;
 for(i = m+1; i <= m+n; i++)
   td = L_i + 1;
 if(ts == td)
   switch = 1;
 else{
   switch = 0;
   r = rand()%(m + n - 2);
   if(ts < td)
     Prufer(r) = rand()%m;
   else
     Prufer(r) = (rand()%n) + m;
 }
while(switch == 0)
```

After generating a feasible Prufer number, a transportation network graph can be determined by using the following decoding procedure:

Procedure: Convert Prüfer Number to Transportation Tree

step 1: Let $P(T)$ be the original Prüfer number and let $\overline{P}(T)$ be the set of all nodes that are not part of $P(T)$ and designed as eligible for consideration.

step 2: Repeat the following process (2.1)-(2.5) until no digits are left in $P(T)$.

2.1 Let i be the lowest numbered eligible node in $\overline{P}(T)$. Let j be the leftmost digit of $P(T)$.

2.2 if i and j are not in the dame set O or D, add the $edge(i,j)$ to tree T. Otherwise, select the next digit k from $P(T)$ that is not included in the same set with i, exchange j with k, and add the $edge(i,k)$ to the tree T.

2.3 Remove j(or k) from $P(T)$, put it into $\overline{P}(T)$. Designate i as no longer eligible.

2.4 Assign the available amount of units to $x_{ij} = \min\{a_i, b_j\}$ (or $x_{ik} = \min\{a_i, b_k\}$) to the $edge(i,j)$ (or (i,k)), where $i \in O$ and j, $k \in D$.

2.5 Update availability $a_i = a_i - x_{ij}$ and $b_j = b_j - x_{ij}$ (or $b_k = b_k - x_{ik}$).

step 3: If no digits remain in $P(T)$ then there are exactly two nodes , i and j, still eligible in $\overline{P}(T)$ for consideration. Add $edge(i,j)$ to tree T and form a tree with $m+n-1$ edges.

step 4: If there no available units to assign, then stop. Otherwise, there are supply r and demand s remaining. Add the $edge(r,s)$ to the tree and assign the available amount $x_{rs} = a_r = b_s$ to the edge. If there exists a cycle, remove the edge that is assigned zero flow. A new spanning tree is formed with $m+n-1$ edges.

3.3 Genetic Operators

Crossover

Crossover is known as the most important recombination operator in GA. There have been many variants of crossover operations given in the literature. For efficiency, we used simple one point crossover. This type of crossover is accomplished by selecting two parent solutions and randomly taking a component from one parent to form the corresponding component of the offspring. The illustrative idea of the one-point crossover can be seen in Figure 3.

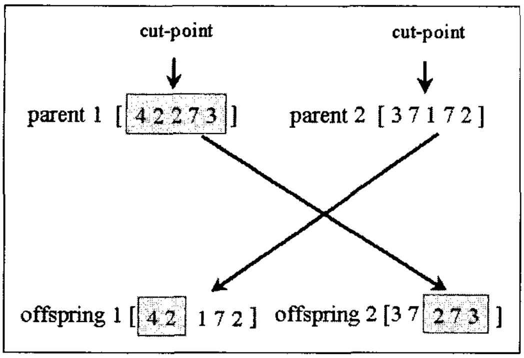

Figure 3 Illustration of crossover operations

Mutation

Mutation is usually used to prevent premature lost of information. It is done by exchanging the information within a chromosome. We used two mutation operations called inversion mutation and displacement mutation as shown in Figure 4. The inversion mutation is done by selecting two positions within a chromosome at random and then inverts the sub-string between these two positions. The displacement mutation selects a sub-string at random and inserts it in a random position.

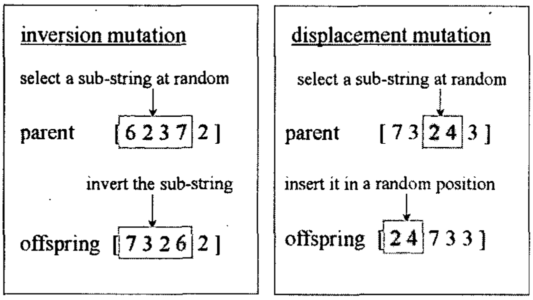

Figure 4 Illustration of mutation operations

3.4 Evaluation

As in nature, it is necessary to provide driving mechanism for better individuals to survive. Evaluation is to associate each chromosome with a fitness value that shows how good it is based on its achievement of the objective function. The higher fitness value of an individual, the higher its chances for survival for the next generation. So the evaluation plays avery important role in the evolutionary process. For this problem, we used the objective function as the fitness value. This fitness value is computed during the decoding of the chromosome. The evaluation procedure of the chromosome consists of two steps as follows:

Procedure: evaluation
Step 1: convert a chromosome into a tree.
Step 2: calculate each objective function.

3.5 Selection

The chromosomes are selected for each subpopulation in the next generation based on their fitness value. Before doing the selection, all chromosome (parent and offspring) in the current generation in combined together. The first and the second subpopulations of chromosome for the next generation are then generated by using the mixed strategy that combined $(\mu+\lambda)$-selection and roulette wheel selection is used.

4. Numerical Experiments

The computational experiments with two problems were carried out. The computational program was coded with C language and ran in the PC with Pentium IV 2.0GHz. The sizes of the problems are 4×5 and 5×10 respectively. The total supplies (or total demands) and the ranges of the fixed costs for two test problems are generated randomly as given in Table 1. The unit variable costs in two test problems are integers ranging from 1 through 8 given in Table 2 and Table 3.

The demands for the each consumer 1 to 5 are

Table 1 Total supply (demand) and ranges of fixed cost

Problem size (m × n)	Total supply	Range of fixed cost
4 × 5	275	[50,100]
5 × 10	1485	[150, 500]

Table 2 Unit variable costs in the 4 ×5 problem

O	Shipping costs c_{ij}					Fixed costs d_{ij}				
	1	2	3	4	5	1	2	3	4	5
1	8	4	3	5	8	60	88	95	76	97
2	3	6	4	8	5	51	72	65	87	76
3	8	4	5	3	4	67	89	99	89	100
4	4	6	8	3	3	86	84	70	92	88

Table 3 Unit variable costs in the 5 ×10 problem

O	Shipping costs c_{ij}										Fixed costs d_{ij}									
O	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
1	8	4	3	5	2	1	3	5	2	6	160	488	295	376	297	360	199	292	481	162
2	3	3	4	8	5	3	5	1	4	5	451	172	265	487	176	260	280	300	354	201
3	7	4	5	3	4	2	4	3	7	3	167	250	499	189	340	216	177	495	170	414
4	1	2	8	1	3	1	4	6	8	2	386	184	370	292	188	206	340	205	465	273
5	4	5	6	3	3	4	2	1	2	1	156	244	460	382	270	180	235	355	276	190

as follows: $b_1=88$, $b_2=57$, $b_3=24$, $b_4=73$, $b_5=33$. And the supplies from the each plant 1 to 4 are as follows: $a_1=57$, $a_2=93$, $a_3=50$, $a_4=75$.

The parameters for this GA in the problem 1 are as follows: crossover rate $P_c=0.2$, mutation rate $P_m=0.4$, population size 100, maximum generation 500 and to run the program 10 times. The following local optimal solution was obtained with total cost 37090 in the almost times.

	D_1	D_2	D_3	D_4	D_5
O_1		33	24		
O_2	60				33
O_3		24		26	
O_4	28			47	

The experiments for the problem 2, the parameters of the GA were set as mutation rate 0.4, crossover rate 0.2, maximum generation 1000, population size 100. The found solution, which is local optimal solution, is given in the following tableau with the objective value 304200.

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
O_1					83				74	
O_2			90					124	79	
O_3		62				88				
O_4	225	88		215	47					
O_5							57		120	133

5. Conclusion

In this paper we proposed the spanning tree-based genetic algorithm to solve the nonlinear fixed charge transportation problem. The proposed method was developed based on the solution structure for the linear transportation problem. The proposed spanning tree-based genetic algorithm requires only $m+n-2$ digits for the chromosome representation but the existing methods require $m\times n$ digits. Especially we proposed new idea about the repairing procedure and feasibility criteria for the chromosome representation. We have done several numerical experiments to show the effectiveness of the proposed method.

Since the algorithm has not absorb the characteristic of the nonlinear structure, so the genetic algorithm based on the spanning tree must to be improved while it is used in the solving the nonlinear fixed charge transportation problem.

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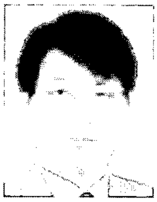

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