A Modified Weighted Least Squares Range Estimator for ASM (Anti-Ship Missile) Application

Ick-Ho Whang, Won-Sang Ra, and Jo-Young Ahn

Abstract: A practical recursive WLS (weighted least squares) algorithm is proposed to estimate relative range using LOS (line-of-sight) information for ASM (anti-ship missile) application. Apart from the previous approaches based on the EKF (extended Kalman filter), to ensure good convergence properties in long range engagement situations, the proposed scheme utilizes LOS rate measurements instead of conventionally used LOS angle measurements. The estimation error property for the proposed filter is investigated and a simple error compensator is devised to enhance its estimation error performances. Simulation results indicate that the proposed filter produces very accurate range estimates with extremely small computations.

Keywords: Anti-ship missiles, EKF, homing guidance, range estimation, WLS.

1. INTRODUCTION

When a missile becomes jammed or when it is equipped with passive seekers, the missile cannot sense relative ranges between itself and the intended target but can only measure LOS information such as LOS angles or LOS rates. However, the range can be estimated from the LOS or LOS rate measurements if the missile maintains appropriate maneuvers to ensure the observability for range estimation.

Over the last few decades, as one of the representative practical nonlinear filtering problems, many researchers have studied this range estimation problem by means of the application of EKF or its variants. One of the major issues of the problem is the bias errors of EKF based filters. To cope with the bias problem, Song proposed MGEKF (modified gain EKF), but its effectiveness was limited [1]. Moorman insisted that the bias of EKF range estimate is caused by the correlation between the EKF gain and its innovation sequence, and he subsequently proposed an unbiased EKF range estimator [2]. Though it works successfully in some cases, it still shows unsatisfactory performance in many real situations. For the another approach to overcome the bias problem of conventional EKF range estimators when the LOS rate is very high or near zero, Peach suggested the RPEKF (range-parameterized EKF), which consists of a set of EKFs with a different initial range estimate. At each update, the filters are weighted along their consistency with the measured LOS angle. In addition, the unreliable filters are chosen and discarded [3]. Although its range estimation performance is improved over others, since the RPEKF requires a complicated filter selection algorithm using the consistency threshold set by the heuristic manner, it might not be a practical solution. More recently, Karsson proposed a range estimator based on the particle filter theory [4]. Apart from the range estimators based on EKF, it does not rely on linearized motion or measurement models and it seems free from Gaussian assumptions concerning the noise sequences. The particle range estimator is likely to tractably deal with the nonlinear range dynamics. However, it is well known that the particle filter based estimation method generally requires large computational burdens making real application quite difficult.

Another main issue is how to ensure effective convergence of the range estimators [5]. This is closely related to the observability and measurement accuracy. So far, most studies have been concentrated on the observability analysis and the means to produce a homing trajectory guaranteeing the observability for good convergence. Most algorithms proposed by these studies use LOS angle measurements. However, the relative range estimator using LOS angle measurements has a critical drawback when the missile is so far from the target.

Fig. 1 illustrates that the accuracy of the range information included in angle measurements is influenced by the engagement condition parameters. In the figure, the missile is assumed to measure the LOS angle with accuracy ε at two different

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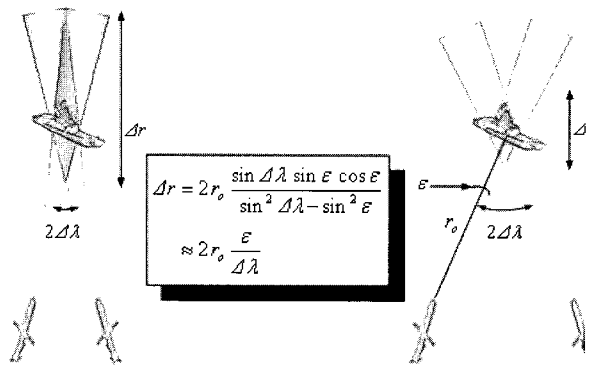


Fig. 1. Range accuracy according to range estimation conditions.

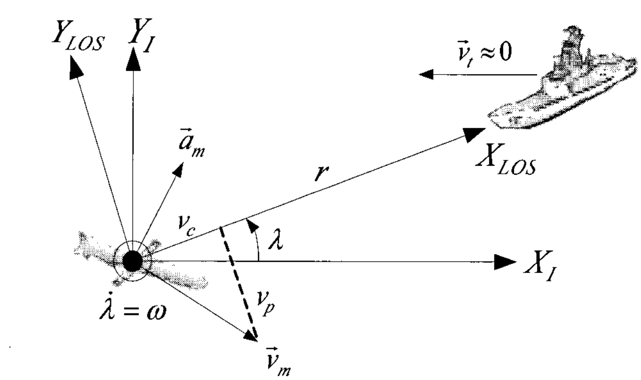


Fig. 2. Range accuracy according to range estimation conditions.

positions to ensure the observability. The figure shows that we can obtain more accurate range information (smaller Δr) as the missile is closer to the target (smaller r_0), and/or the angular difference between the two measuring positions is larger (larger $\Delta\lambda$). This means that, in order to obtain superior range information for long distance situations, the missile has to move farther to ensure large $\Delta\lambda$ and this requires a long flight time. In summary, because it is difficult for the missile to assume good measuring positions guaranteeing good range accuracy in a short time, angle-based range estimators may have poor convergence properties such as long settling time or even divergence. Especially in ASM (anti-ship missile) cases, long-distance range estimation problem has become more important as modern jammers have achieved jamming capability at long range. Therefore, the LOS angle based range estimator may not be appropriate for many ASM cases.

On the other hand, in ASM applications, the relative velocity perpendicular to the LOS vector can easily be approximated from missile velocity because typical ship targets are nearly stationary compared to the ASM motions. Based on the fact that the velocity perpendicular to LOS is equivalent to the product of the range and the LOS rate, a novel WLS range estimator is derived in this paper. Since the LOS rate can be easily amplified by turning the missile heading far from the LOS vector, we can easily and quickly build up the LOS rates large enough to produce good range information. But, as explained above, conventional angle measurement based filters require long periods of time to take the measuring positions sufficiently spaced and obtain clear range information. Hence, the proposed filter is expected to have a great advantage both in convergency and in accuracy.

In addition, the proposed filter enjoys the advantage of simple structure. The conventional EKF based filters generally not only adopt four states to describe full relative geometry but also require nonlinear manipulations to deal with the nonlinearity between the missile kinematics and LOS angle measurements. On the contrary, the proposed filter includes the range state only and it does not require solving nonlinear filter problems. It implies that the newly proposed filter has good convergence properties with drastically reduced computational burdens.

2. WEIGHTED LEAST SQUARE RANGE ESTIMATION

The range estimation problem for the typical sea skimming ASM application is formulated. Since sea skimming ASMs keep constant altitude during most of the flight and are much faster than ship targets, we assume that the missile moves only in a horizontal plane and that the target is stationary. The engagement geometry is depicted in Fig. 2.

Let the closing speed be $v_k^c = -\dot{r}_k$, and r_k be the relative range. Then the range propagation equation can be written by

$$r_{k+1} \cong r_k - \Delta t \cdot v_k^c, \tag{1}$$

where Δt is the sampling interval ($\Delta t = t_{k+1} - t_k$). Define the range difference between t_{k-j} and t_{k-1} as follows:

$$b_{k-j}^k \equiv -\Delta t \sum_{i=1}^{j} v_{k-i}^c = -\Delta t \sum_{i=k-j}^{k-1} v_i^c . \tag{2}$$

Remark that we can easily derive the following relations from (1) and (2).

$$\begin{aligned} \sum_{i=k-j}^{k-1} r_{i+1} &= \sum_{i=k-j}^{k-1} r_i - \Delta t \sum_{i=k-j}^{k-1} v_i^c \\ r_k + \sum_{i=k-j+1}^{k-1} r_i &= r_{k-j} + \sum_{i=k-j+1}^{k-1} r_i + b_{k-j}^k \\ b_{k-j}^k &= r_k - r_{k-j} \end{aligned} \tag{3}$$

Based on the fact that the velocity perpendicular to the LOS vector can be expressed by the product of LOS rate and range, we can establish the recursive relations from (3).

$$v_{k-j}^{p} = \omega_{k-j} r_{k-j} = \omega_{k-j}(r_k - b_{k-j}^{k}) \tag{4}$$

In (4), ω_k and v_k^p represent the LOS rate and the relative velocity perpendicular to the LOS vector, respectively.

The optimal range estimates $\hat{r}_k$ in WLS sense can be obtained by optimizing the weighted quadratic cost function J_k defined by

$$J_k = \frac{1}{2}\sum_{j=0}^{k} q^j (\tilde{\omega}_{k-j}(r_k - \tilde{b}_{k-j}^{k}) - \tilde{v}_{k-j}^{p}). \tag{5}$$

In (5), the tilde symbols represent that the corresponding variables are directly measured or made up of some measurements. For example,

$$\tilde{b}_{k-j}^{k} \equiv -\Delta t \sum_{i=k-j}^{k-1} \tilde{v}_i^{c},$$

where $\tilde{v}_k^c$ is the measurement of v_k^c. The variable q is the forgetting factor to reflect the non-stationary noise statistics.

By differentiating (5), the sufficient condition for optimal range estimates $\hat{r}_k$ is readily obtained.

$$\frac{\partial J_k}{\partial r_k} = \sum_{j=0}^{k} q^j (\tilde{\omega}_{k-j}\hat{r}_k - \tilde{\omega}_{k-j}\tilde{b}_{k-j}^{k} - \tilde{v}_{k-j}^{p})\tilde{\omega}_{k-j} = 0 \tag{6}$$

Rearranging the above equation results the following WLS range estimate.

$$\hat{r}_k^{WLS} = \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}(\tilde{\omega}_{k-j}\tilde{b}_{k-j}^{k} + \tilde{v}_{k-j}^{p}), \tag{7}$$

where it has been defined that

$$\Delta_k \equiv \sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}^{2}.$$

3. MODIFIED WLS RANGE ESTIMATION

3.1. Property of WLS range estimation error

We will investigate the properties of the WLS estimation errors in this chapter. The notation δ will be used for measurement errors. For example, $\delta\omega_k$ represents the error for the measurement of ω_k. Using the notation, the variables in (7) can be expressed with their actual values and measurement errors as follows:

$$\begin{aligned} \tilde{\omega}_k &= \omega_k + \delta\omega_k, \\ \tilde{b}_{k-j}^{k} &= b_{k-j}^{k} + \delta b_{k-j}^{k}, \\ \tilde{v}_k^{p} &= v_k^{p} + \delta v_k^{p} \end{aligned} \tag{8}$$

On the other hand, since (4) and (8) make

$$\begin{aligned} \tilde{v}_{k-j}^{p} &= \omega_{k-j}(r_k - b_{k-j}^{k}) + \delta v_{k-j}^{p} \\ &= \tilde{\omega}_{k-j} r_k - \delta\omega_{k-j} r_k \\ &\quad + \tilde{\omega}_{k-j} b_{k-j}^{k} + \delta\omega_{k-j} b_{k-j}^{k} + \delta v_{k-j}^{p}, \end{aligned} \tag{9}$$

the numerator of (7) can be rewritten as

$$\begin{aligned} &\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}(\tilde{\omega}_{k-j}\tilde{b}_{k-j}^{k} + \tilde{v}_{k-j}^{p}) \\ &= \left[\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}^{2}\right] r_k - \left[\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\delta\omega_{k-j}\right] r_k \\ &\quad + \sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\left(\tilde{\omega}_{k-j}\delta b_{k-j}^{k} + \delta\omega_{k-j} b_{k-j}^{k} + \delta v_{k-j}^{p}\right). \end{aligned} \tag{10}$$

This means that the WLS range estimation errors consist of the scale factor error of $-\alpha_k$ and the bias error of β_k as follows:

$$\hat{r}_k^{WLS} = (1 - \alpha_k) r_k + \beta_k, \tag{11}$$

where α_k and β_k are defined by

$$\alpha_k = \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\delta\omega_{k-j},$$

$$\beta_k = \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j \tilde{\omega}_{k-j}\left(\tilde{\omega}_{k-j}\delta b_{k-j}^{k} + \delta\omega_{k-j} b_{k-j}^{k} + \delta v_{k-j}^{p}\right).$$

3.2. Modified WLS range estimation

Based on the previous results regarding the WLS range estimation errors, we make approximations of α_k and β_k to develop the WLS range estimation error compensation algorithm. To do this, it is assumed that the measurement errors δb_{k-j}^{k}, δv_{k-j}^{p} and $\delta\omega_{k-j}$ are mutually uncorrelated zero mean white noises. Because the denominators of α_k and β_k can be directly evaluated from the actual LOS rate measurements, only the numerators are to be approximated by means of a simple statistical approximation. The basic idea of the approximation is that a sum of random numbers will be equivalent to the sum of their mean if a sufficient amount of random numbers are added. Based on the idea, α_k and β_k are estimated through the following equations.

$$\hat{\alpha}_k = \frac{1}{\Delta_k}\sum_{j=0}^{k} E\left[q^j \tilde{\omega}_{k-j}\delta\omega_{k-j}\right] = \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j Q_{k-j} \quad (12)$$

$$\hat{\beta}_k = \frac{1}{\Delta_k}\sum_{j=0}^{k} E\left[q^j \tilde{\omega}_{k-j}\delta\omega_{k-j} b_{k-j}^k\right] + \frac{1}{\Delta_k}\sum_{j=0}^{k} E\left[q^j \tilde{\omega}_{k-j}(\tilde{\omega}_{k-j}\delta b_{k-j}^k + \delta v_{k-j}^p)\right] = \frac{1}{\Delta_k}\sum_{j=0}^{k} q^j Q_{k-j}\tilde{b}_{k-j}^k$$

In the above equation, Q_k is the measurement error variance of $\tilde{\omega}_k$.

Now, the modified WLS range estimate of $\hat{r}_k^{MWLS}$ is obtained by compensating the WLS range estimate through the following equation.

$$\hat{r}_k^{MWLS} = \frac{\hat{r}_k^{WLS} - \hat{\beta}_k}{1 - \hat{\alpha}_k} \quad (13)$$

3.3. Recursive form of modified WLS algorithm

For easy implementation of the proposed modified WLS (MWLS) range estimator, the recursive forms for the parameters in (12) and (13) are proposed here. Simple arithmetic gives the following recursion of Δ_k.

$$\Delta_{k+1} = q\cdot\Delta_k + \tilde{\omega}_{k+1}^2 \quad (14)$$

Since $\tilde{b}_{k+1}^{k+1} = 0$ and $\tilde{b}_{k+1-(j+1)}^{k+1} = \tilde{b}_{k-j}^k - \Delta t\cdot\tilde{v}_k^c$, we can easily derive the following recursions.

$$\begin{aligned}
&\Delta_{k+1}\hat{r}_{k+1}^{WLS} \\
&= \sum_{j=0}^{k+1} q^j(\tilde{\omega}_{k+1-j}\tilde{v}_{k+1-j}^p + \tilde{\omega}_{k+1-j}^2\tilde{b}_{k+1-j}^{k+1}) \quad (15)\\
&= q\sum_{j=0}^{k} q^j(\tilde{\omega}_{k-j}\tilde{v}_{k-j}^p + \tilde{\omega}_{k-j}^2\tilde{b}_{k+1-j}^{k+1}) + \tilde{\omega}_{k+1}\tilde{v}_{k+1}^p \\
&= q\sum_{j=0}^{k} q^j(\tilde{\omega}_{k-j}\tilde{v}_{k-j}^p + \tilde{\omega}_{k-j}^2\tilde{b}_{k-j}^{k}) + \tilde{\omega}_{k+1}\tilde{v}_{k+1}^p - q\Delta t\tilde{v}_k^c\Delta_k \\
&= q\Delta_k(\hat{r}_k^{WLS} - \Delta t\cdot\tilde{v}_k^c) + \tilde{\omega}_{k+1}\tilde{v}_{k+1}^p
\end{aligned}$$

In a similar way,

$$\begin{aligned}
\Delta_{k+1}\hat{\alpha}_{k+1} &= q\cdot\Delta_k\hat{\alpha}_k + Q_{k+1}, \\
\Delta_{k+1}\hat{\beta}_{k+1} &= q\cdot\Delta_k\hat{\beta}_k - q\Delta t\tilde{v}_k^c\,\hat{\alpha}_k.
\end{aligned} \quad (16)$$

Using (13)-(16), the recursive MWLS range estimation algorithm can easily be implemented. Since the resultant recursive MWLS filter requires just a few flops, it is adequate for the real-time implementation.

4. SIMULATION RESULTS

To investigate the performance of the proposed filter, especially in the sense of estimation errors and filter convergency, simulations for typical anti-ship missile engagement scenarios in horizontal *XY* plane are carried out. The simulation conditions are listed in Table 1. As shown in Fig. 3, four homing trajectories including one stationary target case and three moving target cases are considered. For each scenario, 200 Monte Carlo trials are carried out to produce their error statistics. For comparison, we design two conventional EKFs, EKF using LOS angle measurements only and EKF using both LOS angle and LOS

Table 1. Simulation condition.

Item	Assumption
Target	initial pos. : $\vec{p}_t(0) = [0 \quad 0]^T$ target vel. : $v_t = 0,\ 15.0\,m/s$ dir. of target vel. : $\psi_{TV} = 180°,\ \pm 90°$
Missile	initial pos. : $\vec{p}_m(0) = [-10 \quad 0]^T km$ missile vel. : $300.0\,m/s$ initial heading : $\psi_{MV}(0) = 0°$
Guidance	$a_{mc} = -3.0 v_k^c \omega_k + 20\sin\left(0.315\cdot\frac{r_k}{v_c^k}\right)$
Errors	LOS/LOS rate error std : $0.1°,\ 0.1°/s$
EKF	$\hat{p}(0) = -[49.9 \quad 0.58]km$ $\hat{v}(0) = [300 \quad 0]m/s$ $Q_k = 0.01^2,\ R_k = (0.1°)^2,\ (0.1°/s)^2$
WLS MWLS	initial range : $\hat{r}_0^{WLS} = 50\,km$ $Q_k = (0.1°/s)^2$ $q = 0.999$

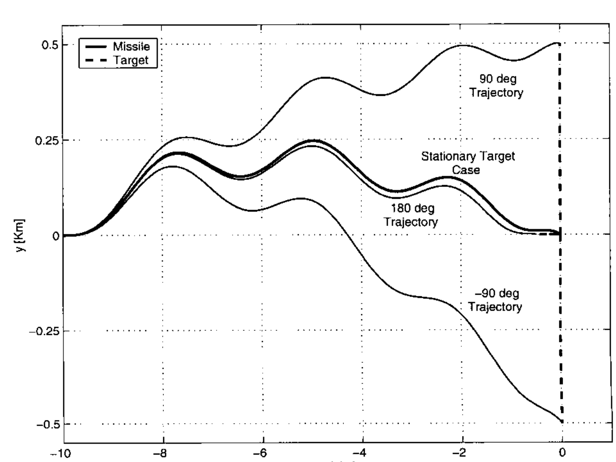
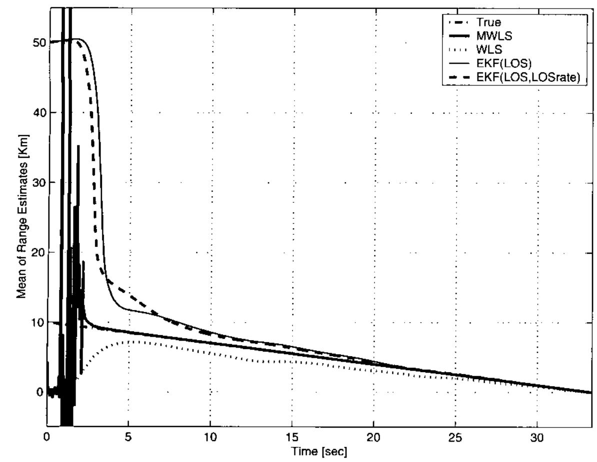
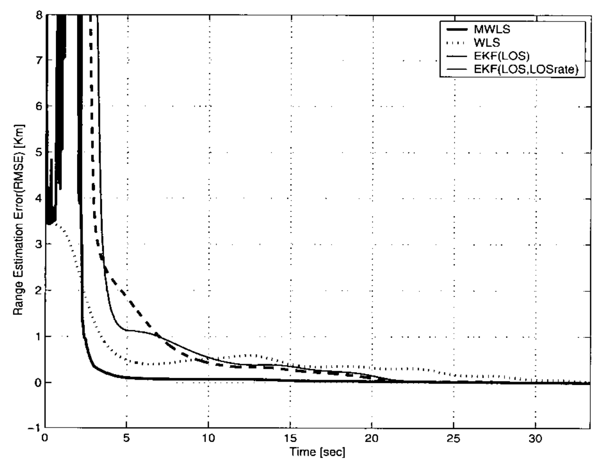


Fig. 3. Trajectories used in the simulations.

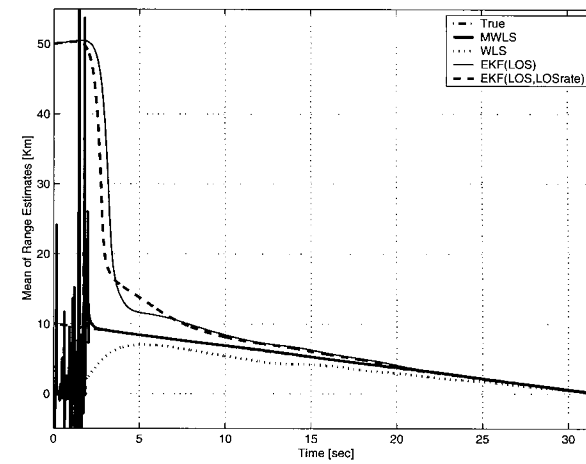


(a) Mean errors.

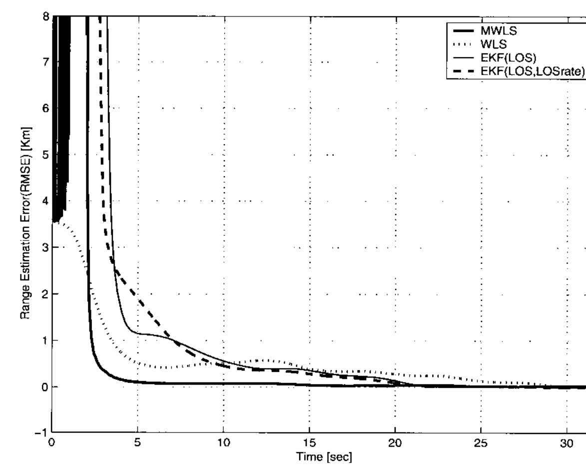


(b) RMS errors.

Fig. 4. Range estimation performance: $v_t = 0$.



(a) Mean errors.



(b) RMS errors.

Fig. 5. Range estimation performance: $\psi_{TV} = 180°$.

rate measurements. The states of the EKFs are relative position and velocity in XY plane. The states are assumed to be driven by zero mean white Gaussian random acceleration with variance Q. The $\Delta t =$ $0.02\,\text{sec}$ sampling interval is applied through all the simulations.

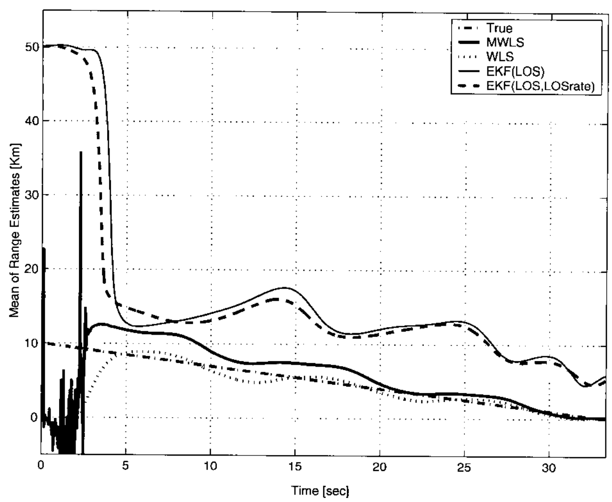
The performances of the filters are compared through the average and RMS (root mean square) behavior of their estimation errors, as shown in Fig. 4– Fig. 7. In these figures, for our convenience, dash-dotted line, thick solid line, dotted line, thin solid line, and dashed line are used to express the results of actual range, MWLS filter, WLS filter, EKF with LOS angle measurement only, and EKF with LOS angle and rate measurement, respectively.

Fig. 4 indicates the estimation error statistics for the stationary target case. As for the mean errors of the range estimators, the proposed filter works very well after some transient period for gathering sufficient measurements to make good $\hat{\alpha}_k$ and $\hat{\beta}_k$. Note that the 'WLS' estimator shows the bias and scale factor errors as discussed in Section 3. In the sense of RMS estimation errors, 'MWLS' also presents the best range estimation performance with excellent convergence property.

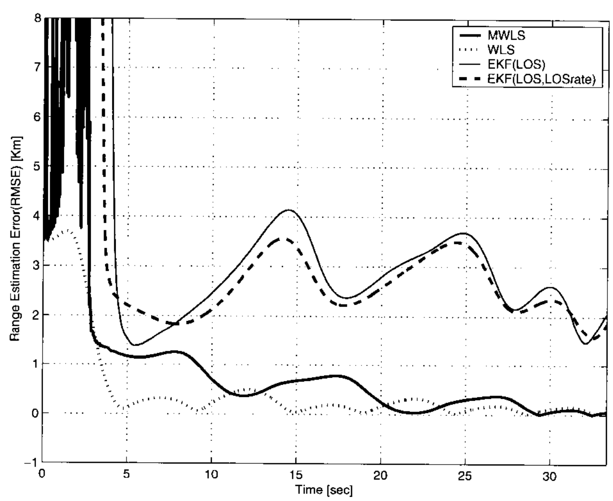
Fig. 5 – Fig. 7 indicate the results for moving target cases. Since the MWLS estimator does not take target motions into account in its derivation, compared to the stationary target case, its performances are degraded. Especially from (15), we can guess that larger v_{k-j}^p will produce larger bias errors. However, we can see that MWLS estimator still maintains relatively good and consistent performances while other filters show drastically different performances depending on the directions of target motions.

In the sense of computational burdens, since the EKFs should update 4 states through many nonlinear operations, the proposed MWLS filter is said to require negligibly small computations compared to the EKF based range estimators.

In summary, the proposed modified WLS range estimator shows extremely superior performance than the conventional EKF in both aspects of estimation errors and computational burdens.

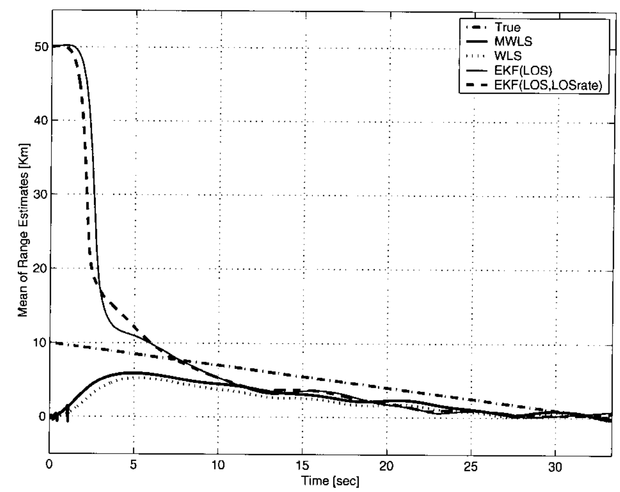


(a) Mean errors.

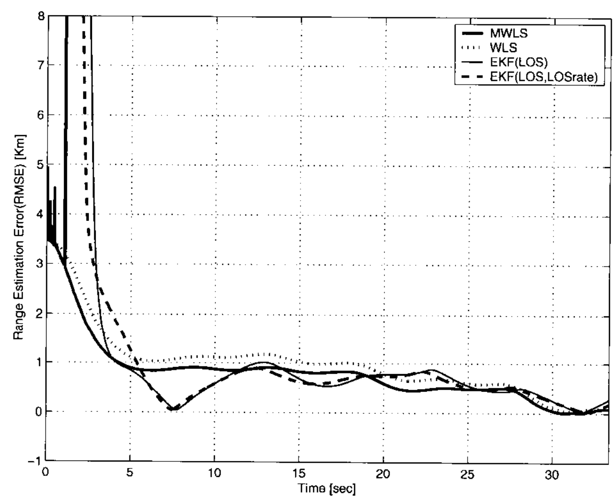


(b) RMS errors.

Fig. 6. Range estimation performance: $\psi_{TV} = 90°$.



(a) Mean errors.



(b) RMS errors.

Fig. 7. Range estimation performance: $\psi_{TV} = -90°$

5. CONCLUSIONS

In this paper, a practical range estimator for ASM (anti-ship missile) application is proposed based on the fact that the velocity perpendicular to LOS vector can be expressed by the product of LOS rate and range. The basic WLS range estimator is derived and investigated to introduce the closed form of its bias and scale factor errors. Then a simple bias and scale factor compensation algorithm is added to propose MWLS range estimator. The proposed filter has a very simple structure with only one state while the conventional EKF based filters generally have a 4 state structure. Consequently, it requires very small computational burden and is suitable for real-time applications.

Simulation results indicate that the MWLS filter possesses very good properties in the aspects of convergence speed, estimation errors, and computational burdens.

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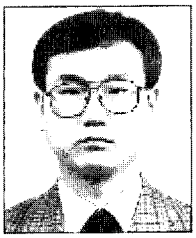

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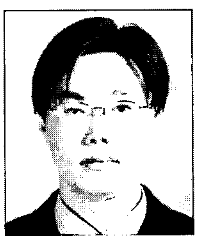

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