OFDM 용량 극대화를 위한 적응 부 반송파 선택에 관한 연구

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Capacity Maximizing Adaptive Subcarrier Selection in OFDM with Limited Feedback

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요 약

본 논문은 제한적인 피드백을 이용하는 OFDM 시스템에서 용량 극대화를 위한 효율적인 적응 부 반송파 선택 방식을 제안한다. 제안하는 방식에서는 데이터 전송에 사용될 부 반송파들과 각 부 반송파에 적용될 변조 및 코딩 방식들이 수신기에서 결정되고, 제한적인 피드백을 통해 송신기로 전달된다. 본 연구에서는 채널 환경에 따라 적절한 수의 높은 신호 대 잡음 비를 갖는 부 반송파들을 선택함으로써 용량이 극대화됨을 이론적으로 유도한다. 또한, 낮은 복잡도로 최적의 부 반송파 집합을 선택하기 위한 정렬 방식을 사용하는 적응 부 반송파 선택 알고리즘을 제안한다. 시뮬레이션 결과는 제안된 적응 부 반송파 선택 방식이 제한된 feedback 정보량만으로 water-filling 방식에 의한 부 반송파 선택 방식이나 water-filling 전력 할당에 의한 용량보다 높은 용량을 제공함을 보여준다.

ABSTRACT

We propose an efficient adaptive subcarrier selection scheme, in which the active subcarriers and their modulation and coding schemes (MCSs) are selected at the receiver, and subsequently conveyed to the transmitter using limited feedback. We theoretically show that capacity maximization can be achieved by selecting subcarriers with highest signal-to-noise ratios (SNRs) and adapting the number of active subcarriers according to channel environments. Furthermore, an ordering based adaptive subcarrier selection algorithm is proposed to select the optimal active subcarriers with low complexity. Numerical results show that the proposed adaptive subcarrier selection scheme provides higher capacity than that obtained by water-filling approaches, even with limited feedback.

키워드

MIMO, precoding, subspace method, limited feedback, spatial multiplexing

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Ⅰ. Introduction

To maximize the information rate of orthogonal frequency division multiplexing (OFDM) transmission, power and bit loading algorithms have been derived to adaptively adjust power and data rates across subcarriers assuming full knowledge of the channel state information at the transmitter (CSIT) [1] or partial CSIT [2]. Due to practical limitations on the feedback rate and delay, it is desirable to allocate equal power to subcarriers in order to limit the overhead in the feedback channel. In [3], the author has proposed equal power allocation to the selected subcarriers with "good" channel gains and has shown that the degradation in capacity is minimal compared with that of the optimal water-filling power allocation. Motivated by several goals including improving system performance based on the diversity obtained by subcarrier selection [4] and reducing delays for real-time wireless communications [5], other studies have also proposed subcarrier selection.

In most efforts to optimize transmission strategies of OFDM, this optimization is made at the transmitter [1]-[5], which results in the requirement of CSIT. Other approaches optimizing transmission strategies without CSIT have been addressed in the context of single carrier multi-input multi-output (MIMO) transmissions [6] and an adaptive modulation and coding (AMC) scheme in OFDM [7]. Transmit optimization at the receiver leads to larger capacity gain, since the receiver can track instantaneous channel realization and exploit full channel state information. Furthermore, since there are typically a small number of transmission modes, the amount of feedback can be limited.

In this paper, we propose an adaptive subcarrier selection scheme, which provides higher capacity than that obtained by water-filling approaches, even with limited feedback. Based on a capacity criterion, the active subcarriers and their MCSs are selected at the receiver, and sent back to the transmitter using limited feedback. Previously, subcarrier selection via a water-filling based subcarrier selection algorithm has been proposed [3]. However, in a realistic case where finite granularity in the transmission rate is required, the water-filling based subcarrier selection scheme is not optimum. In this paper, we theoretically show that capacity maximization can be achieved by selecting subcarriers with highest SNRs and adapting the number of active subcarriers according to channel environments. Furthermore, an adaptive subcarrier selection algorithm considering rate quantization is proposed to select the optimal active subcarriers with considerably reduced complexity. Arranging all subcarriers in order of the SNR, the achievable rate as a function of thenumber of selected subcarriers becomes a concave function. Consequently, an efficient sequential search method such as the golden section search method [8] can be used to find the optimal number of active subcarriers maximizing the transmission rate with low complexity even for a large number of subcarriers.

Ⅱ. System Model

Consider an OFDM system with N subcarriers. Let us define an active subcarrier set A in which K_A subcarriers are selected from N available subcarriers. The high-speed data stream is demultiplexed into several K_A independent substreams. The number of simultaneous substreams is adjusted according to the fading environments. Thesubstreams are then separately coded and mapped to symbols. The coding and modulation are subject to the feedback information $M_{A,i}$, which denotes the specific MCS of the substream transmitted via the i th subcarrier in A. The total transmit power P_T is uniformly distributed over K_A independent substreams. The signal at the receiver is given by

$$\mathbf{y} = \sqrt{\frac{P_T}{K_A}}\mathbf{H}_A\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}_A = \mathrm{diag}\{h_{A,1}, \cdots, h_{A,K_A}\}$. Here, $h_{A,i}$ is the channel gain for the ith subcarrier in A. A $K_A \times 1$ received signal vector is denoted by $\mathbf{y}$ and $\mathbf{n}$ is a $K_A \times 1$ additive white complex Gaussian noise vector. The transmit signal is a $K_A \times 1$ vector denoted by $\mathbf{x}$ in which the ith element represents the symbol multiplied by $\sqrt{\frac{P_T}{K_A}}$ and transmitted from the ith subcarrier in A. The active subcarrier set A and the corresponding MCS set $M_A = \{M_{A,1}, \cdots, M_{A,K_A}\}$ are chosen at the receiver and are sent back to the transmitter using feedback for transmit parameter optimization.

The channel capacity of $\mathbf{H}_A$ when an equal transmit power of $\frac{P_T}{K_A}$ is allocated to K_A subcarriers belonging to A is given by

$$C\left(\mathbf{H}_A, \frac{P_T}{K_A}\right) = \sum_{n \in A} \frac{1}{N} \log_2\left(1 + \frac{P_T}{K_A} H_n\right). \tag{2}$$

The channel gain to noise power ratio for the nth subcarrier is denoted by $H_n = \frac{|h_n|^2}{\sigma_n^2 \frac{B}{N}}$, where σ_n^2 is the variance of additive white Gaussian noise and B is the overall available bandwidth. The optimal active basis set A maximizing the capacity cannot be found analytically and, furthermore, an exhaustive search over all possible $\{A_j\}_{j=1,\cdots,2^N-1}$ is prohibitive for practical implementations for large N [7]. Thus, we derive an ordering based adaptive subcarrier selection algorithm to select the optimal active subcarriers with low complexity.

Ⅲ. Capacity Maximizing Subcarrier Selection

We arrange N subcarriers in order of the channel gain to noise power ratio as follows:

$$H_{k_1} \geq H_{k_2} \geq \cdots \geq H_{k_N}. \tag{3}$$

At low SNR, the capacity supported by the subcarriers belonging to $\mathcal{A}_x = \{k_1, k_2, \cdots, k_x\}$, in which the x subcarriers with the highest channel gain to noise power ratio are selected among N subcarriers, can be expressed by Taylor series approximations as follows:

$$C\left(\mathbf{H}_{\mathcal{A}_x}, \frac{P_T}{x}\right) \approx \frac{1}{N} \sum_{n=1}^{x} \frac{P_T}{x} H_{k_n} = \frac{P_T}{N} F_1(x), \tag{4}$$

where the arithmetic mean of the channel gain to noise power ratio of the x active subcarriers is denoted by $F_1(x) = \sum_{n=1}^{x} \frac{1}{x} H_{k_n}$. Equation (4) above shows that the approximated capacity is a function of the arithmetic mean of the received SNRs of the x active subcarriers. Therefore, the approximated capacity is an nonincreasing function of the number of the active subcarriers because $F_1(1) \geq F_1(2) \geq \cdots \geq F_1(N)$. As a result, the capacity is a quasiconcave function of the number of the active subcarriers because a function is quasiconcave if and only if either it is nondecreasing, or it is nonincreasing [8]. Particularly, in the case of $H_{k_1} > H_{k_2} > \cdots > H_{k_N}$, single-carrier transmission by the k_1th subcarrier achieves the optimal capacity.

At high SNR, the capacity supported by the subcarriers belong to $\mathcal{A}_x$ can be approximated as follows:

$$C\left(\mathbf{H}_{\mathcal{A}_x}, \frac{P_T}{x}\right) \approx \frac{1}{N} \sum_{n=1}^{x} \log_2 \frac{P_T}{x} H_{k_n} = \frac{1}{N} F_2(x), \tag{5}$$

where $F_2(x) = \sum_{n=1}^{x} \log_2 \frac{P_T}{x} H_{k_n}$. According to the concavity property, if and only if $\frac{d^2}{dx^2} f(x) < 0$ then $f(x)$ is a concave function [8], the approximated capacity is a concave function because $\frac{d^2}{dx^2} F_2(x) < 0$ for $x \geq 1$. The capacity gain obtained by selection of the $x+1$ subcarriers over the x subcarriers is defined as $G(x+1, x) = \frac{1}{N}\left(F_2(x+1) - F_2(x)\right)$ and the condition guaranteeing the capacity gain, i.e., $G(x+1, x) > 0$ is given by

$$\frac{P_T}{x+1} H_{k_{x+1}} > \left(1+\frac{1}{x}\right)^x. \qquad (6)$$

The condition shows that increasing the number of the active subcarriers by one can enhance the capacity as long as the received SNR of the newly activated subcarrier, $\frac{P_T}{x+1} H_{k_{x+1}}$ is larger than a threshold, $\left(1+\frac{1}{x}\right)^x$, which is a steadily increasing function of x and increases from 2 to the natural logarithmic base, e, as x increases. Since $\frac{P_T}{x+1} H_{k_{x+1}}$ is steadily decreasing with x, a value may exist at which x satisfies $\frac{P_T}{x+1} H_{k_{x+1}} \le \left(1+\frac{1}{x}\right)^x$, and over which increasing x decreases the capacity, at an interval $[1, N-1]$. Otherwise, at very high SNR where $\frac{P_T}{N} H_{k_N} > e$, the capacity is an increasing function of x, and thus, multi-carrier transmission by all the N subcarriers achieves the optimal capacity. In both approximated cases, the capacity achieved by selecting the subcarriers with the highest SNRs is a quasiconcave or a concave function of the number of the active subcarriers, and thus the capacity can be maximized by optimizing the number of the active subcarriers according to the received SNR.

Based on the above observations, we derive an ordering based subcarrier selection algorithm with low complexity. For rate adaptation per subcarrier, a channel capacity approximation based allocation scheme that rounds off the rate from an approximated Shannon capacity is considered [3]. The available rates are assumed to be $q, 2q$, and so on, where q is the interval between rate quantization levels. The achievable rates obtained by selecting x subcarriers with highest SNRs among all N subcarriers is given by

$$R(x) = \sum_{i=1}^{x} \frac{1}{N} \left\lceil \frac{\log_2\left(1+\frac{\gamma_i}{\Gamma}\right)}{q} \right\rceil \cdot q, \qquad (7)$$

where

$$\gamma_i = \frac{P_T}{x} \cdot H_{k_i}. \qquad (8)$$

Here, Γ is a gap reflecting the capacity loss resulting from a finite-length coding with non-zero error rates and is a function of the target bit error rate (BER) and coding method. $\lceil y \rceil$ is the largest integer that is smaller than or equal to y.

Based on the above proofs of quasiconcavity or concavity of the capacity supported by the active subcarriers, if q is assumed to be small enough for fine granuality, the achievable rate $R(x)$ becomes a quasiconcave or a concave function in an interval $[1, N]$ and has only one maximum in $[1, N]$. Finding x^*, where $R(x)$ reaches a maximum, leads to the optimal active subcarrier set A, as follows:

$$A = \{k_1, k_2, \cdots, k_{x^*-1}, k_{x^*}\}. \qquad (9)$$

Due to quasiconcavity or concavity of $R(x)$, an efficient sequential search method, such as the golden section search method [9], makes it possible to find x^* with low complexity even for a large N.

Ⅳ. Numerical Results

The capacity of the proposed scheme is investigated in this section. An OFDM system with $N = 128$ subcarriers is considered. The interval between rate quantization levels is assumed as $q = 1/2$. The system may use a block turbo coded M-QAM with a target BER 10^{-5}, and therefore $\Gamma = 1.95\,(2.9\text{dB})$ [9]. In the simulations, we use six-path frequency selective Rayleigh fading channels with an exponential power delay profile and a decaying constant of one.

Fig.1 shows the achievable rate $R(x)$ in (9) averaged over 10,000 channel realizations as a function of the ordered subcarrier index x, which represents an active subcarrier set consisting of $N-(x-1)$ subcarriers with highest SNRs among N subcarriers. The results show that

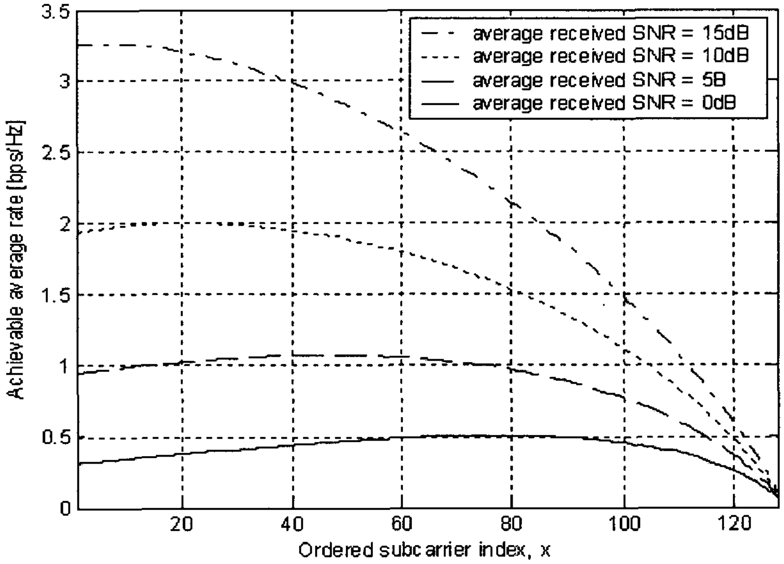

Fig.1. The average achievable rate as a function of the number of active subcarriers.

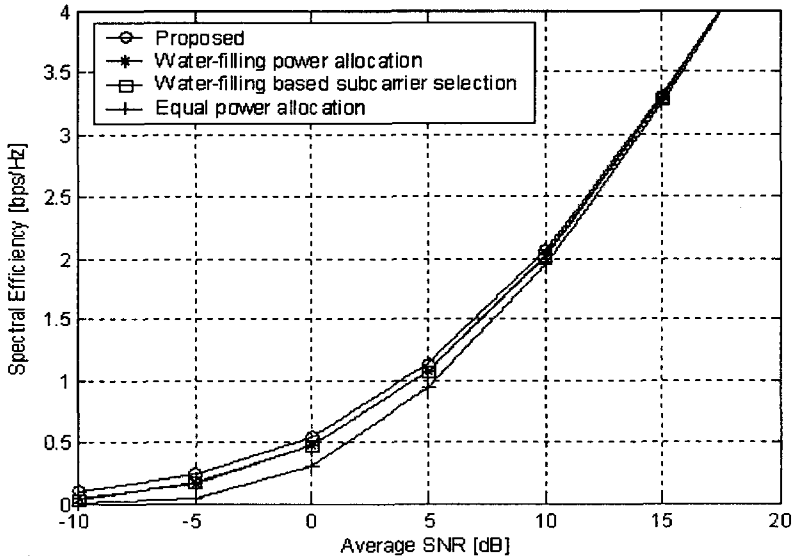

Fig.2. Transmission rate comparison between subcarrier selection schemes.

the average achievable rate is a concave function of the number of selected subcarriers. As the average received SNR increases, the number of active subcarriers maximizing the achievable rate increases, which coincides with the observations at extreme SNR cases in section III.

Transmission rate comparisons between subcarrier selection schemes are shown in Fig. 2. For reference, the transmission rates achieved by the equal power allocation scheme and the water-filling approach, when the capacity loss from the non-zero error rate and the rate quantization is considered, are presented. Equal power allocation with both the proposed ordered subcarrier selection and the water-filling based subcarrier selection [3] schemes are also considered. The proposed ordered subcarrier selection scheme shows an enhancement in the transmission rate over the water-filling based subcarrier selection scheme that provides comparable transmission rates with the water-filling power allocation approach. The results indicate that the proposed ordered subcarrier selection algorithm provides an optimal solution in the realistic case where rate quantizationis required. The proposed subcarrier selection scheme, which compares the quantized achievable rates over possible active subcarrier sets, can choose the optimal active subcarrier set that maximizes the quantized sum rate provided by the active subcarriers. On the other hand, the water-filling based subcarrier selection scheme [3] selects subcarriers with higher SNRs than a certain threshold, which is derived from the water-filling solution. However, in the threshold, the effect of rate quantization cannotbe included. In the case of no rate quantization, both algorithms show identical transmission rates. Furthermore, the capacity difference between the proposed subcarrier selection and the equal power allocation indicates that the proposed subcarrier selection can provide considerable capacity enhancement with only additional feedback information on the active subcarriers.

V. Conclusion

In this paper, we propose an adaptive subcarrier selection scheme, which provides higher capacity than that obtained by water-filling approaches, even with limited feedback. Based on a capacity criterion, the active subcarriers and their MCSs are selected at the receiver, and sent back to the transmitter using limited feedback. Furthermore, an efficient ordered subcarrier selection algorithm considering rate quantization is proposed to select the optimal active subcarriers with considerably reduced complexity. Numerical results show that the proposed ordered subcarrier selection algorithm provides an optimal solution in the realistic case where rate quantization is required. Furthermore, the proposed adaptive subcarrier selection scheme can provide considerable capacity enhancement over the equal power allocation scheme with only additional feedback information on the active subcarriers.

References

[1] A. Scaglione, S. Barbarossa, and G. B. Giannakis, “Filterbank transceivers optimizing information rate in block transmissions over dispersive channels,” IEEE Trans. Inform. Theory, vol.45, no.3, pp.1019-1032, Apr. 1999.

[2] Yao Yingwei and G.B. Giannakis, “Rate-maximizing power allocation in OFDM based on partial channel knowledge,” in Proc. Global Telecommun. Conf., vol.2, pp.863- 867, 2003.

[3] A. Leke and J. M. Cioffi, “Transmit optimization for time-invariant wireless channels utilizing a discrete multitone approach,” in Proc. IEEE ICC 1997, vol. 2, pp. 954-958, 1997.

[4] D. Dardari, “Ordered Subcarrier Selection Algorithm for OFDM-Based High-Speed WLANs,” IEEE Trans. Wireless Commun., vol.3, no.5, pp.1452-1458, July 2004.

[5] A. Kamikura, N. Okada, M. Nakagawa, “OFDM subcarrier selection for real-time wireless communication,” ICCS 2002, vol.1, pp. 52-56, 2002

[6] D. J. Love and R. W. Heath, Jr., “Limited Feedback

Precoding for Spatial Multiplexing Systems," in Proc. Globecomm 2003, pp.1857-1861

[7] S. Simoens, and D. Bartolome, "Optimum performance of link adaptation in HIPERLAN/2 networks," in Proc IEEE VTC 2001, , vol.2, pp.1129-1133, 2001.

[8] W. Cheney and D. Kincaid, Numerical Mathematics and Computing. Third Edition, Brooks/Cole Publishing Company, 1994.

[9] R. Pyndiah, A. Picart, and A. Glavieux, "Performance of block turbo coded 16-QAM and 64-QAM modulations," in Proc. IEEE Globecomm 1995, pp.1039-1043.

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