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Multivariate EWMA Control Charts for Monitoring Dispersion Matrix

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Abstract

In this paper, we proposed multivariate EWMA control charts for both combine-accumulate and accumulate-combine approaches to monitor dispersion matrix of multiple quality variables. Numerical performance of the proposed charts are evaluated in terms of average run length(ARL). The performances show that small smoothing constants with accumulate-combine approach is preferred for detecting small shifts of the production process.

Keywords : multivariate control chart, ARL, accumulate-combine approach

1. Introduction

The basic Shewhart control chart, although simple to understand and apply, uses only the current informations of the sample and is thus inefficient in detecting small changes on the process parameters. In EWMA control chart, the most recent informations of samples are assigned more weights and the older informations are assigned less weights. The performance of EWMA chart is approximately equivalent to that of CUSUM chart and, in some ways, it is more easier to operate and interpret. Vargas et al.(2004) presented a comparative study of the performance of CUSUM and EWMA charts in order to detect small changes of process average.

The charts based on accumulate-combine approach accumulates past sample information for each process parameter and then combines the separate accumulations into a univariate statistic. And multivariate EWMA chart based on accumulate-combine approach can be constructed by forming a univariate statistic into vectors of EWMA's.

A multivariate EWMA(MEWMA) chart for mean vector with accumulate-combine technique was proposed by Lowry et al.(1992). By simulation, they showed that the performances of MEWMA procedure performs better than the multivariate CUSUM procedures of Crosier(1988) and Pignatiello and Runger(1990).

In this paper, we proposed multivariate control statistics and EWMA charts to simultaneously

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monitor the components in dispersion matrix of several related quality variables under multivariate normal process.

2. Evaluating Multivariate Control Statistics

Suppose that p quality variables of interest represented by the random vector $\underset{\sim}{X}=(X_1, X_2, \cdots, X_p)'$ and $\underset{\sim}{X}$ has a multivariate normal distribution with mean vector $\underset{\sim}{\mu}$ and dispersion matrix Σ. We obtain a sequence of random vectors $\underset{\sim}{X}_1, \underset{\sim}{X}_2, \cdots$ to judge the state of the process where $\underset{\sim}{X}_i=(\underset{\sim}{X}_{i1}, \cdots, \underset{\sim}{X}_{in})'$ is an observation vector of each sampling time i and $\underset{\sim}{X}_{ij}=(X_{ij1}, \cdots, X_{ijp})'$. Let $\underset{\sim}{\theta}_0=(\underset{\sim}{\mu}_0, \Sigma_0)$ be the known target process parameters for $\underset{\sim}{\theta}=(\underset{\sim}{\mu}, \Sigma)$. In this section, we propose control statistics of multivariate EWMA chart for monitoring dispersion matrix of related multiple quality variables.

2.1 Control Statistic with Combine-Accumulate Approach

The general statistical quality control chart can be considered as a repetitive tests of significance where each quality characteristic is defined by p quality variables $X_1, X_2, \cdots, X_p$. Therefore, we can obtain a control statistic for monitoring dispersion matrix Σ by using the likelihood ratio test(LRT) statistic for testing $H_0: \Sigma=\Sigma_0$ vs $H_1: \Sigma \neq \Sigma_0$ where target mean vector $\underset{\sim}{\mu}_0$ is known. The region above the upper control limit(UCL) corresponds to the LRT rejection region. For the ith sample, likelihood ratio λ_i can be expressed as

$$\lambda_i = n^{-\frac{np}{2}} \cdot |A_i\Sigma_0^{-1}|^{\frac{n}{2}} \cdot \exp\left[-\frac{1}{2}\, tr(\Sigma_0^{-1}A_i)+\frac{1}{2}\, np\right].$$

Let TV_i be $-2\ln\lambda_i$. Then

$$TV_i = tr(A_i\Sigma_0^{-1}) - n\ln|A_i| + n\ln|\Sigma_0| + np\ln n - np, \tag{2.1}$$

and we use the LRT statistic TV_i as the control statistic for Σ. If the control statistic TV_i plots above the UCL, the process is deemed out-of-control state and assignable causes are sought.

2.2 Control Statistic for Variances with Accumulate-Combine Approach

In the univariate case, the EWMA chart for σ^2 can be constructed by using the chart statistic

$$Y_i=(1-\lambda)Y_{i-1}+\lambda\sum_{j=1}^{n}\left(\frac{X_{ij}-\mu_0}{\sigma_0}\right)^2 \tag{2.2}$$

$i=1,2,\cdots$ where μ_0, σ_0 are known parameters and $0<\lambda\leq 1$. By repeated substitution in (2.2), it can be shown that

$$Y_i=(1-\lambda)^iY_0+\sum_{k=1}^{i}\lambda(1-\lambda)^{i-k}\sum_{j=1}^{n}\left(\frac{X_{kj}-\mu_0}{\sigma_0}\right)^2. \tag{2.3}$$

Multivariate EWMA chart for variances with accumulate-combine approach can be constructed by forming multivariate extension of the univariate EWMA chart. In multivariate case for monotoring variance components of dispersion matrix, we define the vectors of EWMA's

$$\underline{Y}_{1,i}=\begin{bmatrix} Y_{i1}\\ Y_{i2}\\ \vdots \\ Y_{ip}\end{bmatrix}=\begin{bmatrix}(1-\lambda_1)^iY_{10}+\sum_{k=1}^{i}\lambda_1(1-\lambda_1)^{i-k}\left[\sum_{j=1}^{n}\left(\frac{X_{kj1}-\mu_{01}}{\sigma_{01}}\right)^2-n\right]\\ (1-\lambda_2)^iY_{20}+\sum_{k=1}^{i}\lambda_2(1-\lambda_2)^{i-k}\left[\sum_{j=1}^{n}\left(\frac{X_{kj2}-\mu_{02}}{\sigma_{02}}\right)^2-n\right]\\ \vdots \\ (1-\lambda_p)^iY_{p0}+\sum_{k=1}^{i}\lambda_p(1-\lambda_p)^{i-k}\left[\sum_{j=1}^{n}\left(\frac{X_{kjp}-\mu_{0p}}{\sigma_{0p}}\right)^2-n\right]\end{bmatrix}, \tag{2.4}$$

where $0<\lambda_l\leq 1\ (l=1,2,\cdots,p)$ and $i=1,2,\cdots$.

2.3 Control Statistic for Correlation Coefficients with Accumulate-Combine Approach

A univariate EWMA chart for ρ_{12} which is based on $r_{12}=\sum_{j=1}^{n}(x_j-\mu_1)(y_j-\mu_2)\,/\,n\sigma_1\sigma_2$, an estimator of ρ_{12}, can be constructed as

$$Y_i=(1-\lambda)Y_{i-1}+\lambda\frac{\sum_{j=1}^{n}(X_{ij1}-\mu_{10})(X_{ij2}-\mu_{20})}{n\sigma_{10}\sigma_{20}}, \tag{2.5}$$

$i=1,2,\cdots$ and $0<\lambda\leq 1$. By repeated substitution, it can be shown that

$$Y_i=(1-\lambda)^iY_0+\sum_{k=1}^{i}\lambda(1-\lambda)^{i-k}\frac{\sum_{j=1}^{n}(X_{kj1}-\mu_{10})(X_{kj2}-\mu_{20})}{n\sigma_{10}\sigma_{20}}. \tag{2.6}$$

To simultaneously monitor the $s\,(=p(p-1)/2)$ correlation coefficients of p variables, if we let the control statistic for ρ_{lm} be r_{lm} by suitable modification of the simple expression in (2.6), then the vector of EWMA's for $\underline{\rho}=(\rho_{12},\rho_{13},\cdots,\rho_{1p},\rho_{23},\cdots,\rho_{2p},\cdots,\rho_{p-1,p})'$ can be defined as

$$\begin{aligned}\underline{Y}_{2,i}'&=(r_{12},r_{13},\cdots,r_{1p},r_{23},\cdots,r_{2p},\cdots,r_{p-2,p-1},r_{p-1,p})\\ &=(Y_{i1},Y_{i2},\cdots,Y_{i,p-1},Y_{ip},\cdots,Y_{i,2p-3},\cdots,Y_{i,s-1},Y_{i,s}).\end{aligned}$$

Then the vector $\underline{Y}_{2,i}$ can be rewritten as

$$\underline{Y}_{2,i}=\begin{bmatrix}(1-\lambda_1)^iY_{i10}+\sum_{k=1}^{i}\lambda_1(1-\lambda_1)^{i-k}Z_{k12}\\ \vdots \\ (1-\lambda_{p-1})^iY_{i,p-1,0}+\sum_{k=1}^{i}\lambda_{p-1}(1-\lambda_{p-1})^{i-k}Z_{k1p}\\ (1-\lambda_p)^iY_{t,p,0}+\sum_{k=1}^{i}\lambda_p(1-\lambda_p)^{i-k}Z_{k23}\\ \vdots \\ (1-\lambda_{2p-3})^iY_{i,2p-3,0}+\sum_{k=1}^{i}\lambda_{2p-3}(1-\lambda_{2p-3})^{i-k}Z_{k2p}\\ \vdots \\ (1-\lambda_s)^iY_{i,s,0}+\sum_{k=1}^{i}\lambda_s(1-\lambda_s)^{i-k}Z_{k,p-1,p}\end{bmatrix}, \tag{2.7}$$

where $0 < \lambda_a \leq 1\ (a=1,2,\cdots,s)$ and

$$Z_{kmu} = \frac{\sum_{j=1}^{n}(X_{kjm}-\mu_{m0})(X_{kju}-\mu_{u0})}{n\sigma_{m0}\sigma_{u0}} - \rho_{mu0}. \quad (m \neq u)$$

Therefore, we can take the statistics TV_i, $\underline{Y}_{1i}$ and $\underline{Y}_{2i}$ as the control statistics to monitor the components of dispersion matrix of p related quality variables.

3. Multivariate EWMA Control Charts

The EWMA control chart is a good alternative to the Shewhart chart when we are interested in detecting small shift of the process parameters. For the EWMA chart based on LRT statistic TV_i is given by

$$Y_{TV,i} = (1-\lambda)Y_{TV,i-1} + \lambda TV_i, \tag{3.1}$$

where $Y_{TV,0} = \omega\,(\omega \geq 0)$ and $0 < \lambda \leq 1$. This chart signals whenever $Y_{TV,i} > h_{TV}$. When the smoothing constant λ is 1, this chart changes to Shewhart chart.

Since it is difficult to obtain the exact distribution of chart statistic $Y_{TV,i}$, the performances of this chart can be evaluated by simulation when the parameters of the process are on-target or changed. And the process parameter h_{TV} can be obtained to satisfy a specified ARL.

For accumulate-combine procedure, we use two separate multivariate EWMA charts, based on accumulate-combine approach, for variances and for correlation coefficients. This scheme signals if one of the two EWMA charts in (3.5) and (3.7) signals. The multivariate EWMA vectors (2.4) can be expressed as

$$\underline{Y}_{1i} = (I-\Lambda)\underline{Y}_{1,i-1} + \Lambda \underline{Z}_i, \tag{3.2}$$

where smoothing matrix $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_p)$, $0 < \lambda_j \leq 1\ (j=1,2,\cdots,p)$, $\underline{Z}_i = (Z_{i1}, Z_{i2}, \cdots, Z_{ip})'$. By repeated substitution in (3.2), $\underline{Y}_{1,i}$ can be rewritten as

$$\underline{Y}_{1,i} = \sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\underline{Z}_i + (I-\Lambda)^i\underline{Y}_{1,0}, \tag{3.3}$$

$i=1,2,\cdots$ where $\underline{Y}_{1,0} = \underline{0}$.

Then we can obtain the dispersion matrix of $\underline{Y}_{1,i}$ as

$$\Sigma_{\underline{Y}_{1i}} = \sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\Sigma_Z(I-\Lambda)^{i-k}\Lambda, \tag{3.4}$$

where Σ_Z, dispersion matrix of $\underline{Z}$, is given in Theorem 3.1.

Theorem 3.1. When the process is in-control, the dispersion matrix of $\underline{Y}_{1,i}$ is given by

$$\Sigma_{\underline{Y}_{1,i}} = \sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\Sigma_Z(I-\Lambda)^{i-k}\Lambda$$

and

$$\Sigma_{Z}=2nR^{(2)},$$

where $R^{(2)}$ is used to denote the matrix whose *(i,j)*th component is the *2nd* power of the *(i,j)*th component of R which is the correlation matrix of $\underline{X}=(X_1,X_2,\cdots,X_p)$.

Proof. It is easy to show that

$$\begin{aligned}\Sigma_{\underline{Y}_{1,i}} &= Var[(I-\Lambda)\underline{Y}_{1,i-1}+\Lambda\underline{Z}_i] \\ &= \sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\Sigma_{Z_i}(I-\Lambda)^{i-k}\Lambda.\end{aligned}$$

To show the form of Σ_{Z_i}, we obtain the mean vector and dispersion matrix of $\underline{Z}_i$ when the process is in-control and $\underline{Y}_{1,0}=\underline{0}$. Since a random sample $(X_{i1l},X_{i2l},\cdots,X_{inl})'$ at the sampling point i follows a multivariate normal distribution, the statistic $\sum_{j=1}^{n}(\frac{X_{ijl}-\mu_{ol}}{\sigma_{ol}})^2 = Z_{il}+n$ has a chi-squared distribution with n degrees of freedom. Thus $E(Z_{il})=0$ and $Var(Z_{il})=2n$ for $l=1,2,\cdots,p$ and $i=1,2,\cdots$. Now, for $l\neq s$, we can derive as

$$\begin{aligned}Cov[Z_{il},Z_{is}] &= Cov[Z_{il}+n,Z_{is}+n] \\ &= Cov\left[\sum_{j=1}^{n}(\frac{X_{ijl}-\mu_{0l}}{\sigma_{ol}})^2,\sum_{j=1}^{n}(\frac{X_{ijs}-\mu_{0s}}{\sigma_{os}})^2\right] \\ &= nCov\left[(\frac{X_{ijl}-\mu_{0l}}{\sigma_{0l}})^2,(\frac{X_{ijs}-\mu_{0s}}{\sigma_{0s}})^2\right].\end{aligned}$$

If we let $U=\frac{X_{ijl}-\mu_{0l}}{\sigma_{0l}}$ and $V=\frac{X_{ijs}-\mu_{0s}}{\sigma_{0s}}$, then the random variables U and V have a bivariate normal distribution with $N_2(0,0,1,1,\rho_{u,v})$. Using the moment generating function of bivariate normal distribution, $E(U^2V^2)=1+2\rho^2_{u,v}$ and then we can easily obtain that

$$\begin{aligned}Cov(Z_{il},Z_{is}) &= nCov(U^2,V^2) \\ &= n[E(U^2V^2)-E(U^2)E(V^2)] \\ &= 2n\,\rho^2_{u,v}.\end{aligned}$$

Thus, we found that the diagonal elements and corresponding off-diagonal elements of Σ_{Z_i} is $2n$ and $2n\rho^2_{i,j}$ from the above results. Therefore,

$$E(\underline{Z}_i)=\underline{0} \quad\text{and}\quad \Sigma_{Z_i}=2nR^{(2)}.$$ □

This multivariate chart for variances signals whenever

$$T^2_{1,i}=\underline{Y}_{1,i}'\,\Sigma_{\underline{Y}_{1,i}}^{-1}\underline{Y}_{1,i} > h_1, \tag{3.5}$$

where the parameter h_1 can be obtained to achieve a specified in-control ARL. If there is no distinct reason to take different smoothing constants for p diagonal elements of Λ, all diagonal elements of Λ can be set equal values. Under the assumption $\lambda_1=\lambda_2=\cdots=\lambda_p=\lambda$, then we can simplify the dispersion matrix of $\underline{Y}_{1,i}$ as

$$\Sigma_{\underline{Y}_{1,i}}=\frac{\lambda[1-(1-\lambda)^{2i}]}{2-\lambda}\cdot\Sigma_{Z}. \tag{3.6}$$

And EWMA vector for correlation coefficients, based on (2.7), can be expressed as

$$\underline{Y}_{2,i}=\sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\underline{Z}_k+(I-\Lambda)^i\underline{Y}_{2,0}, \tag{3.7}$$

where $\underline{Z}_k{}'=(Z_{k12},Z_{k13},\cdots,Z_{k1p},Z_{k23},\cdots,Z_{k2p},\cdots,Z_{k,p-1,p})$, $\Lambda=diag(\lambda_1,\lambda_2,\cdots,\lambda_s)$ and $0<\lambda_j\le 1$ $(j=1,2,\cdots,s)$.

Under the assumption that $\lambda_1=\lambda_2=\cdots=\lambda_s=\lambda$, the EWMA chart in (3.7) can be written as

$$\begin{aligned}\underline{Y}_{2,i} &= (1-\lambda)\underline{Y}_{2,i-1}+\lambda\underline{Z}_i\\ &=\sum_{k=1}^{i}\lambda(1-\lambda)^{i-k}\underline{Z}_k+(1-\lambda)^i\underline{Y}_{2,0}.\end{aligned} \tag{3.8}$$

This multivariate EWMA chart for correlation coefficients signals whenever

$$T^2_{2,i}=\underline{Y}_{2,i}{}'\,\Sigma_{\underline{Y}_{2,i}}{}^{-1}\underline{Y}_{2,i}>h_2, \tag{3.9}$$

where $\Sigma_{\underline{Y}_{2,i}}$ is the dispersion matrix of $\underline{Y}_{2,i}$ with dimension $s\times s$ and is given in (3.10) below. The parameter h_2 can be obtained to satisfy a specified ARL by simulation and the dispersion matrix of $\underline{Y}_{2,i}$ is given in Kim and Chang(1998) as

$$\Sigma_{\underline{Y}_{ii}}=\left\{\frac{\lambda[1-(1-\lambda)^{2i}]}{2-\lambda}\right\}\cdot\Sigma_Z \tag{3.10}$$

and

$$\Sigma_Z=\begin{pmatrix} Var(Z_{i12}) & Cov(Z_{i12},Z_{i13}) & \cdots & Cov(Z_{i12},Z_{i,p-1,p})\\ & Var(Z_{i13}) & \cdots & Cov(Z_{i13},Z_{i,p-1,p})\\ & & \ddots & \vdots\\ & Sym & & Var(Z_{i,p-1,p})\end{pmatrix}, \tag{3.11}$$

where

$$\begin{aligned}Var(Z_{ipq}) &= \frac{1+\rho^2_{pq0}}{n}\\ Cov(Z_{ipq},Z_{ipr}) &= \frac{\rho_{qr0}+\rho_{pq0}\rho_{pr0}}{n}\\ Cov(Z_{ipq},Z_{irw}) &= \frac{\rho_{pr0}\rho_{qw0}+\rho_{pw0}\rho_{qr0}}{n}\end{aligned}$$

and the subscripts p,q,r and w are different each other.

The multivariate EWMA scheme based on accumulate-combine approach for simultaneously monitoring both variance components and correlation coefficients(or covariance) components signals whenever $T^2_{1,i}>h_1$ or $T^2_{2,i}>h_2$. The overall false alarm probability of the chart based on $(T^2_{1,i},T^2_{2,i})$ is $1-(1-\alpha_1)(1-\alpha_2)$ where the signal probabilities of the chart for variance components and correlation coefficient components are α_1 and α_2 , respectively. Since it is difficult to obtain the exact joint distribution of $T^2_{1,i}$ and $T^2_{2,i}$, we obtain the parameters h_1, h_2 and performances of this scheme by simulation.

4. Computational Results and Conclusion

In this paper, we propose multivariate EWMA control charts for both combine-accumulate and accumulate-combine approaches to monitor dispersion matrix of p multiple quality variables under multivariate normal process. For simplicity, we assume that in-control process mean vector is $\underline{\mu}_0 = \underline{0}'$, all diagonal elements of Σ_0 are 1 and off-diagonal elements of Σ_0 are 0.3. Since the performances of the charts depends on the components of Σ, we consider the following typical types of shifts for comparison in the components of dispersion matrix.

(1) V_i : σ_{10} of Σ_0 is increased to $[1+(4i-3)/10]$.

(2) C_i : ρ_{120} and ρ_{210} of Σ_0 are changed to $[0.3+(2i-1)/10]$

(3) (V_i, C_i) for $i=1,2,3$.

(4) S_i : Σ_0 is changed to $c_i\Sigma_0$ where $c_i=[1+(3i-2)/10]^2$.

To compare the performances of the proposed EWMA charts, the charts should be set up so that both have the same ARL when the process is in-control. And, the design parameters h_{TV}, h_1, h_2 of the proposed charts were evaluated by simulation with 10,000 iterations when the in-control ARL were approximately fixed to be 200 and the sample size for each quality characteristic was five for $p=2\sim4$.

[Table 1] ARL performances for $p=2$

	C-A Approach			A-C Approach		
types of shift	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0.3$	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0.3$
in-control	200.0	200.0	200.0	200.0	200.0	200.0
V_1	153.9	158.4	160.3	45.2	55.5	63.9
V_2	18.6	15.0	14.4	3.7	4.2	4.6
V_3	7.3	5.2	4.6	1.8	2.0	2.1
C_1	167.6	173.4	175.2	83.0	97.2	107.2
C_2	53.5	57.6	64.7	16.6	21.4	26.8
C_3	15.0	12.2	12.6	7.3	8.7	10.5
(V_1, C_1)	139.7	146.4	150.2	33.1	40.1	46.0
(V_2, C_2)	17.0	13.5	12.9	3.4	3.9	4.2
(V_3, C_3)	6.1	4.4	3.8	1.8	1.9	2.0
S_1	123.4	128.4	133.6	26.0	31.8	36.6
S_2	15.6	12.3	11.6	2.9	3.3	3.7
S_3	6.3	4.5	3.9	1.5	1.6	1.7
S_4	3.7	2.6	2.2	1.2	1.2	1.3

[Table 2] ARL performances for $p=3$

types of shift	C-A Approach $\lambda=0.1$	C-A Approach $\lambda=0.2$	C-A Approach $\lambda=0.3$	A-C Approach $\lambda=0.1$	A-C Approach $\lambda=0.2$	A-C Approach $\lambda=0.3$
in-control	200.0	200.1	200.0	200.0	200.0	200.0
V_1	170.7	173.0	176.1	51.6	65.2	74.6
V_2	29.5	24.9	24.8	3.9	4.5	5.1
V_3	11.8	8.1	6.9	1.9	2.1	2.2
C_1	178.8	181.3	183.9	110.2	136.0	149.3
C_2	77.8	84.2	93.7	20.4	30.4	41.7
C_3	24.4	21.3	23.3	8.6	11.0	14.4
(V_1, C_1)	157.9	162.8	167.4	40.7	53.0	62.1
(V_2, C_2)	25.9	21.2	20.9	3.7	4.2	4.7
(V_3, C_3)	9.7	6.5	5.5	1.8	2.0	2.1
S_1	132.2	138.0	143.5	20.9	26.1	30.7
S_2	18.0	13.5	12.7	2.2	2.6	2.8
S_3	7.6	4.9	4.1	1.3	1.3	1.4
S_4	4.4	2.9	2.3	1.1	1.1	1.1

Numerical results show that multivariate EWMA chart based on $Y_{1,i}$ and $Y_{2,i}$ with accumulate-combine approach is more efficient than multivariate EWMA chart based on TV_i with combine-accumulate approach in terms of ARL performance. The ARL performance and comparison for out-of-control state were stated in [Table 1] through [Table 3] when the number of quality variable p is $2 \sim 4$. In Tables, we can see the trends of ARL performances according to smoothing constant λ when the process is out-of-control.

[Table 3] ARL performances for $p=4$

types of shift	C-A Approach $\lambda=0.1$	C-A Approach $\lambda=0.2$	C-A Approach $\lambda=0.3$	A-C Approach $\lambda=0.1$	A-C Approach $\lambda=0.2$	A-C Approach $\lambda=0.3$
in-control	200.0	200.0	200.0	200.0	200.0	200.0
V_1	181.1	184.0	186.0	55.0	72.7	82.9
V_2	44.6	41.7	44.5	4.1	4.8	5.4
V_3	17.7	12.5	11.2	1.9	2.1	2.2
C_1	186.4	189.2	190.0	127.0	154.2	167.4
C_2	104.0	113.9	123.4	23.6	37.8	53.7
C_3	37.7	36.9	42.7	9.6	13.1	18.1
(V_1, C_1)	172.6	176.6	179.7	45.4	61.5	72.9
(V_2, C_2)	38.3	34.6	36.9	3.9	4.5	5.1
(V_3, C_3)	14.5	9.9	8.6	1.9	2.0	2.2
S_1	142.4	149.5	155.6	17.7	22.5	26.3
S_2	21.9	16.9	16.4	1.9	2.1	2.3
S_3	9.4	6.0	4.9	1.1	1.2	1.2
S_4	5.5	3.4	2.7	1.0	1.0	1.0

For simultaneously monitoring both variances and correlation coefficients in Σ of multiple related quality variables, an EWMA procedure based on accumulate-combine feature is more efficient for all proposed types of shift than an EWMA procedure based on combine-accumulate feature. And we also found that small values of λ are preferred for detecting small or moderate shifts and vice versa for various p.

References

[1] Alt, F. B. (1984). Multivariate Control Charts, in *The Encyclopedia of statistical Sciences,* edited by S. Kotz and N.L. Johnson, John Wiley, New York.

[2] Crosier, R. B. (1988). Multivariate Generalization of Cumulative Sum Quality-Control Scheme, *Technometrics,* 30, 291-303.

[3] Kim J. J. and Chang D. J. (1998). Control Chart for Correlation Coefficients of Correlated Quality Variables, *Journal of the Korean Society for Quality Management*, 26, 51-66.

[4] Lowry, C. A., Woodall, W. H., Champ, C. W. and Rigdon, S. E. (1992). A Multivariate Exponentially Weighted Moving Average Control Charts, *Technometrics*, 34, 46-53.

[5] Mason, R. L., Champ, C. W., Tracy, N. D., Wierda, S. J. and Young, J. C. (1997). Assessment of Multivariate Process Control Techniques, *Journal of Quality Technology,* 29, 140-143.

[6] Pignatiello, J. J., Jr. and Runger, G. C. (1990). Comparisons of Multivariate CUSUM Charts, *Journal of Quality Technology,* 22, 173-186.

[7] Prabhu, S. S. and Runger, G. C.(1997). Designing a Multivariate EWMA Control Chart, *Journal of Quality Technology,* 29, 8-15.

[8] Vargas, V. C. C., Lopes, L. F. D. and Sauza, A. M.(2004). Comparative study of the performance of the CuSum and EWMA control charts, *Computer & Industrial Engineering*, 46, 707-724.

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