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Bianshi Problem as the Bridge from "Entering the Way" to "Transcending the Way"[1]: The Cultural Characteristic of Bianshi Problem in Chinese Math Education[2]

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Recently Bianshi problems, math isomorphic problems by changing the conditions, conclusions or deduction process of the three components of the example problem, are consistently identified as an important element in Chinese math education, characteristics in Chinese math education culture and form a hot point in China, 5 main factors related to reasons why is regarded as CULTURE characteristics in Chinese math education are discussed: (a) Exam goal (b) the curriculum objectives (c) traditional mode and conception, (d) Chinese situation, (e) Chinese math tradition.

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[1] "transcending the way" is a better translation than "exiting the way", which is previously used.

[2] This paper will be presented at the Third ICMI-East Asia Regional Conference on Mathematics Education (ICMI-EARCOME-3), Shanghai, Nanjing, and Hangzhou, People's Republic of China, August 7–12, 2005.

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O. INTRODUCTION

Prior studies have shown that Chinese students outperformed their counterparts in the West. These include both international comparisons on mathematics achievements and results at the International Mathematical Olympiad (Wong 2004). The cultural characteristics in Chinese math education have recently attracted the interests of the researchers in mathematics education and cross-cultural psychology. (Biggs & Watkins 1996, 2001; Stevenson & Stigler 1992; Stevenson, Chen & Li 1993, see also Wong 2004). Recently many scholars have contributed to the study of Variation, such as European scholars, Marton and Booth (1997), Marton, Dall'Alba & Tse (1996), Runesson (1999), Marton, Runesson, and Tsui (2003). Hong Kong scholars, Wong (2002), Mok (2003), Ki & Shum (2005), Chinese mainland scholars (Huang 2002; Nie 2004; Bao, Huang & Leung 2004; Gu, Huang & Marton (2004). The study of Variation is forming a hot point in math education fields.

The word of Variation could be translated in to Chinese *Bianshi* "变式", is generally regarded as "a group of isomorphic math problem by changing variable within the math problem"(we use *Bianshi* the word in the whole paper. Because the study focus on Chinese math education characteristic).

More and more studies identified teaching with variation (translated in Chinese Bian shi teaching) as an important element and also a priorities in Chinese math education (Huang 2002; Nie 2004; Bao, Huang & Leung 2004; Gu, Marton & Huang 2004) However, why *Bianshi* problem, the media of Bianshi teaching, is regarded as CULTURE characteristics in Chinese math education is still unclear in these studies.

Inevitably, the tradition of teaching and learning are closely related to the history and culture of the educational region embedded in. In this paper, rather than looking by a fragment angles, the author attempts to introduce:

- What is Bianshi problem?
- What characteristics has *Bianshi* problem?
- Why is *Bianshi* problem regarded as cultural characteristics in Chinese math education?

I. WHAT DOES BIANSHI TEACHING AND BIANSHI EXERCISE MEAN?

Bianshi teaching evolves from the experience and gets its name from the experience of mathematic teaching reform, with Salient practical effect from the passing rate 2.8% in 1979 to 85%in 1986, headed by Professor Gu in Qingpuxian in Shanghai, China (Qingpu

educational reform group 1991). In the 80s, the reform (1977–1992) held by professor Gu, began from filtering Chinese effective teaching experience, rationalized the experience already gained from Chinese mathematics classroom practice, built up the basic teaching principles to let the students to learn effectively.

Today Bianshi teaching is adopted consciously and broadly by classrooms around the country instead of unintentionally before, but till now, Bianshi exercises out of class is a main way to apply. For instance, the Bianshi exercises of synchronization (mean to maintain common timing and coordination between math teaching and math exercise) published by Anhui educational press, has been widely used. And "Bianshi Ttizhen, Huanggang Bingfa" (Zhu et 2003) has been one of the bestsellers in China in educational book series.

1. What is *Bianshi* problem in this study?

We focus on *Bianshi* problems in this study because it is both of the media of *Bianshi* teaching and *Bianshi* exercises.

The 1999 TIMSS Video Study found what the higher-achieving countries have in common does not lie in the organization of classrooms, the kinds of technologies used, or even the types of problems presented to students, but in the way in which teachers and students work on problems as the lesson unfolds. Effective teaching takes many forms, but each form makes connections (Stigler & Hiebert 2004).

Bianshi problems are typical math problems making math questions connected in the "CHC" (Confucian Heritage Cultural learner) Learner phenomenon to some extent.

Math problems predetermine what students learn and how students learn. Math teaching and learning are structured by the content of math problems Since problems are general regarded as the heart of mathematics (Halmos 1980) and problem solving should be the focus of math education recommended by Curriculum and Evaluation Standards for School math (NCTM 1989). I would like focus on *Bianshi* problems only in this study which help to understand CULTURE characteristics in CHC (Confucian-Heritage Culture) environment.

Despite of some confusion about definition of *Bianshi*, in this study, I would like to introduce the definition of *Bianshi* problem by two examples based on the concept from the experimental team on mathematical reformation in Qingpuxian (1991). If a student do not know how to count this problem: How many is 3/4 time 2/3. His teacher perhaps give a group of problems like this:

- How many is 1 time 2
- How many is 2 time 2
- How many is 2 time 1/2

- How many is 1/2 time 2
- How many is 1 time 2/3
- How many is 3 time 2/3
- How many is 1/2 time 2/3
- How many is 3/4 time 2/3

Students may understand this problem step by step despite of changing the number only in the problem; math understanding may be deepened step by step at the same time.

Besides this, changing a variation in the problem below was used broadly:

Another example of *Bianshi* problems

- Factorization of polynomial
- $x^2+5x+6=(x+m)\times(x+n)\quad m+n=5,\quad m\times n=6$
- What is the possible value of a and b, the polynomial can be factored.
- x^2+ax+6
- x^2+5x+b
- x^2+ax+b
- x^3+ax+b
- x^n+ax+b

Students may extend their understanding broader and broader despite of adding a variation in the problem; math understanding may be deepened step by step at the same time. We give this group of problems a name: *Bianshi* problem. Based on this, we generalize the characteristics of *Bianshi* problem and give *Bianshi* problem a definition:

Having solved an example problem, we can get a group of sub problems under students control by changing the conditions, conclusions or deduction process of the three components of the example problem in order to make student construct their own math problem-solving method (The experimental team on mathematical reformation in Qingpuxian 1991). This group of math problems is called *Bianshi* problem. To summarize, Bianshi problems are tools to mediate, created by their teachers, which make the student able to control his understanding from the outside scaffolding.

2. What characteristics has *Bianshi* problem?

Wong (2002) pointed out "entering the way" and "transcending the way": is Cultural characteristic of learner in the CHC (Confucian-Heritage Culture). Wong (2002) mainly clarified that the Chinese views of pedagogy is "from "entering the way" to "transcending the way": by exemplifying two old traditions, calligraphy and martial art. This is well explained in the following words:

"entering the way" is following the stringently the standard way to know the basics of a

certain specialized knowledge ..... through incessant practices and a long period of "hatching", mystical thought may sound ..... repetition by systematically introducing variations could be the key to bring about learning and understanding. Repetition and practices of basic skills form the basis for developing process abilities" (pp. 170).

Bianshi problem just project the Chinese views of pedagogy by its the structure and procedures. *Bianshi* problem act as "the bridge" from "entering the way" to "transcending the way" at same time. We use a picture to clarify the character of *Bianshi* problem above. American scholars identified the characteristic in Japanese classroom in the document of *Principl and stands for school mathematics* in the following words:

The researchers (stigler & Hielbert) found that typical Japanese lessons were designed around one central idea, which was carefully developed and extend; in contrast, typical american lessons included several ideas or topics that was not related and not well developed (Principl and stands for school mathematics, 2000, NCTM, p.15).

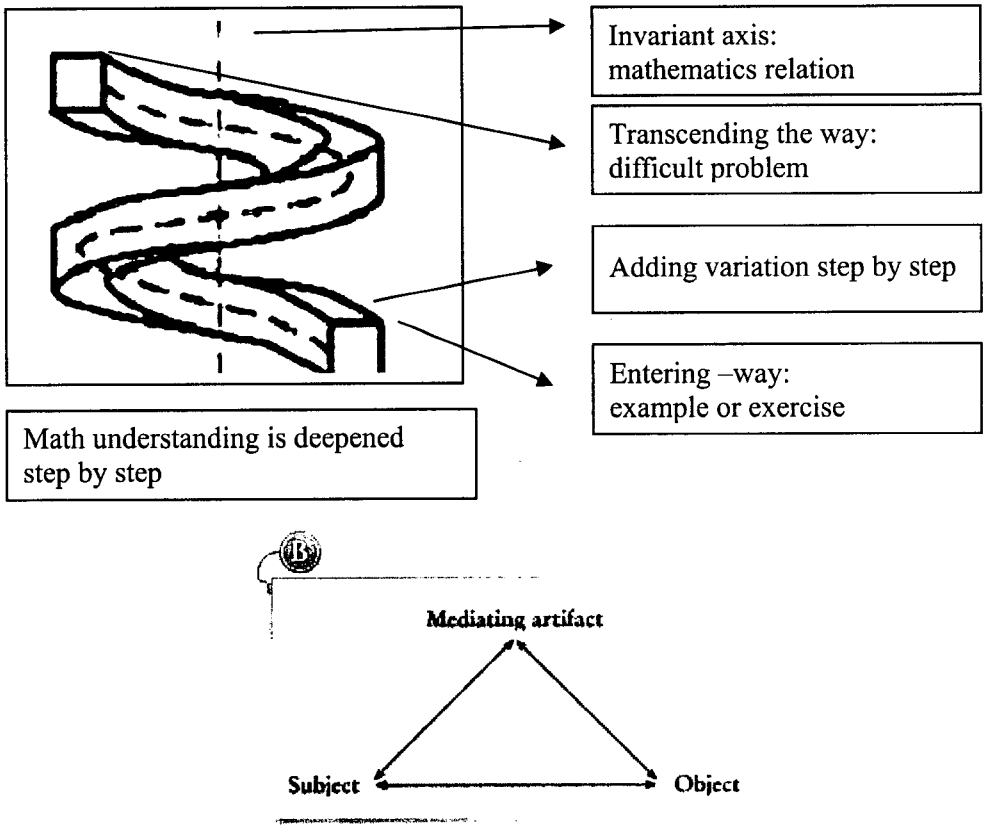


Figure 1. "the mechanism of Bianshi problem

Vygotsky and his colleagues also point out the child may transcend the situation if there is a artifact-mediated and object-oriented action happened between subject and object (Vygotsky 1978, p. 40). Lev Vygotsky created the foundation of cultural-historical psychology, based on the concept of mediation. Collaboration with other humans creates zones of proximal development for individuals, enabling them to go beyond their current capacity by grasping and constructing new mediating tools and signs.

The above analysis detailed the "the mechanism of Bianshi problem" and that is, Bianshi problem is a mediating-artifact and out-of – control between subject, *i.e.*, child and object, *i.e.*, math problem. Which make student able to control him – or herself. Students go beyond their current capacity by grasping and constructing new mediating Bianshi problem.

The Chinese views of pedagogy, that is from "entering the way" to "transcending the way", emphasis "from consolidate" to "develop", emphasis "from familiarity" to "sophistication", emphasis "from imitate" to "grasp", and form the distinctive features which is different from the view of West Ideas of Realistic Mathematics Education (RME) in mathematics education.

In RME, Mathematics should be taught as mathematizing which emphasis leading students from the world of life to the world of symbols in the beginning, *i.e.*, Horizontal Mathematization. However, The Chinese views of Bianshi emphasis Symbols are shaped, reshaped, and manipulated within the mathematics discipline by changing variable step by step, *i.e.*, Vertical Mathematization instead of Horizontal Mathematization.

Human action has a tripartite structure. Leont'ev point out "The relationship between human agent and objects of environment is mediated by cultural means, tools and signs" (Leont'ev 1981, p. 208). We will elaborate below.

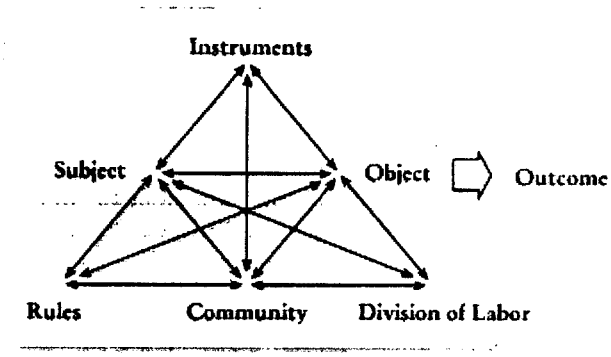


Figure 2. The structure of a human activity system (Engeström 1987, p. 78).

3. Why is Bianshi problem regarded as cultural characteristics in Chinese math education?

3.1. The object of Bianshi problem

3.1.1. Exam problems: The object of *Bianshi* problem

Mathematics occupies a central place in the curriculum of nearly all countries in the world, yet there may be big differences in the importance attached to mathematics as a school subject in Chinese society. Mathematics plays an important role in terms of sifting or filtering students through the education ladder in China (Zhang & Lee 1991). *Bianshi* teaching and learning is resulted from exams and driven by examination, with examination as its root. It was determined by the special position of exams in Chinese society and the distinctive role that Math's exams play in all the exam system.

The special position of exams in Chinese Society.

Exam is not a single education tradition that is formed overnight, there is own root of culture and values beyond exam (Zhang & Lee 1991; Bishop 1998). It is depended on and determined by certain environment. It is the extension of cultural structure and function. China, a country that is rich in population but relatively poor in resources, makes examination crucial in employment, graduation, and ascendance in the society hierarchy. Since 1977, China have held a national exam in the final year of secondary school to select students for university and assign them to different tracks. The culture of examination is one of the characteristics of Chinese community. Examination is still a relatively objective tool in selecting and grading teachers and students.

Math subject is the best differentiator among all the subjects.

Math is highest stakes subject among all subjects with strongly differentiated function at all school phase, to a large extent, determines the pupil's future profession, income, and social standing by highly selective curriculum track (Phelps 2001). Compared with other subjects, Math's is the one that differentiate students most effectively. Exams that characterized by good ruler of distinguishing students should include problems that require higher difficulty and higher level of thinking.

For the sake of reaching the goal, Bian shi teaching corresponds to teaching strategies developed under the pressure and motivation of society, schools and parents. Therefore, the culture of Bian shi teaching is derived from "teaching for exams" and "learning for exams", (*i.e.*, the so called examination oriented education). In some sense, "teaching for exams" and "learning for exams" constitute a special "Math's teaching culture" which realizes the unique "differentiation" in Chinese society system. The distinguished position of examination in Chinese society and the distinctive role of Math's exams plays in the exam system combined gave rise to the evolvement from "Math's teaching culture"

to "Bian shi teaching culture to a certain degree.

The exam problems are "from textbooks, but above textbooks".

Because the exam problems are "from textbooks, but above textbooks" (words from The Illustration of 2003 National College Entrance Examination for Science Majors in china 2003). For instance, it is stipulated by the National College Entrance Examination (hereafter referred as NCEE) Committee that examinations should have necessary differentiation ratio and proper difficulty, stressing on the hierarchy of problems. Based on testing basic knowledge, the emphasis of the examination is on testing students' mathematical thinking and methods, together with their mathematical abilities, which lead to the fact that problems appeared in NCEE are from textbooks after progressive variation, rather than the recurrence of exercises in the textbooks. According to some researches, although math's in NCEE does not have problems that simply request memorizing or that had appeared on textbooks, the archetype of many problems can be found in textbooks. That is to say, many problems appeared in NCEE are the variation, transformation or combination of the problems appeared in textbooks.

Bianshi problems link textbook problems to exam problems.

Math teaching under the pressure and motivation of society, schools and parents, has developed into the teaching and exercise of Bianshi problem, linking textbook problems to exam problems. Math teaching under the pressure and motivation of society, schools and parents, has become, intentionally or unintentionally, in the hope of accomplishing "economical" and "efficient" teaching, a process that making problems based on the mastered exercise in textbooks. The problems are given out according to the standards in the textbooks, but are changed in forms. However, the essential relations rules and theorems in the problem are invariable, enabling students to understand quickly. Therefore, the teaching that emphasizes laying strong foundation and improving thinking abilities is developed "economically and efficiently" through examinations in China. This requires that math teachers in junior and middle schools not to stay in copying or simply imitating the axioms, formulas, examples and exercises, but to apply Bian shi teaching based on textbooks, and develop several problems from one problem and several types from one type, by changing difficulties, variables, symbols through applying reversal thinking and creativity. As a result, students' thinking agility and adaptability can be developed by studying Bianshi problems and drawing inferences about other cases from one instance through to form a complete knowledge system, which are the gist of Bianshi problems.

An example.

For example, the group of Bianshi problem below (designed by Shen, 2001), made up a textbook problem and 4 exam problems, show the root of Bianshi problems tie with

exams.

The textbook problem:

There are 25 questions in a test. Each of them has 4 answers. Only one is right among them. Pupil would score 4 if he chooses the right one. Pupil would score-1 if he chooses the wrong one and does not choose. How many questions did student chose right if he scores 90? How many questions did student chose right if he scores 60? (Algebra 1 new edition Vol. 1 the fifth question in B group).

***Bianshi* problem 1:**

During a football competition in china in 1996, Dalian Wanda Team scored 3 points and had no defeat record. How many times did it win? According to the rule of competition: The team would score 3 points if it wins 1 time otherwise it would score 1 point if it ended with a draw (Ningxia upper secondary entrance exam problem 1996).

***Bianshi* problem 2:**

During a football competition (each team should have 11 times), Beijing Guoan Team scored 14 points. The times of wins are twice of defeats. How many times did it end with a draw? according to the rule of competition: The team would score 2 points if it wins 1 time otherwise it would score 1 point if it endes with a draw and it would score 0 point if it defeats (Wuhan upper secondary entrance exam problem 1997).

***Bianshi* problem 3:**

During a middle school football competition (each team should have 8 times), Menghu Team scored 7 points. The times of ends with a draw are twice of defeats. How many times did it win? According to the rule of competition: The team would score 3 points if it wins 1 time otherwise it would score 1 point if it ended with a draw and it would score 0 point if it defeats (Nanjing upper secondary entrance exam problem 1997).

***Bianshi* problem 4:**

During a middle school football competition (each team should have 4 times). The Team scored 17 points and .had no defeat record. How many times did it win? According to the rule of competition: The team would score 3 points if it wins 1 time otherwise it would score 1 point if it ended with a draw and it would score 0 point if it defeats (Zhuhai upper secondary entrance exam problem 2001).

The example contrasts the *Bianshi* problems, upper secondary entrance exam problems, textbook problems and show the relation of the *Bianshi* problems, upper secondary entrance exam problems, textbook problems. *Bianshi* problems are like the bridge to link textbook problems to exam problems.

The *Bianshi* exercises after school plays an important role in China.

Bianshi problems serve as the bridge between the original problems in the textbooks and real problems in examinations. Besides, exercise is highly regarded as an ideal learning way by Chinese society, which makes *Bianshi* problem an even more crucial part in mathematical curriculum and a major activity in math's learning after school. But the exercise problems applied are primarily *Bianshi* problem that are simultaneous with school teaching, progressive in their difficulties.

For example, "*Bianshi problem zhen, Huanggang bingfa*" (a success way of examination out by a package of *Bianshi problem*), the book granted National Excellent Educational Books Award and National Bestseller Book Award, has been one of the bestsellers in China in educational series (Zhu et 2003). The preface on the book cover reads "If you can solve examples in the textbook, you have learned 50% of the solutions. If you can solve *Bianshi* problems, you have learned 100% of the solutions — grasp the laws". It also shows Chinese math beliefs beyond surface.

Examinations: the quality control of the education systems.

It is well-known the exam is the culture characteristics of East Asia culture (zhang 1991; Wong 2004). But it is pity that the role of exams are not identified by scholars of East Asia at all. Examinations have been criticized for the reasons that narrow down curriculum and limit students' interest by most scholars for a long time.

In fact, exams' advantage were taken for granted and neglected. Some study (Phelps 2001; Mullis 1997; Bishop 1997) found that examinations are education quality control systems and mechanics to enforce the curriculum and instruction system. Examinations are superstructure of the education systems that support curriculum and instruction and hold implemented and attained system together. The relationship between the degree of the quality control and student achievement appears to be positive and exponential conclusion: The more quality control measures employed in an education system, the greater is students` academic achievement.

Bianshi problem may be regarded as the means to achieve the "backwash" effect of exams.

The rule could be generalized to cross-cultural comparisons. The top-performing country at the 8th grade level was more likely to have high-stakes examination systems than were others (Mullis 1997). For example, Curriculum objective — "Double Foundation" the list of top-performing counties: Singapore, Korea, Japan, Hong Kong, The Netherlands, all of these countries have high-stakes examination systems. Phelps (2001) mentioned the discovery is all remarkable even in states and province. Bishop (1997) further called the alleged effect of the high stakes upper secondary exit exams on the behavior of students and teachers at the lower secondary level a "backwash" effect.

Bianshi problem may be regarded as the means to achieve the "backwash" effect.

Bianshi problem may be regarded "the bridge" for exams to take step on.

Examinations set a "national achievement standards" for students to pursuit. China maintains curriculum quality horizontal coherence primarily through frequent administrations of nationally standard high-stakes examinations. At the same time Exams also keeps curriculum quality vertical coherence primarily by adjusting curriculum between the intended and attained curriculum. So the exam with high reasoning load and high order thinking-added play a important "ladder" role to make students to "climb". From this angle, Bianshi problem may be regarded "the bridge" for students to take step on.

Thus it can be seen it is the special position of exams in the Chinese society and the special position of mathematics among all the subjects double predetermine the cultural characteristics of Bianshi problem.

3.1.2. Curriculum objective — "Double Foundation"

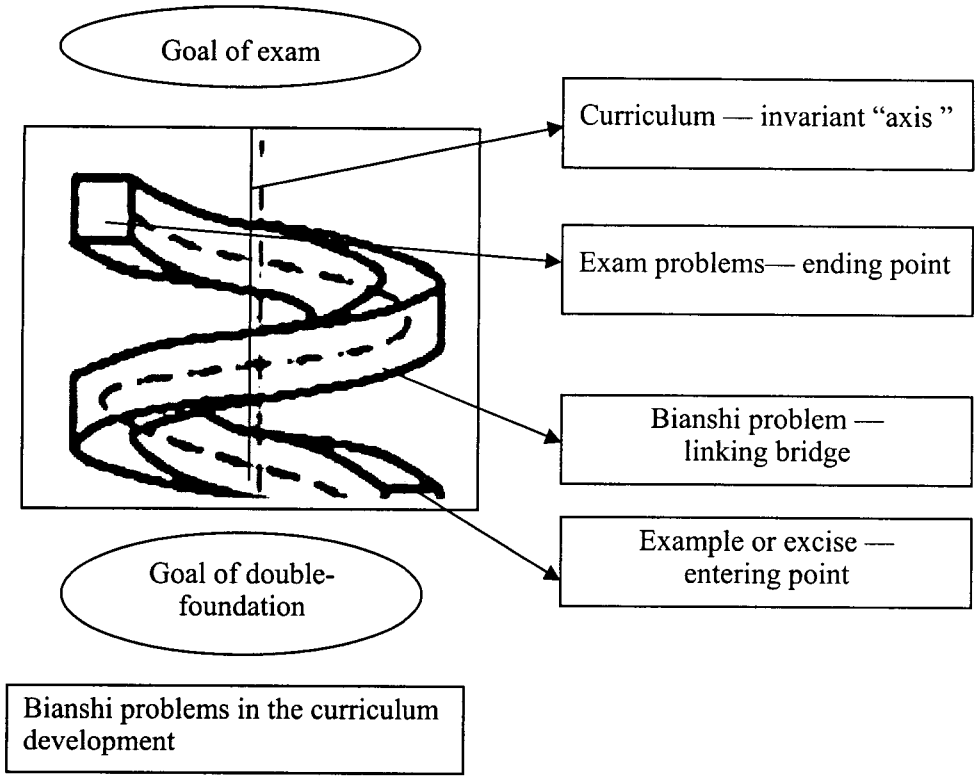
"Double Foundation" stipulated.

Double Foundation" is official content standard and core curriculum. It refers to basic knowledge and basic skill stipulated by Ministry of education. "Double Foundation" as the objective of Chinese mathematical teaching is stipulated in the General Outline of National Mathematical Teaching (jiaoxue dagang, now known as curriculum standard). Based on the general national math curriculum standard and textbooks, and oriented to examinations, each element involved in the education process, from national central authorities (Jiaoyu bu) to student's works together to carry out this education goal. "Double Foundation" means to recognize the invariables in the changing situation, and apply each basic skill in answering problems in different conditions. For example, the General Outline of National Mathematical Teaching (Ministry of education 1999) stipulated: the concepts, formulae, theorems are foundation for continued learning. Students should understand completely. Therefore, *Bianshi* problems solving are effective curriculum implementing models characterized by "dispersed and progressive difficulties" based on curriculum standard. "Bian shi teaching and exercise" has become the pathway to the "double foundation" (Zheng 2003).

We drew a picture clarify the character of *Bianshi* problem in the curriculum development above.

Based on the analysis mentioned, Bianshi problems should be designed to the "bridge problems" with depth under the vertical direction. So as that students to realize the goal of exam by solving it. On the other hand, Bianshi problems should also be designed to the "bridge problems" with width under the horizontal direction. So as that students to realize the goal of double base by solving it. All the Bianshi problems form a Bianshi

problem space with the spiral uptrend by circling math understanding.



3.2. The Conditions of Bianshi problem: the Chinese situation

3.2.1. The centralization of the education system

The centralization of the education system guarantees the implementation of Bian shi teaching without discount among different parts in the country. Mathematical education in China follows strictly the national general outline (jiaoxue dagang, now known as curriculum standard). In the centralized education system, mathematical teaching is carried out according to the "three unifications" which means unified national plans, general national outlines, and national textbooks and reference books. The textbooks written according to the teaching plans and curriculum standard made by government authorities are strictly followed by local schools and teachers in the process of education. Starting from curriculum standard and orienting to examinations decide the starting point and ending point of Bian shi teaching and learning. Together with the centralized unified management, all the factors make sure that Bian shi teaching will be carried out as expectation.

3.2.2. Bian shi teaching: the teaching strategy that deals with individual difference

Biansi problems are regarded double scaffolding in the individual level and in the classroom level.

Biansi problems are regarded scaffolding in the individual level (Bao Huang & Leung 2004). Besides this, The scaffolding are also regarded scaffolding in the classroom level. For example, Factorization of polynomial:

- $x^2+5x+6 = (x+m)\times(x+n) \quad m+n=5, \quad m\times n=6$
- What is the possible value of a and b, the polynomial can be factored.
- x^2+ax+6
- x^2+5x+b
- x^2+ax+b
- x^3+ax+b
- x^n+ax+b

Vygotsky showed that *bianshi* problem is first used in the interaction between teacher and student as a means of shared actions of help to overcome difficulty. Gradually it is internalized into a means of teacher's teaching and guide of students activity. Vygotsky formulated "the genetic law of cultural development" according to which the child's cultural development appears twice or on two planes (Vygotsky 1981, p. 163). First it appears interpsychologically, in interaction between individuals, and secondly within the child as an intrapsychological achievement.

Bianshi problem is the evolution of collective activity to mediate students thinking from. individual action under exam, object-oriented activity, control in conditions of joint.

Labour appears from the very beginning as a process mediated by tools (in the broad sense) and at the same time mediated socially (Leont'ev 1981, p. 208).

The group of *Bianshi* problems includes tree stages of "factorization" concept development: interiorization, condensation and reification, according to Sfard (1991) classification. Each pupil should learn "factorization" concept and extend their understanding by breaking away from surface content to structure content through the tree stages. At last grasp the abstract essence.

On the other hand, there are tree levels of difficulty problem. The students in different level of cognition would find the start points from the group of *Bianshi* problems. So the group of Biansi problems may be regarded double scaffolding in the individual level and in the classroom level.

There are so many students in China that each class has many students. Therefore, teaching in China is different from that in foreign countries where classes contain relatively less students so that the teachers can adopt different teaching instruments to

meet the needs of different students. Bian shi teaching, however, helps students in each level to find how to learn and think by gradually dividing them into different levels according to their thinking abilities and by changing the way of teaching, giving students in each level the opportunities to develop their abilities accordingly.

This is a special means to handle individual difference in classes with relatively large number of students. it is the special position of exams in the Chinese society and the special position of mathematics among all the subjects double predetermine the cultural characteristics of *Bianshi* problem.

Today Bian shi teaching is adopted consciously and broadly by classrooms around the country instead of unintentionally before, but till now, *Bianshi* exercises out of class is a main way to apply.

For instance, the *Bianshi* exercises of synchronization (Tongbu *Bianshi* exercise, *Bianshi* exercises of synchronization, maintaining common timing and coordination between math teaching and math exercise) published by Anhui educational press, has been widely used. And *"Bianshi Ttizhen, Huanggang Bingfa", written by Zhu Tongbiao et and* published by Shanxi Nomal School publisher has been one of the bestsellers in China in educational book series. Nowadays, it is more and more clear that Bian shi teaching and learning is suitable for Chinese society, and it is effective and time proving (The experimental team on mathematical reformation in Qingpuxian 1991).

3.3. The conception and mode of Chinese Teaching

3.3.1. The conception of Chinese Teaching

Bianshi teaching coincides with the conception of Chinese traditional teaching which emphasizes teachers as learning examples and problems in the textbook as a workable example to follow. This traditional Chinese teaching conception that epitomized as "initiating principles, imparting knowledge, and solving problems" ("传道授业解惑") affirms the dominant role teachers play in classrooms. Bian shi teaching also coincides with the teaching pattern after the liberation of China, a pattern expressed as a process of preparation — introduction — explanation — exercise — conclusion(复习-引入-讲授-练习-总结). All these make Bian shi teaching easily popularize in Chinese classrooms.

3.3.2. The mode of Chinese Teaching

The mode of Chinese Teaching mainly was influenced by Russian Kailuofu mode since the liberation of China. And Chinese traditional teaching ideas and methods (specially the method of elicitation, teaching benefits teachers as well as students) combine a united mode that emphasizes on system of knowledge and teacher-control in all junior and middle schools (Cao 2000). This mode made it easy common model that workable example enter and *Bianshi* problems consolidate in the country. The mode confirms thinking exercise tradition in the Chinese math's teaching.

3.4. Chinese math tradition

***Bianshi* showed the traditional continuity of structure and function of Chinese ancient math tradition to some extent in the modern Chinese classrooms.**

One of the intentions of education is to support the continuity of structure and function of a special tradition. There are significant differences between ancient Chinese mathematics and ancient Greek mathematics. The former is more algorithm-oriented and application-oriented, whereas the latter orients towards deductive reasoning. Without question, math teaching is always related with math history. Since the first SUANXUE GUAN was founded in 1876, Chinese have started to learn western math, and used western math textbooks. All Chinese math education systems began to copy western system and mode, namely, math content included Euclid geometry, classroom teaching instead of individually one by one teaching. From one tradition into another, it is not a simple import-export business, but *Bianshi* displays the traditional continuity of structure and function of Chinese ancient math tradition to some extent in modern Chinese classrooms.

The emphasis on *Bianshi* is the main feature of ancient Chinese math system. Fundamental *pattern* for the ancient Chinese mathematics is an *inductive and*

Bianshi system.

The Chinese ancient math developing system has its differentia-tion from the western traditional teaching system, "The Origin of Geometry" 《几何原本》as main teaching materials, which stresses on the systematicness and formalization mould-through-formality of math knowledge, (Li 1996). As stated by Josef Lee (李约瑟) in his works, The East has its own science and math and develops different paths and ideological methods from the West. There is a certain fundamental pattern for the Oriental mathematics. "That is to produce new methods from practical problems, promote them up to the level of general method, generalize them into shu (similar to"arithmetic" or general methods) and deploy these shu to solve various similar problems which are more complicated, more important and more abstruse" (Li 1996). The basic characteristic of Chinese mathematics is to conclude "shu" ("术" the general methods) from various derivatives, to distinguish the applicable conditions of various problems and to apply the "arithmetic" (shu "术" the general methods) to solve more difficult practical problems. According to the definition of Bianshi problem, all the problems can be called Bianshi problems, Accordingly, we call this certain fundamental pattern for the Oriental mathematics inductive and Bianshi system.

Almost all of the ancient math books take the same way.

All of the ancient math books such as "Zhoubbi Suanjing"《周髀算经》"Jiuzhang Suanshu"《九章算术》 "Haidao Suanjing"《海岛算经》"Zhangqiujian Suanjing"《张丘建算经》"Wuchao suanjing"《五曹算经》"Wujing Suanshu" 《五经算术》"Figu Suanjing"《缉古算经》 "Shushu jiyi"《数术记遗》"Xiahouyang Suanjing"《夏侯阳算经》(Wu 1998) took the same way known as the "Suanjing Shishu"《算经十书》which has thousands of years of history and has been the major teaching materials of ancient Chinese math teaching and contemporary Chinese Arithmetic teaching. All of them present the characteristic of inductive and Bianshi systems, and constitute the overall features of ancient Chinese mathematics (Wu 1998).

What is inductive and Bianshi system Jiuzhang Suanshu 《九章算术》as a example.

Jiuzhang Suanshu《九章算术》is the typical case of Chinese ancient math curriculum, and it has a far-reaching influence on Chinese ancient math. Back to Tang dynasty and Song dynasty, "Jiuzhang Suanshu" was cited as the textbook, and was the earliest math textbook in print (Zhang 1996). It plays a vital part in Chinese and even the world math history. It had been the most important textbook in the long history. Its configuration and ideological methods has deeply influenced the dissemination and teaching of mathematics (Wu 1982). Its problems were summarized in nine chapters, 206 problems, 202"shu". The Pythagorean theory was one of the shu concluded as quadratic

equation. The volume problem was also one of the shu concluded as method of volume, To use those shu to solve various complicated Bianshi problems in all book.

There are 18 problems in the eight volume of "Jiuzhang Suanshu" which are all linear equation problems. For example the fifth problem below follows a route like this "the example problem–answer — solution–the Bianshi problem".

Upgraded from the ideology of *inductive and Bianshi system to* the focal concept of *Bianshi.*

Take another case as an example, "Xiangjie Jiuzhangsuanfa" ("详解九章算法") written by Yang Hui in the early 1260s' chose 80 problems that typifies the basic types of problems as independent workable examples. He gave detailed explanations, summarized basic methods of calculation and analyzed through compare and contrast, Generalization like this: knowing 3 based on 1 (ju yiyu er sanyufan "举一隅而三隅反"), understanding 10 based on 1 (wenyi zhishi "闻一知十") (Zhang 1996). All these are upgraded concept of Bianshi: to pay attention to what varies and what is invariant in a learning situation, as patterns of variation mentioned by many scholars (Marton et al 2003; Mok 2003). It extended from the ideology of inductive and Bianshi system to the focal concept of Bianshi.

The Chinese math teaching formed the tendency of focal concept of Bianshi.

The Chinese math teaching was directly or indirectly influenced by Chinese math history, and formed the tendency of focal concept of Bianshi, such as emphasis the inductive, critical and distinguishing perspectives. For instance, in Chinese math classes, teachers usually ask students to solve Bianshi problems of different features, and unconsciously generalizing the solving tactics into methods, such as area method in geometric, Jiaoguifa (交轨法), which reflects this Chinese teaching characteristic of the rest may be referred (Yinfa leitui,yilei xiangcong, "因法类推" "以类相从") For example, the teaching materials for problem solving in Chinese math classes tend to be classified according to different types of problems, which has always been one characteristic of Chinese math education (Wong & Sun 2002).

Recurrence of mathematics tradition.

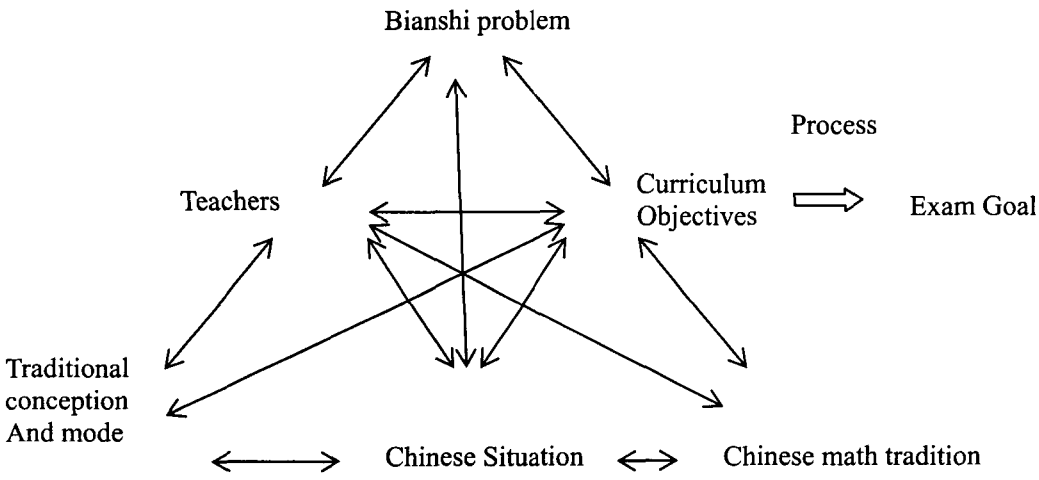
Bian shi teaching and exercise, in some sense, explicates a Chinese tradition of summarization in mathematics. It is also a recurrence of mathematics tradition under current western deduction system. In other words, it is the "replay" of a specific form of education, a 2000-year mathematical teaching culture that works under today's mathematical teaching environment.

II. INDICATIONS

In short, Bianshi problem, not just an isolated teaching actions and skills , but a collective system driven by an curriculum object and exam object. It is realized through individual actions driven by goals. It in turn is realized by means of routinized operations, dependent on the conditions such as Exam Goal, Curriculum Objectives, Chinese Situation, Traditional conception and mode, math traditions. So Bianshi problems are regarded double scaffolding in the individual level and in the classroom level to deal with individual difference. In order to portray the character of Bianshi problem in the implementation of curriculum in Mainland China, We integrate the various features of historical and cultural context of Bianshi problems behind into a network by a holistic view.

The purpose of this paper however is not to justify the practice of Chinese mathematics education The intention of the paper is to show that there exist distinctive features of Bianshi problem in china as the byproduct of exam culture, and that those features are distinctive expressions underlying cultural and historical context

The model of Bianshi problem



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