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Student Conceptual Understanding and Application on Algebra-problem-based Curricula

Lee, Kwangho
Department of Science and Mathematics Education, Weniger Hall 249,
Oregon State University, Corvallis, OR 97331 U.S.A.;
Email: leekw@onid.orst.edu

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This paper investigates student conceptual understanding and application on algebra using problem-based curricula. Seven principles which National Research Council announced were considered because these seven principles all involved in the development of a deep conceptual understanding. A problem-based curriculum itself provides a significant contribution to improving student learning. A problem-based curriculum encourages students to obtain a more conceptual understanding in algebra. From the results the national curriculum developers in Korea consider the problem-based curriculum.

Keywords: problem-based curricula, algebra, conceptual understanding.
ZDM Classification: H10
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INTRODUCTION

When students do not take algebra, they greatly limit their future career choices. In most schools, algebra is typically a prerequisite for geometry, and nearly all students who plan to attend college take geometry in high school (Pelavin & Kane 1988). Learning algebra improves student's ability to think clearly; it provides an excellent mental exercise that strengthens the mind. Also, since algebra requires creative problem-solving ability, learning algebra bolsters student ability to solve all kinds of problems. The concepts, principles, and techniques of algebra are important tools for describing and reasoning about patterns in all branches of mathematics. In spite of the importance of the algebra, traditional curricula treat only algebraic skills.

The National Council of Teachers of Mathematics (NCTM 2000) has addressed the algebra standards for instructional programs from pre-kindergarten through grade 12, stating that, "all students should enable: understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts" (p. 37).

Many studies on learning and cognition indicate that learning is an active process of constructing knowledge, rather than a passive process of memorizing information. Information is stored in memory in meaningful, dynamic structures and adding new information to a meaningful structure results in a richer structure in which many relations exist among the concepts included in the structure. Recently, new standards-based curricula have been developed in order to enhance student learning in mathematics. Standard-based mathematics curriculum is considered as curriculum aligned with the content standards prepared by the NCTM (2000). Since the passage of the Education Reform Act in 1993, as school districts align curriculum and teaching practices with the frameworks, standard-based mathematics programs are beginning to replace a more traditional curriculum.

The new curricula programs are developed to meet the increasingly diverse needs of mathematics classrooms, student conceptual understanding, and active learning. Many of those new curricula are designed based on a problem-based curriculum. Problem-based curricula offer engaging problem situations based on real world problems to help students come to their own understanding of mathematical concepts and their connections. For example, consider the problem "How many handshakes will occur at a party if every one of the 15 guests shakes hands with each of the others?" (NCTM 1989, p. 77). Does the use of such a problem-based curriculum positively affect students' conceptual understanding and ability to apply these new concepts of algebra content as expressed in the NCTM 2000 Standards?

To further this study, seven principles which National Research Council (NRC 2002) announced were considered because these seven principles all involved in the development of a deep conceptual understanding. Several research studies (Boaler 2002; Huntley, Rasmussen, Villarubi & Fey 2000; Krebs 2003; Lloyd & Wilson 1995; Remillard & Bryans 2004; Thompson & Senk 2001) are aligned with these seven principles. Other research studies (Caroll 1997; Briars & Resnick 2000; Riordan & Noyce 2001) show the direct effect of the curriculum on students' conceptual understanding. In conclusion this paper reveals the position of the question with the principles and the direct effect of the curriculum to student conceptual understanding.

The importance of whether teachers' beliefs are influenced by the curriculum that they use and whether they can implement a new curriculum is essential prior to consideration of the research studies. Nathan and Koedinger (2000) indicated that the discord between teachers' reform-based beliefs of students' algebra development and their instructional decisions appears to be influenced by textbook. According to the Nathan and Koedinger study, teachers' instructions are affected by the curricula. Obviously student achievement is affected by teacher instructions. So, student achievement is affected by both curriculum and instruction.

Most standard-based curricula are problem-based curriculum. Many empirical studies (Boaler 2002; Reys, Reys, Lapan, Holliday & Wasman 2003; Riordan & Noyce 2001) show that teachers' use of these curricula have a positive effect on students' mathematics learning in ways that increase students' mathematics achievement.

WHAT DO RESEARCHERS SAY?

Because of the numeric, graphic, and symbolic tools provided by today's calculators and computers, many mathematics educators (Davis 1993; Fey 1989; Kaput 1997) have suggested that students need to know only how to plan and interpret algebraic calculations, not to be proficient in the procedures themselves. Contemporary researchers on mathematics learning have suggested that students might be able to develop conceptual understanding of important algebraic ideas without prior acquisition of proficiency in procedural skills (Hiebert & Carpenter 1992). But many mathematicians and teachers are convinced that the ability to perform symbolic manipulations is an essential correlate of (if not prerequisite to) conceptual understanding and problem solving with algebraic expressions and equations. The various assessments used in studies allowed researchers to gain insight into this crucial issue from a variety of perspectives.

Several recent studies have focused on issues of problem-based curriculum implementation, providing details of successes and difficulties of the reform curriculum. Teachers' ideas about mathematics and how it is learned as well as their views about teaching contribute to their use of curriculum materials. Reviewing the research is considered for alignment with NRC's seven principles.

The first principle indicates that learning with understanding is facilitated when new and existing knowledge is structured around the major concepts and principles of the discipline (Lloyd & Wilson 1995; Remillard & Bryans 2004). When emphasizing conceptual understanding and connections between concepts and representations, teachers facilitate students' conceptual understanding. Highly proficient performance in a subject domain requires knowledge that is both accessible and usable-knowledge that is connected with other concepts. Lloyd & Wilson (1995) indicated that teachers' well-established conceptions contributed to instruction that emphasize conceptual connection, powerful representations, and meaningful discussions in teaching a function unit using

Core-Plus Mathematics Project (CPMP). The teacher allowed students to explore multiple representations of problem situations in their group and then had a whole-class summary discussion highlighting connections between concepts and representations. Remillard and Bryans (2004) also suggested that a problem based curriculum (Investigations) consists of teachers in examining unfamiliar curriculum resources and developing new approaches to using these materials. The problem-based curriculum influenced teachers emphasized the importance of conceptual understanding and verbalizing one's knowledge by using the curriculum. The teachers' emphases of conceptual understanding and communication by using curricula will directly influence students' conceptual understanding according to Nathan and Koedinger (2000).

The second principle indicates that learners use what they already know to construct new understandings. People construct meaning for a new idea or process by relating it to ideas or processes they already understand (Kreb 2003). Krebs found that students possessed some strategies to help them formalize generalizations in Connected Mathematics Project (CMP). When students described a problem as an "algebra task" they used a numeric approach generating a table of values and studying the numerical data for patterns. When the students confronted a situation requiring them to frame patterns of data as a function, they demonstrated a robust understanding of an important slice of algebra using their prior knowledge.

In the third principle, learning is facilitated through the use of metacognitive strategies that identify, monitor and regulate cognitive processes. Students, discussing in small groups and reflecting on their concepts followed by a whole-class summary discussion using CPMP, lead to a deep understanding of function (Lloyd & Wilson 1995). From the whole-class summary discussion students had an opportunity to evaluate their thinking processes. Students planned and monitored their ideas before explaining their thinking or understanding in University of Chicago School Mathematics Project (UCSMP) (Thompson & Senk 2001). Also, students selected thinking strategies deliberately to find strategies for formalizing generalizations in CMP (Krebs 2003).

The fourth principle indicates that learners have different learning styles. Educators need to be sensitive to such differences so that instruction and curricular materials are suitably matched to students' developing abilities, knowledge base, preferences, and styles. A significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes. The degree of students' conceptual understanding is related to the richness and extent of the connections they have made (NRC 2001). In Krebs' (2003) study, using CMP students demonstrated their conceptual understanding in different ways. These students demonstrated their understanding of quadratic functions and their ability to make connections among representations (tables, symbols, and graphs) of different functions.

The fifth principle describes learners' motivation as important in the learning process. Students' positive attitudes are productive for reaching conceptual understanding and application in mathematics. Students' positive dispositions toward the curriculum increase motivation to attack problems. In Boaler's (2002) study the students taking Interactive Mathematics Program (IMP) expressed satisfaction with their mathematics experience. In Krebs' (2003) study, when students using CMP confronted challenging problems, their response was to persevere with the problem. When students did not write generalizations in algebra problems, they wanted to continue toward a solution. All students had sufficient confidence to pursue the problems, even after they recognized them as challenging or unfamiliar.

According to the sixth principle, the practices and activities, in which people engage while learning, shape what is learned. In a comparison study of one traditional and one reform-oriented school, Boaler (1998) found that students in the traditional school were much less able to apply their mathematical knowledge to novel or real-life situations than students in the reform-oriented school.

Teachers can engage learners in important practices that can be used in different situations by drawing upon real-world exercises, or exercises that foster problem-solving skills and strategies that are used in real-word situations. Problem-based curriculum involves approaches that create opportunities for students to engage in real word practice. Real world application problems are effective in developing students' understanding (Huntley et al. 2000; Thompson & Senk 2001).

Huntley et al. (2000) investigated the growth of student understanding, skill, and problem solving ability in algebra. The results indicated that the CPMP curriculum emphasis on multiple representations of algebraic ideas is more effective than conventional curricula in developing a student's ability to solve algebraic problems when those problems are presented in realistic contexts. Thompson and Senk (2001) compared student achievement in second-year algebra on two different curricula, UCSMP and traditional curriculum. In the study, UCSMP students outperformed comparison students on multi-step problems and problems involving applications or graphical representations. Also UCSMP classes outperformed the comparison classes on the Problem-Solving and Understanding Test which included real world applications and required students' explanations.

Finally, the seventh principle indicates that learning is enhanced through socially supported interactions. Through meaningful discussion and explanation of their knowledge or ideas, students enhance their conceptual understanding. Many studies (Lloyd & Wilson 1995; Remillard & Bryans 2004; Thompson & Senk 2001) used students' communication for better understanding. Using CPMP teachers in Lloyd and

Wilson's study had meaningful discussion for the development of tables, graphs and equations. Most of the lesson in Remillard and Bryans' (2004) study involved students' explanations to articulate their understanding to the class or teacher. Students were engaged in class discussion and encouraged to explain their thinking or justify their reasoning in UCSMP advanced algebra (Thompson & Senk 2001).

Although changing teacher practice is difficult (Hiebert 1999; Richardson 1990), teachers can implement the problem-based mathematics curriculum in their instruction. This change would allow them to increase their students' conceptual understanding through teachers' different approaches. The materials may encourage the teacher to have students work at explaining their reasoning and coming up with multiple approaches to solving a problem. In standard-based programs, lessons are often introduced by presenting students with an unfamiliar problem rather than a worked example (Goldsmith & Mark 1998). Thus the problem-based curriculum indirectly affects students' conceptual understand and application.

Reys, Reys, Lapan Holliday & Wasman (2003) reported that there were significant differences on the Missouri Assessment Program with Algebra which included students describing patterns and relationships using algebraic inequalities. Students using problem-based curriculum materials (*Math Thematics or Connected Mathematics Project*) for at least 2 years did better than students using traditional curriculum materials. The achievement test items included conceptual understanding in algebra. The results in Riordan and Noyce (2001) also added that a longer implementation of problem-based curriculum in the school was associated with a greater score advantage for students. According to the results, the problem-based curricula influenced student conceptual understanding in algebra over a long term.

Many studies have provided data concerning student achievement based on standardized test. Carroll (1997) reported higher test scores for students who had experienced *Everyday Mathematics* for one or two years, even though the test format was not aligned with the curriculum. Briars & Resnick (2000) also examined the impact of *Everyday Mathematics* on achievement in the Pittsburgh public school. They compared scores on a statewide test from 1996 to 1998 for all fourth-grade students.

They found overall improvement during this time in all competency levels designated as skills, concepts, and problem solving. Riordan and Noyce (2001) indicated that students in schools using the problem-based curricula (*Everyday Mathematics* for elementary level and CMP for middle grade level) performed significantly better on the 1999 statewide mathematics test which is based on student reasoning and conceptual understanding in Patterns, Relations and Functions, Geometry and Measurement, Statistics and Probability, and Number Sense than did students in traditional programs attending matched comparison schools. The students using either of the programs performed better on solving higher order mathematics problems. According to this, a problem-based curriculum itself provides a significant contribution to improving student learning.

CONCLUSION

Many studies (Boaler 1998; Boaler 2002; Huntley et al. 2000; Krebs 2003; Lloyd & Wilson 1995; Remillard & Bryans 2004; Thompson & Senk 2001) reveal that problem-based curricula are aligned with NRC's seven principles for learning with deep understanding. This alignment suggests that problem-based curriculum encourage students to obtain a more conceptual understanding. From the results the national curriculum developers in Korea consider the problem-based curriculum.

Although teacher practice change is difficult, problem-based curriculum influences teachers to change their practices in order to directly influence student conceptual learning. However, it takes time to implement in the classroom to enhance student conceptual understanding. Korean teachers have many kinds of professional development programs in vocation. Their practice will change faster than USA teachers'. Their practice will more focus on student conceptual using problem-based curriculum.

Achievement tests also reveal that students using problem-based curriculum have a better conceptual understanding than others. One study showed that students' reasoning and conceptual understanding were better for students using a problem-based curriculum. Given this evidence, it is reasonable to suggest that problem-based curriculum positively affects students' conceptual understanding and ability to apply these new concepts of algebra content as expressed in the NCTM 2000 standards.

Many of the studies do not directly reveal students' conceptual understanding using a problem-based curriculum. These studies provide general information about the implementation of a problem-based curriculum in mathematics not specifically in algebra. According to some of the research, the effectiveness of the problem-based curriculum did not appear immediately rather the effectiveness demonstrating took at least two years. So, before implementing the curriculum, teachers need to be prepared to deal with the curriculum. Also, developers of problem-based curricula need to address more carefully how teachers can learn to develop their instruction along the lines of reform. These studies indirectly dealt with algebra while assessing the problem-based curricula. So, more studies are needed to specifically investigate the direct impact of problem-based curriculum to students' conceptual understanding in algebra.

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