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Testing for Exponentiality Against Harmonic New Better than Used in Expectation Property of Life Distributions Using Kernel Method

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Abstract. A new test for testing that a life distribution is exponential against the alternative that it is harmonic new better (worse) than used in expectation upper tail HNBUET (HNWUET), but not exponential is presented based on the highly popular "Kernel methods" of curve fitting. This new procedure is competitive with old one in the sense of Pitman's asymptotic relative efficiency, easy to compute and does not depend on the choice of either the band width or kernel. It also enjoys good power.

Key Words : *test exponentiality, convex ordering, life distributions classes, kernel method, asymptotic normality, asymptotic relative efficiency, power estimate.*

1. INTRODUCTION AND DEFINITIONS

In reliability theory, various concepts of aging have been proposed to study lifetimes of components or systems. Therefore, statisticians and reliability analysts have shown a growing interest in modeling survival data using classification life distributions (i.e. distribution function F with $F(0-) = 0$, survival function $\overline{F} = 1-F$ and finite mean $\mu = \int_0^\infty \overline{F}(u)du$) based on some aspects of aging. Among these aspects are IFR (increasing failure rate), IFRA (increasing failure rate average), NBU (new better than used), NBUE (new better than used in expectation) and HNBUE (harmonic new better than used in expectation). For definitions and further details see, for example, Haines (1973), Barlow and Proschan (1981) and Zacks (1992). Deshpande *et al.* (1986) introduced a new class of life distribution named HNBUE(3) (HNWUE(3)) (harmonic new better (worse) than used in expectation of

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third order) which is larger than HNBUE (HNWUE) class. Note that Abouammoh and Ahmad (1989) renamed the class HNBUE(3) by HNBUET (harmonic new better than used in expectation upper tail).

The implications among the above classes of life distributions are

$$IFR \Longrightarrow IFRA \Longrightarrow NBU \Longrightarrow NBUE \Longrightarrow HNBUE \Longrightarrow HNBUET.$$

Similar implications hold for the corresponding dual classes

$$DFR \Longrightarrow DFRA \Longrightarrow NBU \Longrightarrow NBUE \Longrightarrow HNBUE \Longrightarrow HNBUET.$$

Many test statistics have been developed for testing exponentiality against various aging alternatives. Testing exponentiality against the classes of life distributions has received a good deal of attention. For testing against new better than used (NBU), we refer to Hollander and Proschan (1972) and Koul (1977) among others. For testing against decreasing mean residual life (DMRL) we refer to Hollander and Proschan (1975), Ahmad and Li (1992) and Abu-Youssef (2002) among others. For new better than used in expectation (NBUE), we refer to Hollander and Proschan (1975) and Ahmad et al. (1999) among others. For harmonic new better than used in expectation (HNBUE), we refer to Klefsjo (1982) and Hendi et al. (1998) among others.

The classes HNBUE and HNBUET may be defined on the basis of a variability definitions due to Stoyan (1983), which is the following.

Definition 1. Let X and Y be two random variables with marginal distributions F and G, respectively. We say that X is less variable than Y (or X is smaller than Y in convex ordering) and write $X \leq_{VR} Y$ if $E[h(X)] \leq E[h(Y)]$ for all increasing convex functions h. Clearly $X \leq_{VR} Y$ if and only if $\int_x^\infty \overline{F}(u)du \leq \int_x^\infty \overline{G}(u)du$ for all $x \geq 0$ where $\overline{F}(x) = 1 - F(x)$ and $\mu = \int_0^\infty \overline{F}(u)du$.

It is not difficult to see that X is HNBUE if and only if $X \leq_{VR} X_0$, cf. Ahmad (1995), where X_0 is a random variable with the exponential distribution with mean μ. Then from definition (1.1) we have the following:

(i) $F \epsilon$ HNBUE iff

$$\int_x^\infty \overline{F}(u)du \leq \mu \exp\left(\frac{-x}{\mu}\right); \qquad x \geq 0\ ,\ \mu > 0. \tag{1.1}$$

Integrating both sides of Equation (1.1) with respect to x, from t to ∞, we obtain

$$\int_t^\infty \int_x^\infty \overline{F}(u)dudx \leq \mu^2 \exp\left(\frac{-t}{\mu}\right); \qquad x, t \geq 0\ ,\ \mu > 0. \tag{1.2}$$

Equation (1.2) is the definition of the class "harmonic new better than used in expectation upper tail (HNBUET)".

Example 1. Consider the survival function $\overline{F}(x)$ given by

$$\overline{F}(x) = \begin{cases} e^{-x} & \forall\, 0 \le x \le 1 \\ 0.7e^{-1} & \forall\, 1 \le x < 1.5 \\ 0 & \forall\, x > 1.5 \end{cases}$$

It is easy to prove that $\overline{F}(x)\epsilon$ HNBUET. Al-Ruziza et al (2003) derived a moment inequality for HNBUET, based on this inequality they introduced testing procedure for exponentiality against HNBUET. Al-Ruziza (2003) derived test statistics based on U-test and demonstrated applications for HNBUET.

This article proposes a new test statistic, based on Kernel method , for testing H_0 : F is exponential (μ) against H_1 : F is HNBUET and not exponential. This approach is based on defining a measure of departure from H_0 in favor of H_1 that depend on pdf $f(x)$ and then estimating this measure empirically. The empirical version of this measure require estimating $f(x)$ and thus one may use the celebrated kernel method. For a background material on this method, we refer to the books by Scott (1992) and Jones and Wand (1995). Using Kernel method in reliability appears in early work of Watson and Leadbetter (1964) and Ahmad (1976) among others. While using kernal method for testing NBUC, NBUE, and HNBUE are given by Ahmad, et al (1999). In section 2, conditions under which $\sqrt{n}(\hat{\Delta}_{KF_n} - \Delta_{KF})$ is asymptotically normal are given and the null and non-null variance are obtained. The test based on $\hat{\Delta}_{KF_n}$ is shown to be consistent. Monte Carlo null distribution critical points for sample sizes $n = 5(1)40$ are presented. In section 3, efficiency of the test statistic are calculated for some common alternatives and compared to other procedures. Finally the power estimate of the test is given for some well known alternatives in section 4.

2. TESTING THE HNBUET CLASS

2.1 The test procedure

The test here depends on a random sample $X_1, \ldots, X_n$ from a population with distribution F. We wish to test the null hypothesis H_0 : $\overline{F}$ is exponential against the alternative hypothesis H_1 : $\overline{F}$ is HNBUET and not exponential, that is,

$$\int_t^\infty \int_x^\infty \overline{F}(u)dudt \le \mu^2 \exp\left(\frac{-t}{\mu}\right); \qquad x, t \ge 0\ ,\ \mu > 0.$$

In order to test H_0 against H_1 we use the following measure of departure from H_0 as

$$\delta_{KF} = \int_0^\infty f(x)[\mu^2 \exp(-x/\mu) - \int_x^\infty \nu(t)dt]dF(x) \tag{2.1}$$

where

$$\nu(x) = \int_x^\infty \overline{F}(u)du.$$

We have

$$\int_x^\infty \nu(t)dt = -x\nu(x) - \frac{1}{2}x^2\overline{F}(x) + \frac{1}{2}\int_x^\infty t^2 I(x > t)dF(t). \tag{2.2}$$

Using (2.2), (2.1) becomes

$$\begin{aligned}\delta_{KF} = \int_0^\infty f(x)[\mu^2 e^{\frac{-x}{\mu}} + x\nu(x) + \frac{1}{2}x^2\overline{F}(x) \\ -\frac{1}{2}\int_0^\infty t^2 I(x > t)dF(t)]dF(x),\end{aligned} \tag{2.3}$$

where

$$I(y > t) = \begin{cases} 1, & x > t \\ 0, & o.w \end{cases}$$

Note that under $H_0 : \delta_{KF} = 0$, while under $H_1 : \delta_{KV} > (<)0$. To estimate δ_{KF}, let $X_1, X_2, \ldots, X_n$ be a random sample from F, let $\overline{F}_n(x) = \frac{1}{n}\sum_{j=1}^n I(X_j > x)$ denotes the empirical distribution of the survival function $\overline{F}(x), dF_n(x) = \frac{1}{n}$,μ is estimated by sample mean $\overline{X}$ and pdf $f(x)$ is estimated by $\hat{f}_n(x) = \frac{1}{na_n}\sum_{j=1}^n k(\frac{x-X_j}{a_n})$, where $k(.)$ be a known pdf, symmetric and bounded with 0 mean and variance $\sigma_k^2 > 0$. Symmetric uniform, normal, double exponential are examples of such pdf. Let $\{a_n\}$ be a sequence of reals such that $a_n \to 0$ and $na_n \to \infty$ as $n \to \infty$. Other conditions on k and a_n will be stated when needed. We propose to estimate δ_{KV} by

$$\begin{aligned}\hat{\delta}_{KF_n} = \int_0^\infty \hat{f}_n(x)[\overline{X}^2 \exp(\frac{-x}{\overline{X}}) + x\hat{\nu}_n(x) + \frac{1}{2}x^2\overline{F}_n(x) \\ -\frac{1}{2}\int_0^\infty t^2 I(t > x)dF_n(t)]dF_n(x),\end{aligned} \tag{2.4}$$

i.e.,

$$\begin{aligned}\hat{\delta}_{KF_n} = \frac{1}{n^4 a}\sum_{i=1}^n\sum_{j=1}^n\sum_{k=1}^n\sum_{l=1}^n K(\frac{X_i - X_l}{a})\{X_j X_k e^{-X_i/\overline{X}} \\ +(X_i X_j - \frac{1}{2}X_i^2 - \frac{1}{2}X_j^2)I(X_j > X_i)\}.\end{aligned} \tag{2.5}$$

Let us rewrite(2.5) as

$$\hat{\delta}_{kV_n} = \frac{1}{n(n-1)(n-2)(n-3)}\sum\sum\sum\sum_{i\neq j\neq l\neq k\neq n} \phi_n(X_i, X_j, X_k, X_l). \tag{2.6}$$

To make the test statistics scale invariant, we take

$$\hat{\Delta}_{kV_n} = \frac{\hat{\delta}_{kV_n}}{\overline{X}^2} \tag{2.7}$$

with measure of departure $\hat{\Delta}_{KV_n} = \frac{\delta_{kV_n}}{\mu^2}$.

Set

$$\phi(X_1, X_2, X_3, X_4) = k(\frac{X_1 - X_4}{a})\{X_2 X_3 e^{-X_1/\overline{X}} + (X_1 X_2 - \frac{1}{2}X_1^2 - \frac{1}{2}X_2^2)I(X_2 > X_1)\}, \quad (2.8)$$

and define the symmetric kernel

$$\xi(X_1, X_2, X_3, X_4) = \frac{1}{4!}\sum_R \phi_n(X_{i1}, X_{i2}, X_{i3}, X_{i4}),$$

where the sum over all arrangements of ($X_1, X_2, X_3$and X_4). Then $\hat{\delta}_{KV_n}$ is equivalent to the U-statistic. Since $\hat{\Delta}_{KV_n}$ and $\frac{\hat{\delta}_{kV_n}}{\mu^2}$ have the same limiting distribution, we use $\sqrt{n}(\hat{\delta}_{KF_n} - \delta_{KF})$ and the following theorem summarizes the large sample properties of $\hat{\delta}_{KF_n}$ as U_n.

Theorem 1. If $na_a^4 \to 0$ as $n \to \infty$, if f has bounded second derivative and if $V(\psi_n(X_1)) < \infty$, where $\psi_n(X_1)$ is as given (2.15), then $\sqrt{n}(\hat{\delta}_{KF_n} - \delta_{KF})$ is asymptotically normal with mean 0 and variance $\lim_n V(\psi_n(X_1)$. Under H_0, the variance $= 0.244$

The following simple lemma is needed in the proof of Theorem 1.

Lemma 1. Let $\theta_n = E\hat{\delta}_{KF_n}$, then

$$\theta_n = \int_0^\infty E[\hat{f}_n(x)] \int_0^\infty f(x)[\mu^2 e^{\frac{-x}{\mu}} + x\nu(x) + \frac{1}{2}x^2\overline{F}(x) - \frac{1}{2}\int_0^\infty t^2 I(x > t)dF(t)]dF(x). \quad (2.9)$$

Proof. Note that $E\hat{f}_n(x) = \frac{1}{a}\int(\frac{x-y}{a})f(y)dy$. Set $g_n(x) = E\hat{f}_n(x)$, thus

$$E\hat{\delta}_{KF_n} = \theta_n = E[\phi_n(X_1, X_2, X_3, X_4)] \quad (2.10)$$

where

$$\phi(X_1, X_2, X_3, X_4) = k(\frac{X_1 - X_4}{a})\{X_2 X_3 e^{-X_1/\overline{X}} + (X_1 X_2 - \frac{1}{2}X_1^2 - \frac{1}{2}X_2^2)I(X_2 > X_1)\},$$

Hence

$$\theta_n = Eg_n(X_1)\{X_2 X_3 e^{-X_1/\overline{X}} + (X_1 X_2 - \frac{1}{2}X_1^2 - \frac{1}{2}X_2^2)I(X_2 > X_1)\}$$

$$= \int_0^\infty g_n(x)[\mu^2 e^{\frac{-x}{\mu}} + x\nu(x) + \frac{1}{2}x^2\overline{F}(x) - \frac{1}{2}\int_0^\infty t^2 I(x > t)dF(t)]dF(x). \quad (2.11)$$

Proof of theorem 1. Note that

$$\sqrt{n}(\hat{\delta}_{KF_n} - \delta_{KF}) = \sqrt{n}(\hat{\delta}_{KF_n} - \theta_n) + \sqrt{n}(\theta_n - \delta_{KF}) \quad (2.12)$$

But

$$\begin{aligned} E\hat{f}_n(x) &= \frac{1}{a}\int k(\frac{x-y}{a})f(y)dy = \int k(w)f(x-aw)dw \\ &\simeq f(x) + \frac{a^2}{2}f''(x)\sigma_k^2, \end{aligned}$$

under the condition assumed on k. Hence

$$\begin{aligned} \theta_n &\simeq \delta_{KF} + \frac{a^2}{2}\sigma_k^2\Big\{\int_0^\infty f''(x)[\mu^2 e^{\frac{-x}{\mu}} + x\nu(x) + \frac{1}{2}x^2\overline{F}(x) \\ &-\frac{1}{2}\int_0^\infty t^2 I(x > t)dF(t)]dF(x)\Big\}. \end{aligned} \quad (2.13)$$

Thus $\sqrt{n}(\theta_n - \delta_{KF}) = O(a^2\sqrt{n}) = 0(1)$ by assumptions. Note also $\hat{\delta}_{KF}$ is unbiased estimate of $\theta_n = E\hat{\delta}_{KF}$ and is asymptotically unbiased estimate of δ_{KF_n}. Next, note that

$$\begin{aligned} \sqrt{n}(\hat{\delta}_{KF_n} - \theta_n) &= \sqrt{n}(\frac{1}{n}\sum_{i=1}^n \psi_n(X_i)) + (n(n-1)(n-2)(n-3)^{-1} \\ &\sum\sum\sum_{i\neq j\neq l+k} \xi_n(X_i, X_j, X_k, X_l,) \end{aligned} \quad (2.14)$$

where

$$\begin{aligned} \psi_n(X_1) &= E[\phi_n(X_1, X_2, X_3, X_4)|X_1] + E[\phi_n(X_1, X_2, X_3, X_4)|X_1] \\ &+E[\phi_n(X_2, X_3, X_1, X_4)|X_1] \\ &+E[\phi_n(X_2, X_3, X_4, X_1)|X_1] - 4\theta_n \end{aligned} \quad (2.15)$$

and

$$\xi_n(X_1, X_2, X_3, X_4) = \phi_n(X_1, X_2, X_3, X_4) - \psi_n(X_1) - 3\psi\theta_n. \quad (2.16)$$

Now, by Layaponouff's central theorem, the first term in the right hand side of (2.16) is asymptotically normal if $L_n = \frac{E[\psi_n(X_1)]^{2+\delta}}{\sqrt{n}}[V(\psi_n(X_1))]^{1+\delta/2} \to 0$ as $n \to \infty$. Now using (2.12) it is easy to see for large n

$$\begin{aligned} E[\phi_n(X_1, X_2, X_3, X_4)|X_1] &= f(X_1)[\mu^2 e^{\frac{-X_1}{\mu}} + X_1\int_{X_1}^\infty udF(u) \\ &-\frac{1}{2}X_1^2\overline{F}(x) - \int_{X_1}^\infty u^2 dF(u)], \end{aligned} \quad (2.17)$$

$$
\begin{aligned}
E[\phi_n(X_2, X_1, X_3, X_4)|X_1] &= \mu X_1 \int_0^\infty f(u) e^{\frac{-y}{\mu}} dF(u) + X_1) \int_0^{X_1} u f(u) dF(u) \\
&\quad -\frac{1}{2}\int_0^{X_1} u^2 f(u) dF(u) \\
&\quad -\frac{1}{2} X_1^2 \int_0^{X_1} f(u) dF(u), \qquad (2.18)
\end{aligned}
$$

$$
\begin{aligned}
E[\phi_n(X_2, X_3, X_1, X_4)|X_1] &= \mu X_1 \int_0^\infty f(y) e^{\frac{-y}{\mu}} dF(y) + \int_0^\infty y f^2(y) \int_y^\infty u f(u) du dy \\
&\quad -\frac{1}{2}\int_0^\infty y^2 f^2(y) \overline{F}(y) dy \\
&\quad -\frac{1}{2}\int_0^\infty f^2(y) \int_y^\infty u^2 f(u) du dy. \qquad (2.19)
\end{aligned}
$$

Observe that $E[\phi_n(X_2, X_3, X_4, X_1)|X_1]$ has the same representation as (2.17). Set $\eta(X_1)$ to be the sum of twice of right hand side of (2.17) plus that of (2.18) and that of (2.19).

Thus

$$\psi_n(X_1) = \eta(X_1) + O_p(a^2). \qquad (2.20)$$

Hence

$$V(\psi_n(X_1)) = Var(\eta_1(X_1)) + O(a^2)$$

and for $p > 2$,

$$E|\psi_n(X_1)|^p \leq C_p E|\eta(X_1)|^p = O(1).$$

Hence, $L_n \to 0$ as $n \to \infty$ provided that $na^4 \to 0$ as $n \to \infty$.

Next, look at

$$
\begin{aligned}
& E\left[\frac{\sqrt{n}}{n(n-1)(n-2)(n-3)} \sum\sum\sum\sum\sum_{i\neq j\neq k\neq l\neq m} \xi_n(X_i, X_j, X_k, X_l)\right] \\
&\doteq \frac{1}{n(n-1)^2(n-2)^2(n-3)^2} \sum\sum\sum\sum_{i\neq j\neq k\neq l} \\
&\qquad E[\xi_n(X_i, X_j, X_k, X_l) \times \xi_n(X_i, X_j, X_k, X_l)] \\
&= \frac{1}{(n-1)} E\xi_n^2(X_1, X_2, X_3, X_4) = O(na)^{-1} = O(1). \qquad (2.21)
\end{aligned}
$$

Under H_0, $\overline{F}(x) = e^{-x}$ and

$$\eta(X_1) = \frac{-11}{24} + \frac{11}{12} X - \frac{1}{4} X^2 + \frac{1}{8} e^{-2X}. \qquad (2.22)$$

Thus $E_0[\eta(X_1)] = 0$ and $\sigma_0^2 = Var[\eta(X_1] = 0.24441$ by direct calculation. The theorem is proved.

2.2 Monte Carlo null distribution critical points

We have simulated the upper percentile points for 95%, 98% and 99%. Table 2.1 gives these percentile points of the statistic $\hat{\Delta}_{KF_n}$ in (2.6). The calculations are based on 5000 simulated samples of sizes n =5(1)40. It is clear from Table 2.1 that percentile values change slowly as n increases. To perform the above test, calculate $\sqrt{\frac{1000n}{244}}\hat{\Delta}_{KF_n}$ and reject H_0 if this values exceeds Z_α the standard normal variate.

Table 2.1 Critical values of $\hat{\delta}_{KF_n}$

n	95%	98%	99%
5	1.1116	1.9625	2.9074
6	0.7376	1.5085	2.2226
7	0.5067	0.7512	0.9450
8	0.4108	0.6422	0.9790
9	0.3174	0.5569	0.7182
10	0.3033	0.4205	0.6207
11	0.2481	0.3994	0.5462
12	0.2683	0.4323	0.5614
13	0.2105	0.3227	0.3888
14	0.2051	0.3121	0.4345
15	0.2116	0.2654	0.3576
16	0.1688	0.2476	0.2760
17	0.1609	0.2385	0.2972
18	0.1527	0.2357	0.2766
19	0.1684	0.2268	0.2775
20	0.1335	0.1901	0.2259
21	0.1437	0.2044	0.2402
22	0.1363	0.1856	0.2321
23	0.1305	0.1705	0.2189
24	0.1136	0.1681	0.2116
25	0.1262	0.1737	0.2164
26	0.1178	0.1528	0.1963
27	0.1101	0.1444	0.1751
28	0.1160	0.1588	0.1788
29	0.1096	0.1468	0.1596
30	0.1004	0.1376	0.1566
31	0.1079	0.1370	0.1546
32	0.0916	0.1300	0.1645
33	0.0930	0.1244	0.1520
34	0.0955	0.1301	0.1590
35	0.0866	0.1147	0.1285
36	0.0860	0.1208	0.1469
39	0.0844	0.1052	0.1301
40	0.0805	0.1056	0.1216

3. PITMAN ASYMPTOTIC RELATIVE EFFICIENCY (PARE)

In this section, we compare the statistic $\hat{\Delta}_{KF_n}$, given in equation (2.7), with the statistics $\hat{\Delta}_{F_n}$, proposed by Al-Ruzaiza (2003) and $\hat{\Delta}_3^{(1)}$, proposed by Al-Ruzaiza *et al.* (2003) The construction of this statistic is based on the moments inequalities for harmonic new better than used in expectation property. The comparisons are achieved by using the Pitman asymptotic relative efficiency (PARE), which is defined as follows:

Let T_{1n} and T_{2n} be two test statistics for testing $H_0 : F_\theta \epsilon \{F_{\theta_n}\}$, $\theta_n = \theta + Cn^{-1/2}$ where C is an arbitrary constant. Then the PARE of T_{1n} relative to T_{2n} is defined by

$$e(T_{1n}, T_{2n}) = \{\mu_1'(\theta_0/\sigma_1(\theta_0)\}/\{\mu_2'(\theta_0)/\sigma_2(\theta_0)\}$$

where $\mu_i'(\theta_0) = \lim_{n\to\infty}\{\frac{\partial}{\partial\theta}E(T_{in})\}_{\theta\to\theta_0}$ and $\sigma_i^2(\theta_0) = \lim_{n\to\infty} Var_0(T_{in}),\ i = 1, 2$ is the null variance.

We consider the following three alternative distributions:

(i)	The linear failure rate family	: $\overline{F}_1(x) = e^{-x-\frac{\theta x^2}{2}}$,	$x \geq 0, \theta \geq 0$
(ii)	The Makeham family	: $\overline{F}_2(x) = e^{-x-\theta(x+e^{-x}-1)}$,	$x \geq 0, \theta > 0$
(iii)	The Weibull family	: $\overline{F}_3(x) = e^{-x^\theta}$,	$x \geq 0, \theta > 0.$

Direct calculations give the efficiencies as follows:

Table 3.1 Efficiencies of the test

Distribution	$\hat{\Delta}_{KF_n}$	$\hat{\Delta}_3^{(1)}$	$\hat{\Delta}_{F_n}$
F_1 Linear failure rate	0.974	0.9486	0.9584
F_2 Makham	0.239	0.1976	0.228
F_3 Weibull	1.889	0.791	-

By direct calculations , the asymptotic relative efficiencies (PARE) of the test $\hat{\Delta}_{KF_n}$ with respect to $\hat{\Delta}_3^{(1)}$ are 1.029, 1.21 and 2.39 for F_1, F_2 and F_3 respectively and with respect to $\hat{\Delta}_{F_n}$ are 1.0177 and 1.209 for F_1 and F_2. Then our U-statistic $\hat{\Delta}_{KF_n}$ performs better than the statistic $\hat{\Delta}_3^{(1)}$ and $\hat{\Delta}_{F_n}$ for all the alternative distributions considered.

4. THE POWER ESTIMATES

We calculate the estimate of the statistic $\hat{\Delta}_{KF_n}$ defined in (2.6) at 95% upper percentile level and for the following alternative distributions:

(i)	The linear failure rate family	: $\overline{F}_1(\theta) = e^{-x-\frac{\theta x^2}{2}}$,	$x \geq 0, \theta \geq 0$
(ii)	The Makeham family	: $\overline{F}_2(\theta) = e^{-x-\theta(x+e^{-x}-1)}$,	$x \geq 0, \theta > 0$
(iii)	The Weibull family	: $\overline{F}_3(\theta) = e^{-x^{\theta}}$,	$x \geq 0, \theta > 0.$

All these distributions are IFR (for an appropriate restriction on θ), hence they all belong to a wider class. Moreover, all these distributions reduce to exponential distribution for (i) and (ii) when the value $\theta = 0$ and for (iii) when the value of $\theta = 1$. Table 4.1 contains the power estimate for the $\hat{\Delta}_{KF_n}$ test statistic with respect to these distributions. The estimates are based on 5000 simulated samples of size $n =$ 10, 20 and 30 at level 95% upper percentile.

Table 4.1 Power estimate for $\hat{\Delta}_{KF_n}$-statistic

Distribution	Parameter	Sample size		
	θ	n=10	n=20	n=30
$F_1(\theta)$	1	0.937	0.975	0.982
(Linear failure	2	0.964	0.990	0.995
Rate)	3	0.980	0.995	0.996
$F_2(\theta)$	1	0.898	0.904	0.921
Makeham	2	0.930	0.942	0.971
	3	0.965	0.980	0.977
$F_3(\theta)$	1	0.762	0.697	0.652
Weibull	2	1.000	1.0000	1.0000
	3	1.0000	1.0000	1.0000

The power estimate in Table 4.1 shows clearly the departure from exponentiality towards (HNBUET) properties as θ increases.

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