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Modeling, Analysis of Flexible Manufacturing System by Petri Nets

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유연제조시스템을 Petri Nets으로 구현하고, 결과를 다른 시뮬레이션과 비교, 검토

이 종 환

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페트리 네트(Petri Nets)는 이산 사건 시스템을 모델링할 수 있는 그래픽하고, 수학적인 도구이다. 본 연구는 유연제조 시스템을 확률적인 페트리 네트(Stochastic Petri Nets)중의 하나인 임베디드 마코프 체인(Embeded Markov Chain)에 도입하고, 임베디드 마코프 체인의 방법 중에 하나인 일반화된 확률적 페트리 네트(Generalized Stochastic Petri Nets)에 적용시켰다. 그리고 결과치의 정확성을 알아내기 위하여, 페트리 네트 시뮬레이션과 아레나를 사용하여 실행하였다.

Keywords : eneralized Stochastic Petri Nets, Embedded Markov Chain, Manufacturing System

1. Introduction

Petri Nets (PNs) were named after Carl A Petri, who created a net-like mathematical tool for the study of communication with automata in 1962. They were developed to meet the need in specifying process synchronization, asynchronous events, concurrent operations, and conflicts or resource sharing for a variety of industrial automated systems at the discrete event-level.

In any physical net, two basis elements were founded : nodes and links. Both nodes and links play their own roles. For example, forces could be transferred from one end to another through nodes and links. A PNs divides nodes into two kinds : aces and transitions. Places are used to represent condition or status of a component in a system. They are pictured by circles. Transition represents the events or operations. They are pictured by empty rectangles or solid bars. A PNs utilizes directed arcs to connect from places to transitions or from transitions to places. Places, transitions, and directed arcs make a PNs a directed graph, called the PNs structure. The dynamic is introduced by allowing a place to hold either none or positive number of tokens pictured by small solid dots. These dots could represent the number of resources or indicate whether a condition is true or not in a place. When all the input places hold enough number of tokens, an event modeled by a transition can happen, called transition firing. This firing changes the token distribution in the places, signifying the change of system states.

The development of man-made systems requires that both functional and performance requirements be met. The ordinary PNs do not include any concept of time. With this class of nets, it is possible only to describe the logical structure and behavior of the modeled system, but not its evolution over time. Responding to the need for the temporal performance analysis of discrete-event systems, time has been introduced into PNs in a variety of ways. There are two fundamental types of timed PNs in the context of performance evaluation and scheduling. They are deterministic timed PNs, and stochastic ones.

When time delays for operations in a concurrent choice-free system are fixed, we can model the system as a deterministic timed PNs. When time delays are modeled as random variables, probabilistic distributions are added to the timed PNs models for conflict resolution. Stochastic timed PNs models are yielded. In such models, it has become a convention to associate time delays with the transitions only. When the random variables are of general distribution or both deterministic and random variables are involved, the resulting net models cannot be solved analytically for general cases. Thus simulation or approximation methods are required. The Stochastic Petri Nets(SPN) in which time delay for transition is assumed to be random and exponentially distributed are called SPN. The SPN models which allow for immediate transitions, i.e., with zero time delay, are called Generalized Stochastic Petri Nets(GSPN).

2. Analysis of Generalized Stochastic Petri Nets(GSPN)

The existence of immediate transitions makes the analysis of Generalized Stochastic Petri Nets(GSPN)s more complicated than that of SPN. Immediate transitions produce multiply simultaneous events in the process that describe the time behavior of a GSPN (due to a sequence of immediate transition firings) and possibly an infinite number of events in a finite-time interval (if such a sequence starts and ends in the same marking). Such a process can be characterized as a continuous-time stochastic point process (SPP) with one-to-one correspondence between the markings of the GSPN and the SPP states.

In order to ensure the existence of a unique steady-state probability distribution for the marking process of a GSPN, the following simplifying assumptions are made :

(1) The GSPN is bounded. That is, the reachability set is finite.
(2) Firing rates do not depend on time parameters. This ensures that the equivalent Markov chain is homogeneous.
(3) The GSPN model is proper and deadlock free. That is, the initial marking is reachable with nonzero probability from any marking in the reachability set and also there is no absorbing marking.

These assumptions further specify the nature of the SPP that can thus be classified as a finite state space, and a stationary (homogeneous), irreducible, and continuous-time stochastic point process.

There are three methods for the analysis of GSPN models in the reference book. One of the methods chose for applying flexible manufacturing system. This method is called Embedded Markov Chain(EMC). This method is based on identifying an embedded Markov chain with the SPP for the evaluation of the steady-state probability distribution of the GSPN markings. To get a utilization of machines, we follow the formula from (2.1) to (2.11) and there are already given in the reference book.

To better understanding of EMC method, we will reiterate this formula again. The EMC of the marking process comprises tangible marking as well as vanishing markings. The transition probability matrix of this EMC can be computed using the firing rates and the random switches.

Let S = state space of the SPP, $|S|=k_s$, S_t = set of tangible states in SPP, $|S_t|=k_t$, S_v = set of vanishing states in SPP, $|S_v|=k_v$, With $S=S_t \cup S_v$, $S_t \cap S_v=\varnothing$ and $k_s=k_t+k_v$.

Disregarding for the time being the concept of time, and focusing attention on the set of states in which the process is led because of a transition out of a given states, it is observed that a stationary EMC can be recognized within the SPP. The transition probability of the EMC can be written as follows :

$$U = A + B = \begin{bmatrix} C & D \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ E & F \end{bmatrix} \quad \cdots\cdots (2.1)$$

The elements of matrix A will go to a vanishing state (C) or to a tangible state (D) given that it is at a vanishing state. And the elements of B will go to a vanishing state (E) or to a tangible state (F) given that the process is in a tangible state. The transition probability matrix $U' = [u'_{ij}]$ between tangible states only be computed as follows :

$$u'_{ij} = f_{ij} + \sum_{r \in v} e_{ir} \Pr[r \to j] \quad i, j \in T, r \in V \quad \cdots\cdots (2.2)$$

where f_{ij} is the transition probability from tangible state i to tangible state j, e_{ir} is the transition probability from i to vanishing state r, and Pr[r→ represents the probability that the SPP moves from the state r to the state j in an arbitrary number of steps following a path through vanishing state only.

Now we consider the computation of matrix Q. The k_{th} power of matrix A can be written as

$$A^k = \begin{bmatrix} C^k & C^{k-1}D \\ 0 & 0 \end{bmatrix} \quad \cdots\cdots (2.3)$$

Each component of the upper of portion of the matrix A^k represents the probability of moving from any state $r \in S_v$ to any other state of the original SPP in exactly k steps, such that intermediate states can be of the vanishing type only. The matrix G^k, defined as

$$G^k = \sum_{h=0}^{k-1} C^k D \quad \cdots\cdots (2.4)$$

provides the probability of reaching any tangible state i ($\in S_t$), moving from any vanishing state r($\in S_v$), in no more than k steps, visiting intermediate states of the vanishing type only.

The limit of the sum $\lim_{k\to\infty}\sum_{h=0}^{k} C^h$ exists, and it is finite. The irreducibility property of the SPP ensures that the spectral radium of the matrix C is less than one. This implies that the limit of the sum

Whenever loops among vanishing states do not exist, a suitable ordering of these states can be found that allows writing C as an upper triangular matrix, so that there exists a value $k_o \leq k_v$ such that

$$Ck = 0 \text{ for any } k \geq \quad \cdots\cdots (2.5)$$

and the previous infinite sum reduces to a sum of finite number of terms. If instead such loops among vanishing states exists, the infinite sum has the asymptotic value

$$\sum_{h=0}^{\infty} C^h = [I - C]^{-1} \quad \cdots\cdots (2.6)$$

These two possible forms of the same infinite sum can be used to provide an explicit expression for the matrix G :

$$G^{\infty} = [I - C]^{-1} D \quad \cdots\cdots (2.7)$$

represents loops among vanishing states, whose elements represent the probabilities that the SPP reach for the first time a given tangible state, moving out of a given vanishing state in no matter how many steps. We can thus conclude that an explicit expression for the desired total transition probability among any two tangible states is

$$U' = F + EG^{\infty} \quad \cdots\cdots (2.8)$$

Now we consider the steady state probability distribution. Let Y = (y_1, y_2,...,y_S) be a vector of real numbers with $s = k_t$. Then the solution of the system of linear equations

$$Y = Y \cdot \quad \cdots\cdots (2.9a)$$

$$\sum_{i=1}^{s} y_i = 1 \quad \cdots\cdots (2.9b)$$

gives the stationary probabilities of the reduced EMC. y_i gives the relative number of visits to M_i by the marking process. To obtain the steady state probabilities of the marking process, we then use the expression

$$\pi_i = \frac{y_i m_i}{\sum_{j=1}^{s} y_j m_j} \quad \cdots\cdots (2.10)$$

for i = 1,2,...,s, where i is the steady-state probability of marking Mi in the marking process (proportion of time the marking process spends in M_i) and mi is the mean sojourn time of the marking M_i, which is given by

$$m_i = \frac{1}{\sum_{t_k \in E(M_i)} \lambda_k} \quad \cdots\cdots (2.11)$$

machine and inspector queues : ype 1, Type 2, and Repairable.

The inspection-time for each workpiece, regardless of type or rework status, has an exponential distribution with a minimum time of 2 minutes. Of the jobs inspected at this station, 80 percent are classified as Good and depart the system, 10 percent are classified as Reject and depart the system, and 10 percent are classified as Repair and are returned to the machine queue for rework. The same percentages apply to both types the inspection station will be recycled through the machining station.

3. Describing the Process

The following summary describes the process steps and entity flow for this problem.

1. Create arriving jobs.
2. Assign Job Type = 1 or 2, Status = 1, and Priority = Job Type.
3. Wait according to Priority for the machine to be idle.
4. Seize the machine.
5. Delay by the processing time.
6. Release the machine for the next waiting entity, if any.
7. Wait according to Priority for the inspection station.
8. Seize the inspector.
9. Delay by the inspection time.
10. Release the inspector for the next waiting entity, if any.
11. Branch with the following probabilities

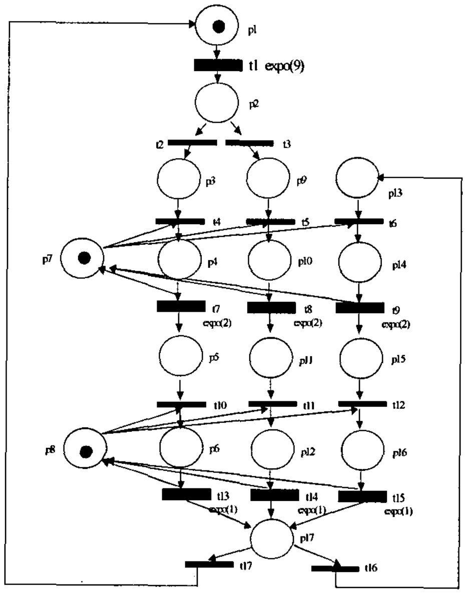
11-1. 0.8 probability go to step 12(Go)
11-2. 0.1 probability go to step 13(Reject)
11-3. 0.1 probability go to step 14(Repair)

11-1, 11-2, and 11-3 can be described as follows :) percents of parts are classified as Good and depart the system, 10 percents are classified as Reject and leave the system, and 10 percents are classified as Repair and are returned to the machine queue for the rework.

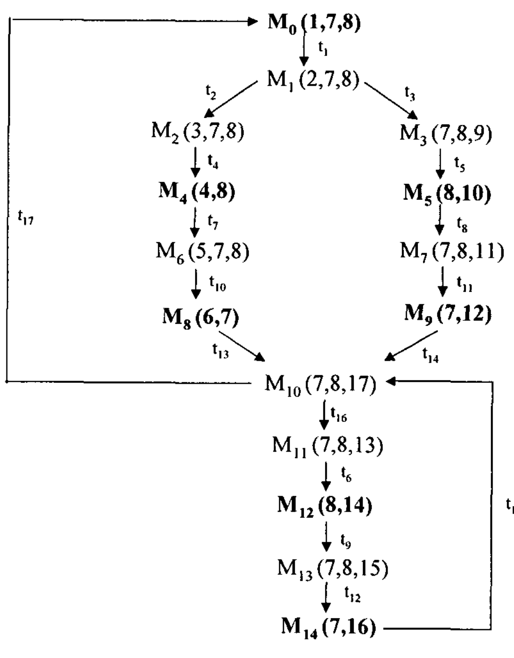
4. Modeling and Analysis of Manufacturing System

Figure 1. shows the GSPN model of our manufacturing system. The GSPN model has 17 places, 7 timed transitions $\{t_1, t_7, t_8, t_9, t_{13}, t_{14}, t_{15}\}$ and 10 immediate transitions. The initial marking is $M_0 = (1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)^T$, which will, for simplicity, be denoted as $p_1p_7p_8$ by specifying those places having a token in this paper. There is a random switch comprising the transitions t_2, t_3, and t_{16}, t_{17} with the corresponding probabilities 0.3, 0.7, 0.1,and 0.9, respectively.

Graphically, timed transitions are drawn as thick bars, and immediate transitions as thin bars. In the initial marking $p_1p_7p_8$, the exponentially timed transitions t_1 is enabled and hence this is a tangible marking. When t1 fires, the new marking is p_2, p_7, p_8, which is a vanishing marking as the immediate transition t_2 and t_3 are enabled in it. At this stage, the random switch is invoked to choose the next transition to fire. The evolution of the marking process proceeds as described above and the reachability graph of the GSPN model can be constructed in this way. In the Figure 2. shows the reachability graph of GSPN model.



<Figure 1> GSPN model of an FMS



<Figure 2> Reachability graph of the GSPN model of Figure 1.

There are 7 tangible markings(bold nodes in figure) and 8 vanishing markings. The transition probability matrix of the reduced EMC can be computed (6.9). The individual transition probabilities are labeled on the directed arcs connecting the tangible markings.

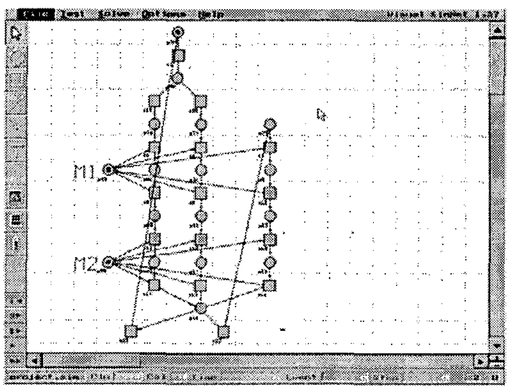
Solving the linear system of (2.9b) gives
y_0 = 0.310, y_4 = 0.093, y_5 = 0.217,
y_8 = 0.093, y_9 = 0.217, y_{12} = 0.035,
y_{14} = 0.035.
By (2.11), the mean sojourn times are
M_0 = 9, M_4 = 1, M_5 = 1,
M_8 = 2, M_9 = 2, M_{12} = 1, M_{14} = 2.

Using (2.10), we obtain :
p_0 = 0.7294, p_4 = 0.024, p_5 = 0.057,
p_8 = 0.049, p_9= 0.113, p_{12} = 0.009,
p_{14} = 0.018.

From the steady state probabilities, we can calculate some performance indices of the system. For example,

- achine Utilization
  $M1 = Pr\{M(p7) = 1\} = p_4 + p_5 + p_{12}$
  $= 0.024 + 0.057 + 0.009 = 0.09$
  $M2 = Pr\{M(p8) = 1\} = p_8 + p_9 + p_{14}$
  $= 0.049 + 0.113 + 0.018 = 0.18$

- ıroughput rates of timed transitions
  $TR(t_7)=p_4*1=0.024$, $TR(t_8)=p_5*1=0.057$,
  $TR(t_9)=p_{12}*1=0.009$, $TR(t_{13})=p_8*0.5=0.0245$,
  $TR(t_{14})=p_9*0.5=0.056$, $TR(t_{15})=p_{14}*0.5= 0.009$,
  $TR(t_1)=p_0*(1/9)= 0.081$.



<Figure 3> Petri Nets Simulation (Simnet)

With EMC method, several results are obtained. In Figure 3. the manufacturing system is modeled with PNs software Simnet. By running this software, machine utilization is acquired.

5. Conclusion

This paper modeled manufacturing system with three methods. First method is EMC, which is one of GSPN method. By following the EMC method step by step, machine utilization was obtained. Second, modeling the system with PNs software and get a solution. Lastly, popular simulation software ARENA was used to verify the previous two method.

<Table 1> Utilization Result of comparison of EMC, PNs and ARENA

	EMC	PNs	ARENA
Machine1	0.09	0.088	0.093
Machine2	0.18	0.176	0.182

Fortunately, results of three methods are almost the same. Results of utilization of machine 1 and machine 2 by three methods are obtained and compared in Table 1. For a PNs simulation, Simnet V. 1.37 was used and Arena software was used to get a simulation result.

Petri nets recently emerged as a powerful tool and methodology for the modeling, analysis, simulation, and performance evaluation of manufacturing systems. This paper presents the fundamental concepts of Petri nets. By applying Petri Nets analysis, we can show that when we apply example in the SIMAN to Petri nets, it is reachable and get the same answer as we calculate by following equations in the reference book.

The manufacturing system example in the SIMAN is also can be done by Petri nets. Through the three methods, EMC, PN simulation, and Arena, the same results were obtained. With these results, we can conclude that modeling of manufacturing system to Petri nets was properly used. In the manufacturing system, normal and triangular distributions are commonly used. But, SPN supports only exponential distribution; therefore, transformation from the normal distribution to the exponential distribution was needed.

Due to the random number generation, there is an insignificant difference between the results of SIMAN and Petri nets, and also there is some difference between Petri nets modeling and SIMAN modeling when they modeling the manufacturing system. Nevertheless, to compute the performance of a SPN, it is necessary to generate all states to obtain its underlying Markov process. It may not be feasible to apply the method to large systems. As a result, simulation is often used in real-world cases.

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