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Extension of L-Fuzzy Topological Tower Spaces

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Abstract

The purpose of this paper is to introduce the notions of L-fuzzy topological towers by using a completely distributive lattic L and show that the category **L-FPrTR** of L-fuzzy pretopoplogical tower spaces and the category **L-FPsTR** of L-fuzzy pseudotopological tower spaces are extensional topological constructs. And we show that **L-FPsTR** is the cartesian closed topological extension of **L-FPrTR**. Hence we show that **L-FPsTR** is a topological universe.

Key words : completely distributive lattice, L-fuzzy pseudotopological tower, L-fuzzy pretopological tower, cartesian closed, topological extension, topological universe.

1. Introduction

In [3], P. Eklund and W. Gahler defined the notions of generalized limit and Cauchy structures from the standpoint of general structure theory. Each of these structures is related to a certain covariant functor $\Phi=(\phi, \leq)$ from the category **Set** of sets to the category **SLat** of semilattices satisfying three conditions (P), (Pr) and (D) respectively concerning the existence, preservation and distributivity of Φ-products which generalize the usual notions of filter products. Under the assumption that ϕ is connected i.e. $\phi(1)$ (with $1=\{0\}$) is a singleton set which gives a natural transformation $\eta: id \to \phi$, they defined the notions of Φ-limit spaces and showed the category Φ-**Lim** of Φ-limit spaces is cartesian closed. And in [10], K. C. Min and Y. J. Lee introduced the notions of L-prefilter (In this paper, it is called L-fuzzy filter) and P_L-limit structure (L-fuzzy limit structure) generalizing the neighborhood system of points in L-fuzzy topological space in the scheme of Φ-limit structure in [10].

On the other hand probabilistic convergence spaces were defined in 1989 by Florescu [4], in terms of net convergence. The filter formulation considered was introduced by G. D. Richardson and D. C. Kent [13]. The fundamental idea is to assign a probability to the convergence of a given filter to a given point. In 1997, P. Brock and D. C. Kent [2], introduced the limit tower spaces whose objects resemble probabilistic convergence spaces.

In this paper, we introduce the notions of L-fuzzy limit tower, L-fuzzy pseudotopological tower and L-fuzzy pretopological tower. And show that the category L-**FLTR** of L-fuzzy limit tower spaces and L-fuzzy continuous maps is a cartesian closed topological construct and the category **L-FPsTR** of L-fuzzy pseudotopological tower spaces and the category L-**FPrTR** of L-fuzzy pretopological tower spaces are extensional topological construct and the category **L-FPsTR** is the cartesian closed topological extension of the category **L-FPrTR**. Hence **L-FPsTR** is the topological universe extension of **L-FPrTR**.

2. Preliminaries

Throughout this paper, we let L be a completely distributive complete lattice with different least and last elements denoted by 0_L, 1_L, respectively. Recall that the complete distributive law $\vee(\alpha_i \wedge \alpha)=(\vee \alpha_i) \wedge \alpha$ holds and with an order reversing involution $x \to x'$ ($x \in L$). If X is any set, the map $f: X \to L$ is called an *L-fuzzy subset* of X.

L^X denotes the set of all L-fuzzy subsets of X.

Let X be a set. A non-empty subset F of L^X is called an *L-fuzzy filter* on X if

(LF1) if $A \in F$ and $A \subseteq B$, then $B \in F$,

(LF2) for all $A, B \in F$, we have $A \cap B \in F$,

(LF3) $\underline{\alpha} \in F$, $\forall \alpha > 0_L$ where $\underline{0_L} \notin F$.

For a set X, let $F_L(X)$ be the set of all L-fuzzy filters on X.

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For any set X, Y and a map $f: X \to Y$ and the map $F_L(f(F)): L^Y \to \{0,1\}$ defined by $F_L(f(F))(g) = F(g \circ f)$ $\forall g \in L^Y$ is an L-fuzzy filter on Y and we write $F_L(f(F))$ simply by $f(F)$.

A subset **B** of an L-fuzzy filter F is called a *base* for F if $\forall A \in F, \exists B \in$ **B** such that $B \leqq A$. For a given set X, a subset **B** of L^X is called an *L-fuzzy filter base* on X if it satisfies the following :

(LFB1) $\underline{\alpha} \in$ **B**, $\forall \alpha > 0_L$ and $\underline{0_L} \notin$ **B**,

(LFB2) $\forall A, B \in$ **B**, $\exists C \in$ **B** s.t $C \leqq A \wedge B$.

Definition 2.1. An L-fuzzy filter U on X is an *ultraL-fuzzy filter* if there is no other L-fuzzy filter finer than U.

Definition 2.2.[10] An *L-fuzzy limit structure* t is a subset of $F_L(X) \times X$, subject to the following axioms : where $F \in t(x)$ means $(F, x) \in t$.

(LFL1) $<x> = \{A \in L^X : \mu_A(x) > 0_L\} \in t(x)$, for all $x \in X$,

(LFL2) if $F \in t(x)$ and $F \subseteq G$ then $G \in t(x)$,

(LFL3) if $F, G \in t(x)$, then $F \cap G \in t(x)$.

The pair (X, t) is called an *L-fuzzy limit space.* If $F \in t(x)$, then we say that F converges to x and x is a limit point of F.

Definition 2.3. A map $f: (X, t_X) \to (Y, t_Y)$ between L-fuzzy limit spaces is said to be *L-fuzzy continuous* if $F \in t_X(x)$, then $f(F) \in t_Y(f(x))$.

Let **L-FLim** denote the category of all L-fuzzy limit spaces and all L-fuzzy continuous maps.

Theorem 2.4.[10] *The category* **L-FLim** *is a topological construct.*

Theorem 2.5.[10] *The category* **L-FLim** is *a cartesian closed category.*

Theorem 2.6.[10] *The category* **L-FLim** *is a quasitopos.*

3. L-fuzzy limit tower spaces

Definition 3.1. An *L-fuzzy limit tower* $\bar{t}$ is a family $\{t_\alpha : 0 \leqq \alpha \leqq \infty\}$ of L-fuzzy limit structures such that :

(LFTR1) t_α is an L-fuzzy limit structure, for each $\alpha \in [0, \infty]$,

(LFTR2) t_∞ is the indiscrete topology,

(LFTR3) $t_\alpha = \cap_{\beta > \alpha} t_\beta$, for each $\alpha \in [0, \infty)$.

The pair $(X, \bar{t})$ is called an *L-fuzzy limit tower space.*

Definition 3.2. For any L-fuzzy limit tower spaces $(X, \overline{t_X})$ and $(Y, \overline{t_Y})$, a map $f: (X, \overline{t_X}) \to (Y, \overline{t_Y})$ is *L-fuzzy continuous* if $F \in t_\alpha^X(x)$ then $f(F) \in t_\alpha^Y(f(x))$ for each $0 \leqq \alpha \leqq \infty$ where $\overline{t_X} = (t_\alpha^X)$, $\overline{t_Y} = (t_\alpha^Y)$.

Let **L-FLTR** denote the category of all L-fuzzy limit tower spaces and all L-fuzzy continuous maps.

Theorem 3.3. *The category* **L-FLTR** *is topological category.*

Proof. To show that **L-FLTR** is initially complete, let $(f_i: X \to (Y_i, \bar{t}_i))_{i \in I}$, be a source such that $(Y_i, \bar{t}_i)$ is an L-fuzzy limit tower space for each $i \in I$. We define $\bar{t} = (t_\alpha)$ by $F \in t_\alpha(x)$ iff $f_i(F) \in t_\alpha^i(f(x))$ for all $i \in I$, $x \in X$ and for each $\alpha \in [0, \infty)$. And $t_\infty(x) = F(X)$. Then $\bar{t}$ is the initial L-fuzzy limit tower on X with respect to $(f_i)_{i \in I}$ and the remainder is trivial.

Function spaces

Let $(X, \overline{t_X})$ and $(Y, \overline{t_Y})$ be L-fuzzy limit tower spaces and let $C(X, Y)$ be the set of all L-fuzzy continuous maps from X into Y. Let H be a prefilter on $C(X, Y)$ and F be an L-fuzzy filter on X. For $\mathrm{H} \in H$ and $\mathrm{F} \in F$, define an L-fuzzy set $\mathrm{H(F)}: Y \to L$ by

$$\mu_{H(F)}(y) = \begin{cases} sup_{(x,g) \in ev^{-1}(y)} \mu_{H(g)} \wedge \mu_F(x) & \text{if } ev^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$

where $ev: X \times C(X, Y) \to Y$ is the evaluation map. We note that if $\mathrm{H}_1 \subseteq \mathrm{H}_2$ in H and $\mathrm{F}_1 \subseteq \mathrm{F}_2$ in F, then $\mathrm{H}_1(\mathrm{F}_1) \subseteq \mathrm{H}_2(\mathrm{F}_2)$. Thus **H(F)**$= \{B \in L^Y : H(F) \subseteq B$, for some $H \in$ **H**, $F \in$ **F**$\}$ is an L-fuzzy filter on Y. Define a $\bar{t} = (t_\alpha)$ as follows : for each $f \in C(X, Y)$, $H \in t_\alpha(f)$ if and only if for any $x \in X$, if $F \in t_\alpha^X(x)$

then $H(F) \in t_\alpha^Y(f(x))$ for each $\alpha \in [0, \infty)$. And $t_\infty = F_L(C(X, Y))$.

Proposition 3.4. *For objects* $(X, \overline{t_X})$, $(Y, \overline{t_Y})$ *in* **L-FLTR**, *the L-fuzzy limit tower* $\overline{t}$ *on* $C(X, Y)$ *is an L-fuzzy limit tower.*

Proof. Let $f \in C(X, Y)$, $H \in F(C(X, Y))$.

(LFTR1): $\dot{t} = (t_\alpha)_\alpha$, $0 \leq \alpha \leq \infty$ is an L-fuzzy limit structure.

(LFL1) Let $F \in t_\alpha^X$. Then $f(F) \subseteq <f>(F)$ and since f is L-fuzzy continuous, $f(F) \in t_\alpha^Y(f(x))$, $<f>(F) \in t_\alpha^Y(f(x))$. By definition of t_α, $<f> \in t_\alpha(f)$.

(LFL2) Suppose $H \in t_\alpha(f)$ and $H \subseteq K$. Then for $F \in t_\alpha^X(x)$, $H(F) \in t_\alpha^Y(f(x))$ and $H(F) \subseteq K(F)$. Thus $K(F) \in t_\alpha^Y(f(x))$. Hence $K \in t_\alpha(f)$.

(LFL3) Suppose $H, K \in t_\alpha(f)$. Then for $F \in t_\alpha^X(x)$, $H(F), K(F) \in t_\alpha^Y(f(x))$. $H(F) \cap K(F) \in t_\alpha^Y(f(x))$. But $(H \cap K)(F) = H(F) \cap K(F)$. So $(H \cap K)(F) \in t_\alpha^Y(f(x))$. Thus $H \cap K \in t_\alpha(f)$.

(LFTR2): It is given by definition of $\dot{t}$ on $C(X, Y)$.

(LFTR3): To show that $t_\alpha = \cap_{\beta > \alpha} t_\beta$, let $H \in t_\alpha(f)$ then for $F \in t_\alpha^X(x)$, $H(F) \in t_\alpha^Y(f(x))$. Since $\overline{t_Y}$ is an L-fuzzy limit tower, for all $\beta > \alpha$, $H(F) \in \cap t_\beta^Y(f(x))$. So $H \in \cap t_\beta(f)$ for all from $\beta > \alpha$. Conversely let $H \in \cap t_\beta(f)$ for all $\beta > \alpha$ then for $F \in \cap t_\beta^X(x)$, $H(F) \in \cap t_\beta^Y(f(x))$ and hence $H(F) \in t_\alpha^Y(f(x))$. Thus $H \in t_\alpha(f)$.

The space $C(X, Y)$ equipped with L-fuzzy limit tower is denoted as $[X, Y]$.

Theorem 3.5. *The evaluation map* $ev : X \times [X, Y] \rightarrow Y$ *is L-fuzzy continuous.*

Proof. Let $F \rightarrow (x, f)$ in $(X, \overline{t_X}) \times (C(X, Y), \dot{t})$ then for each α, $Pr_2(F) \in t_\alpha(f)$ in $C(X, Y)$, $Pr_1(F) \in t_\alpha^X(x)$ in X, $Pr_2(F)(Pr_1(F)) \in t_\alpha^Y(f(x))$ in Y. Thus it is enough to show that $Pr_2(F)(Pr_1(F)) \subseteq ev(F)$. Take any $T \in Pr_2(F)(Pr_1(F))$. Then there exist $K \in Pr_2(F)$ and $L \in Pr_1(F)$ with $K(L) \subseteq T$. Take any $A, B \in F$ such that $Pr_2(A) \subseteq K$ and $Pr_1(B) \subseteq L$. Let $C = A \cap B$. Then $C \in F$ and $ev(C) \subseteq Pr_2(C)(Pr_1(C))$, by routine work. Hence $ev(C) \subseteq Pr_2(B)(Pr_1(A)) \subseteq K(L) \subseteq T$.

Theorem 3.6. *Let* $(X, \overline{t_X})$, $(Y, \overline{t_Y})$, $(Z, \overline{t_Z}) \in$ **|L-FLTR|** *and* $f : X \times Z \rightarrow Y$ *be L-fuzzy continuous. Then there exists a unique fuzzy continuous map* $f^* : Z \rightarrow C(X, Y)$ *such that* $ev \circ (id \times f^*) = f$.

Proof. Define a map $f^* : Z \rightarrow C(X, Y)$ by $f^*(z)(x) = f(x, z)$ $(z \in Z,\ x \in X)$. Then we can show that this map is unique L-fuzzy continuous, where $[z] : X \rightarrow Z$ is a constant map with value z.
To show that f^* is L-fuzzy continuous, let $G \in t_\alpha^Z(z)$ and $F \in t_\alpha^X(x)$. Then for each F$\in F$, G$\in G$, define an L-fuzzy set [F, G]:$X \times Z \rightarrow L$ by $\mu_{[F,G]}(x, z) = \mu_F(x) \wedge \mu_G(z)$ and B={[F, G] : F$\in F$, G $\in G$}. Then this forms a basis for an L-fuzzy filter V on $X \times Z$. Since f is L-fuzzy continuous, $f(V) \in t_\alpha^Y(f(x))$ in Y. Thus it is enough to show that $f^*(G)(F) = f([F, G])$: Suppose $ev^{-1}(y) \neq \varnothing$ and $S = \{g \in C(X, Y) : (x, y) \in ev^{-1}(y)$ for some $x \in X,\ f^{*-1}(y) \neq \varnothing\} \neq \varnothing$. Then $f^{-1}(y) \neq \varnothing$ and

$$\begin{aligned} &\mu_{f^*(G)(F)}(y) \\ &= sup_{(x,y) \in ev^{-1}(y)} \mu_{f^*(G)}(g) \wedge \mu_F(x) \\ &= sup_{(x,z) \in ev^{-1}(y)} (sup_{z \in f^{*-1}(y)} \mu_G(z)) \wedge \mu_F(x) \\ &= sup_{(x,z) \in ev^{-1}(y)} sup_{z \in f^{*-1}(y)} (\mu_G(z) \wedge \mu_F(x)) \\ &= sup_{(x,z) \in f^{-1}(y)} \mu_G(z) \wedge \mu_F(x) \\ &= \mu_{f([F,G])}(y). \end{aligned}$$

If $ev^{-1}(y) = \varnothing$ or $S = \varnothing$, then $f^{-1}(y) = \varnothing$ and $\mu_{f(G)}(F)(y) = \mu_{f([F,G])}(y) = 0$. So $f^*(G) \in t_\alpha(f^*(z))$. Obviously $ev \circ (id \times f^*) = f$ and a map f^* is a unique.

Categorical viewpoints

A category **A** is said to be cartesian closed [5] if (1) for any pair A, B of objects there exists a product A×B, and (2) for any object A, the functor A×_: $A \rightarrow A$ has a right adjoint. Therefore by above Proposition 3.4., Theorem 3.5. and Theorem 3.6., we obtain :

Theorem 3.7. *The category* **L-FLTR** *is a cartesian closed topological category.*

Throughout this paper, we refer to [5] for category theory.

4. L-fuzzy pretopological tower spaces

Definition 4.1. An L-fuzzy limit tower is called a *L-fuzzy pretopological tower* on X if for an L-fuzzy limit tower $\bar{t}=(t_\alpha)$, $0 \leqq \alpha \leqq \infty$ and for all $F \in t_\alpha(x)$, $\cap F \in t_\alpha(x)$ for any $x \in X$.

The pair $(X, \bar{t})$ is called an *L-fuzzy pretopological tower space.*

Let **L-FPrTR** be the category of all L-fuzzy pretopological tower spaces and all L-fuzzy continuous maps.

Proposition 4.2. *The category* **L-FPrTR** *is a topological construct.*

Proof. The initial structures in **L-FPrTR** can be described in the same way as those in **L-FLTR**.

Proposition 4.3. *The category* **L-FLTR,** and **L-FPrTR** are well-fibred topological constructs and **L-FPrTR** is initially closed in **L-FLTR**, hence bireflective.

Definition 4.4. [6]. Let **A** be a well-fibred topological construct:

(1) A partial morphism from X to Y is a morphism $f: Z \to Y$ whose domain Z is a subspace of X.
(2) Partial morphism to Y are representable provided Y can be embedded via the addition of a single point p into an object $Y^{\#}$ with the property that for every partial morphism $f: Z \to Y$, the map $f^X: X \to Y^{\#}$ defined by $f^X(x) = f(x)$ if $x \in Z$, $f^X(x) = p$ if $x \notin Z$ is a morphism.
The object $Y^{\#}$ is called *the one point extension* of Y.
(3) **A** is called *extensional* if partial morphisms into all **A**-objects are representable.

Theorem 4.5. **L-FPrTR** *is extensional.*

Proof. Suppose $(X, \overline{t_X})$ is an object in **L-FPrTR**. Let $X^{\#} = X \cup \{p\}, p \notin X$ and define $\overline{t_{\#}}$ on $X^{\#}$ as follows that for all $x \in X^{\#}$ and $F \in F_L(X^{\#})$, if $x = p$ or $F = < p >$ then $F \in t_0^{\#}(x)$ and if $x \neq p, F \neq < p >$, $0 \leqq \alpha < \infty$ then $F \in t_\alpha^{\#}(x)$ if and only if $F|_X \in t_\alpha^X(x)$. And $t_\infty^{\#}(x) = F_L(X)$. Then (1) $(X^{\#}, \overline{t^{\#}})$ is an object in **L-FPrTR**. Clearly $(\overline{t_\alpha^{\#}})_\alpha$ is an L-fuzzy limit tower. Enough to show that it is L-fuzzy pretopological. For all $F \in t_\alpha^{\#}(x)$, if $x = p$ or $F = < p >$ then $\cap F \in t_0^{\#}(x) \subseteq t_\alpha^{\#}(x)$ and if $x \neq p$, $F \neq < p >$ then $F|_X \in t_\alpha^X(x)$ and since t_α^X is an L-fuzzy pretopological tower and $\{\cap F\}|_X = \cap\{F|_X\}$, $\cap\{F|_X\} \in t_\alpha^X(x)$. Thus $\cap F \in t_\alpha^{\#}(x)$. (2) The inclusion $i: X \to X^{\#}$ is initial because any L-fuzzy filter F on X and $x \in X$, $i(F)|_X = F$, and so $F \in t_\alpha^X(x)$ if and only if $i(F) \in t_\alpha^{\#}(i(x))$. (3) For every partial morphism from $f: A \to X$, $A \subseteq Y$, the function $f^Y: Y \to X^{\#}$ defined by $f^Y(y) = f(y)$ if $y \in A$ and $f^Y(y) = p$ if $y \notin A$ then it is obviously L-fuzzy continuous. Therefore **L-FPrTR** is extensional.

5. L-fuzzy pseudotopological tower spaces

By weakening the axioms of convergence in an L-fuzzy pretopological tower spaces, we will obtain a cartesian closed topological extension of **L-FPrTR.**

Definition 5.1. An L-fuzzy limit tower is called an *L-fuzzy pseudotopological tower* on X if for L-fuzzy limit tower $\bar{t}=(t_\alpha)$, $F \in t_\alpha(x)$ if and only if every ultra L-fuzzy filter U finer than F, $U \in t_\alpha(x)$ for any $x \in X$, for each $\alpha \in [0, \infty]$.

The pair $(X, \dot{t})$ is called *an L-fuzzy pseudotopological tower space.*

Let **L-FPsTR** be the category of all L-fuzzy pseudotopological tower spaces and all L-fuzzy continuous maps.

Remark 5.2. Since every L-fuzzy filter is the intersection of all finer ultra L-fuzzy filters, **L-FPrTR** is a subcategory of **L-FPsTR.**

Proposition 5.3. *The category* **L-FPsTR** *is a topological construct.*

Proof. The initial structures in **L-FPrTR** can be described in the same way as those in **L-FPsTR.**

Proposition 5.4. *The category* **L-FPrTR** *is initially closed in* **L-FPsTR,** *hence* **L-FPrTR** *is a bireflective subcategory of* **L-FPsTR.**

Proposition 5.5. *For objects* $(X, \overline{t_X})$, $(Y, \overline{t_Y})$ *in* **L-FPsTR,** *the L-fuzzy limit tower* $\bar{t}$ *on* $C(X, Y)$ *is an L-fuzzy pseudotopological tower.*

Proof. We enough to show that this L-fuzzy limit tower t on $C(X, Y)$ is pseudotopological.: if K is a ultra L-fuzzy filter containing H such that $K \in t_\alpha^H(f)$ then for each $F \in t_\alpha^X(x)$, $K(F) \in t_\alpha^Y(f(x))$ and since

$(Y, \overline{t_Y}) \in$ |**L-FPsTR**| and $H(F) \subseteq K(F)$, $H(F) \in t_\alpha^Y(f(x))$. Thus $H \in t_\alpha(f)$.

As a corollary of Theorem 3.7., we have

Theorem 5.6. *The evaluation map* $ev : X \times [X, Y] \rightarrow Y$ *is L-fuzzy continuous.*

Theorem 5.7. *Let* $(X, \overline{t_X})$, $(Y, \overline{t_Y})$, $(Z, \overline{t_Z}) \in$ |**L-FPsTR**| *and* $f : X \times Z \rightarrow Y$ *be L-fuzzy continuous. Then there exists a unique fuzzy continuous map* $f^* : Z \rightarrow C(X, Y)$ *such that* $ev \circ (id \times f^*) = f$.

Theorem 5.8. **L-FPsTR** *is a cartesian closed topological category.*

By the above theorem, **L-FPsTR** is a cartesian closed topological extension of **L-FPrTR**.

Proposition 5.9. **L-FPsTR** *is extensional.*

Proof. Suppose (X, $\overline{t_X}$) is an object in **L-FPsTR**. Let $X^\# = X \cup \{p\}, p \not\in X$ and define $\overline{t^\#} = (t_\alpha^\#)$ on $X^\#$ as follows : if $x = p$ or $F = < p >$ then for $F \in F_L(X^\#)$, $F \in t_0^\#(x)$ and if $x \neq p, F \neq < p >$ then for $F \in F_L(X^\#)$, $F \in t_\alpha^\#(x)$ if and only if $F|_X \in t_\alpha^X(x)$ for each $\alpha \in [0, \infty)$. And $t_\infty^\# = F_L(X^\#)$. Using the Theorem 4.5, it is sufficient to show that $\overline{t^\#} = (t_\alpha^\#)$ is L-fuzzy pseudotopological. Let $F \in F_L(X^\#)$ and for every ultra L-fuzzy filter $U \supset F$, $U \in t_\alpha^\#(x)$. Then if $x = p$ or $F = < p >$ then by definition of $\overline{t^\#}$, $F \in t_0(x) \subset t_\alpha^\#(x)$, for all $\alpha \in [0, \infty]$. Otherwise i.e., if $x \neq p, F \neq < p >$ then $F|_X$ is a proper L-fuzzy filter on X. Since $j(F|_X) \supset F$, every ultra L-fuzzy filter V on X with $V \supset F|_X$ can be represented as $U|_X$ of some ultra L-fuzzy filter $U \supset F$ with $U \neq < p >$: from $U \in t_\alpha^\#(x)$, $V = U|_X \in t_\alpha^X(x)$. Since $\overline{t_X} = (t_\alpha^X)$ is an L-fuzzy pseudo topological tower, $F|_X \in t_\alpha^X(x)$. Thus $F \in t_\alpha^\#(x)$. Therefore **L-FPsTR** is extensional.

Definition 5.10.[11] A well fibred topological construct is called a topological quasitopos or a topological universe if it is both cartesiian closed and extensional.

By Theorem 5.8. and Theorem 5.9., we have

Theorem 5.11. **L-FPsTR** is a topological universe.

Question : We proved **L-FPsTR** is cartesian closed extensional topological construct of **L-FPrTR.** Hence **L-FPsTR** is topological universe of **L-FPrTR**. So now we have a question that 'is **L-FPsTR** the "smallest" topological universe extension of **L-FPrTR** ? ' In topological case [7] it was already proved i.e., the category **PsTop** of pseudotopological spaces and continuous maps is topological universe of the category **PrTop** of pretopological spaces and continuous maps. Moreover it was proved that **PsTop** is the topological universe hull of the construct **PrTop**. Thus I guess this answer is "yes". But there are some problems to prove it. First of all, we should obtain L-fuzzy topological tower $\dot{t}$ on $[0, \infty]$ which plays a similar role in L-fuzzy topological tower space as the Sierpinski space **2** in topological space. We will investigate the topological universe hull of **L-FPrTR**.

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