퍼지 및 지능시스템학회 논문지 2005, Vol. 15, No. 3, pp. 343-348

# 퍼지이론을 이용한 FEM 모델링을 위한 자동 요소분할 시스템

# Automatic Mesh Generation System for a Novel FEM Modeling Based on Fuzzy Theory

이양창* · 이준성** · 최윤종* · 김남용*
Yang-Chang Lee, Joon-Seong Lee, Yoon-Jong Choi and Nam-Yong Kim

* 경기대학교 대학원 기계공학과
** 경기대학교 기계시스템디자인공학부

**Abstract**

This paper describes an automatic finite element (FE) mesh generation for three-dimensional structures consisting of free-form surfaces. This mesh generation process consists of three subprocesses: (a) definition of geometric model, i.e. analysis model, (b) generation of nodes, and (c) generation of elements. One of commercial solid modelers is employed for three-dimensional solid structures. Node is generated if its distance from existing node points is similar to the node spacing function at the point. The node spacing function is well controlled by the fuzzy knowledge processing. The Delaunay method is introduced as a basic tool for element generation. Automatic generation of FE meshes for three-dimensional solid structures holds great benefits for analyses. Practical performances of the present system are demonstrated through several mesh generations for three-dimensional complex geometry.

**Key words** : Finite Element Analysis, Solid Model, Fuzzy Theory, Bucket Method

## 1. Introduction

The finite element method(FEM) has been widely utilized in simulating various engineering problems such as structural deformation, thermal conduction, electromagnetics and so on. The main reason for this is its high capability of dealing with boundary-value problems in arbitrarily shaped domains. On the other hand, a mesh used influences computational accuracy as well as time so significantly that the mesh generation process is as much important as the FEM analysis itself. Especially, in such large scale nonlinear FEM analyses that approach the limitation of computational capability of so-called supercomputers, it is highly demanded to optimize the distribution of mesh size under the condition of limited total degrees of freedom. Thus, the mesh generation process becomes more and more time-consuming and heavier tasks.

Loads for pre-processing and post-processing are increasing rapidly in accordance with an increase of scale and complexity of analysis models to be solved. Particularly, the mesh generation process, which influences computational accuracy as efficiency and whose fully automation is very difficult in three- dimensional cases, has become the most critical issue in a whole process of the FE analyses. In this respect, various researches [1-13] have been performed on the development of automatic mesh generation techniques. Among mesh generation methods, the tree model method[14] can generate graded meshes and it uses a reasonably small amount of computer time and storage. However, it is, by nature, not possible to arbitrarily control the changing rate of mesh size with respect to location, so that some smaller projection and notch etc. are sometimes omitted. Also, domain decomposition method[15] does not always succeed, and a designation of such sub-domains is very tedious for uses in three-dimensional cases.

In recent years, much attention has been paid to fuzzy knowledge processing techniques [17], which allow computers to treat “ambiguous” matters and processes. In this paper, we explain an FE mesh generation system based on fuzzy knowledge processing and computational geometry techniques. Here, the node density distribution, which is a kind of a node spacing function, was well controlled by means of the fuzzy knowledge processing technique [18], so that even beginners of the FE analyses are able to produce nearly optimum meshes through very simple operations as if they were experts.

접수일자 : 2005년 4월 1일
완료일자 : 2005년 6월 2일
감사의 글 : 본 연구는 한국과학재단 목적기초연구 (R01-2004-000-10469-0)지원으로 수행되었습니다.

## 2. Outline of the System

A flow of this system is shown in Fig. 1. Each subprocess will be described below.

Geometric modelers are utilized to define geometries of analysis domains. One of commercial geometric modelers, Designbase [18]. In the present system, nodes are first generated, and then a finite element mesh is built. In general, it is not so easy to well control element size for a complex geometry. A node density distribution over a whole geometry model is constructed as follows. The present system stores several local nodal patterns such as the pattern suitable to well capture stress concentration, the pattern to subdivide a finite domain uniformly, and the pattern to subdivide a whole domain uniformly. A user selects some of those local nodal patterns, depending on their analysis purposes, and designates where to locate them.

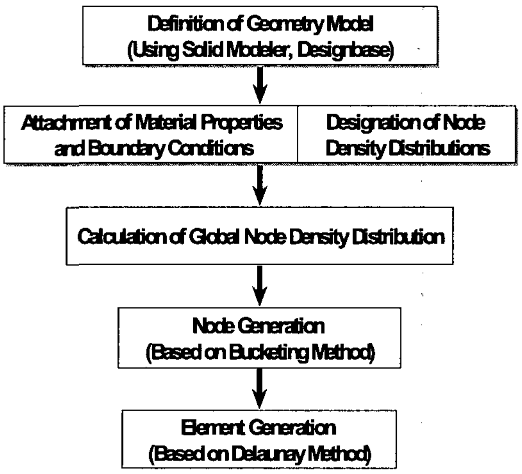


Fig. 1. Flow of the present system

## 2.1 Desigination of node density distribution

In this section, the connecting process of locally-optimum mesh images is dealt with using the fuzzy knowledge processing technique [17,18].

Performances of automatic mesh generation methods based on node generation algorithms depend on how to control node spacing functions or node density distributions and how to generate nodes. The basic concept of the present mesh generation algorithm is originated from the imitation of mesh generation processes by human experts on FE analyses. One of the aims of this algorithm is to transfer such experts' techniques to beginners.

In the present system, nodes are first generated, and then a finite element mesh is built. In general, it is not so easy to well control element size for a complex geometry. A node density distribution over a whole geometry model is constructed as follows. The present system stores several local nodal patterns such as the pattern suitable to well capture stress concentration, the pattern to subdivide a finite domain uniformly, and the pattern to subdivide a whole domain uniformly. A user selects some of those local nodal patterns, depending on their analysis purposes, and designates where to locate them.

## 2.2 Superposition of Mesh Pattern

In the present method, the field A close to the crack-tip and the field B close to the hole are defined in terms of the membership functions used in the fuzzy set theory as shown in Fig. 2(c).

For the purpose of simplicity, each membership function is given a function of one-dimension in the figure. In practice the membership function can be expressed as $\mu(x, y)$ in this particular example, and in 3D cases it is a function of 3D coordinates, i.e. $\mu(x, y, z)$.

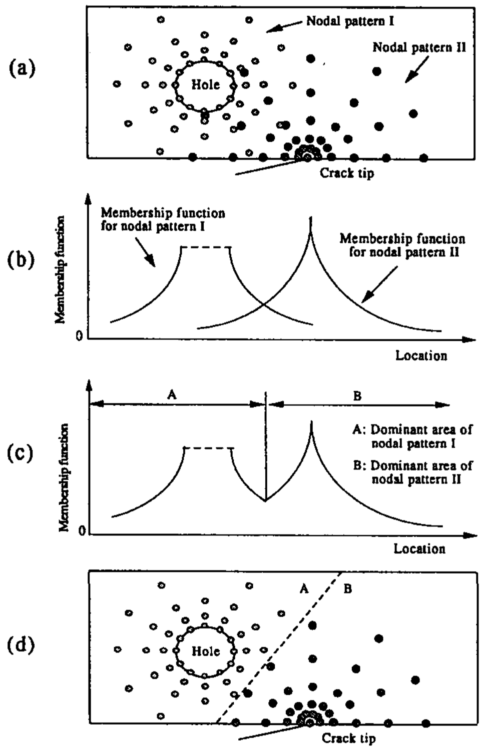


Fig. 2. Superposition of node patterns based on knowledge processing

In Fig. 2(c), the horizontal axis denotes the location, while the vertical axis does the value of membership function, which indicates the magnitude of "closeness"of the location to each stress concentration field. That is, a nodal location closer to the stress concentration field takes a larger value of the membership function. As for Fig. 2(b), choosing the mesh pattern with a larger value of the membership function in each location, one can obtain an overlapped curve of both membership functions, and the domain can be automatically divided into the following two sub-domains A and B as shown in Fig. 2(c) : the sub-domain close to the crack-tip and that of the hole. Finally, both node patterns are smoothly connected as shown in Fig. 2(d). This procedure of node generation, i.e. the connection procedure of both node patterns, is summarized as follows :

- If $\mu_A(x_p, y_p) \geq \mu_B(x_p, y_p)$ for a node p $(x_p, y_p)$ belonging to the pattern A, then the node p is generated, and otherwise p is not generated.

- If $\mu_A(x_q, y_q) \geq \mu_B(x_q, y_q)$ for a node q $(x_q, y_q)$ belonging to the pattern B, then the node q is generated, and otherwise q is not generated.

It is apparent that the above algorithm can be easily extended to 3D problems and any number of node patterns. In addition, since finer node patterns are generally required to place near stress concentration sources, it is convenient to let the membership function correspond to node density as well.

### 2.3 Node Generation

Node generation is one of time consuming processes in automatic mesh generation. In the present study, the novel bucketing method [19] is adopted to generate nodes which satisfy the distribution of node density over a whole analysis domain. Fig. 3 shows the generated appearance of nodes for a half of piston head. The distance of two neighboring candidate nodes is set to be smaller than the minimum distance of nodes to be generated in the relevant bucket. Next, candidate nodes are pick up one by one, starting from the left-bottom corner of the bucket, and are put into the bucket. A candidate node is adopted as one of the final nodes when it satisfies the two criteria [9].

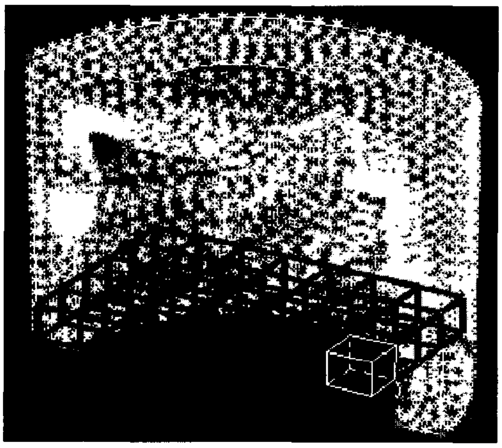

Fig. 3. Generated appearance of nodes

### 2.4 Element Generation

The Delaunay triangulation method [1,3] is utilized to generate tetrahedral elements from numerous nodes given in a geometry.

Let **N** be a set of nodes, it has the property that the circumcircle of any triangle in the triangulation contains no point of **N** in its interior. The remaining points in **N** will be iteratively added to the triangulation. After each point is added, it will be connected to the vertices of its enclosing triangle. (See Fig. 4) All internal edges of a triangulation of a finite set **N** are locally optimal if no point of **N** is interior to any circumcircle of a triangle.

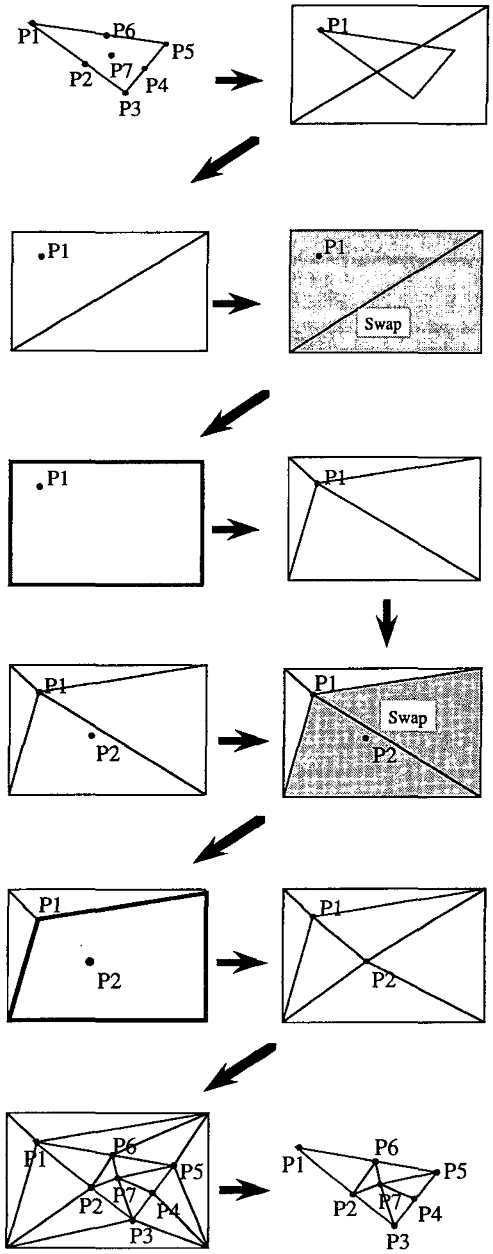


Fig. 4. Example showing a Delaunay triangulation

The speed of element generation by the Delaunay triangulation method is proportional to the number of nodes. If this method is utilized to generate elements in a geometry with indented shape, elements are inevitably generated even outside the geometry as shown in Fig. 5. However, such mis-match elements can be removed by performing the IN / OUT check for gravity center points of such elements. In addition, it is necessary to avoid the generation of those mis-match elements crossing domain boundary by setting node densities on edges to be slightly higher than those inside the domain near the boundaries.

### 2.5 Smoothing Operation

The algorithm of element generation mentioned above works well in most cases. However, element shapes obtained are sometimes distorted in a superposed region of several node patterns or near domain boundary. The smoothing method called "Laplacian operation" is here applied to remedy such distorted elements as shown in Fig. 6. In this operation, the location of each node is replaced with a mean value of locations of its neighboring nodes. This operation is iterated several times.

The algorithm of element generation mentioned above works well in most cases. However, element shapes obtained are sometimes distorted in a superposed region of several node patterns or near domain boundary. The smoothing method is here applied to remedy such distorted elements.

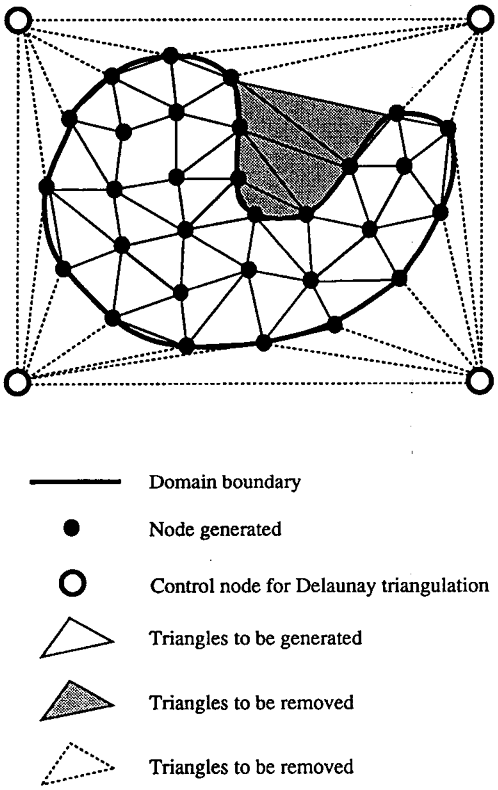


Fig. 5. Techniques of avoiding mis-match elements

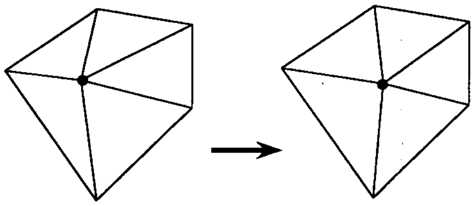

Fig. 6. Smoothing operation

## 3. Examples and Discussions

The performance of the system is demonstrated through the mesh generation of several three- dimensional structures. Fig. 7 to 12 show the examples of the application of this mesh generator for three-dimensional geometry. As shown in figures, a uniform mesh and a nonuniform mesh were connected very smoothly. In case of a half of piston head as shown in Fig. 7, it took about 40 minutes to define this geometry model by using Designbase. The mesh consists of 16,430 tetrahedral elements and 28,356 nodes. Nodes and elements are generated in about 14 minutes and in about 2 minutes, respectively.

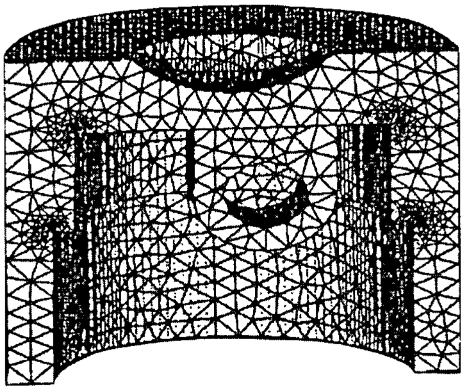

Fig. 7. Mesh for a half of piston head

To complete this mesh, the following two node patterns are utilized ; (a) the base node pattern in which nodes are generated with uniform spacing over a whole analysis domain, (b) a special node pattern for stress concentration of four corners.

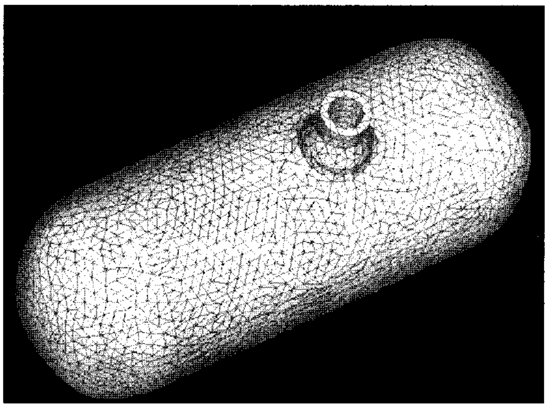

Fig. 8. Mesh for a pressure vessel

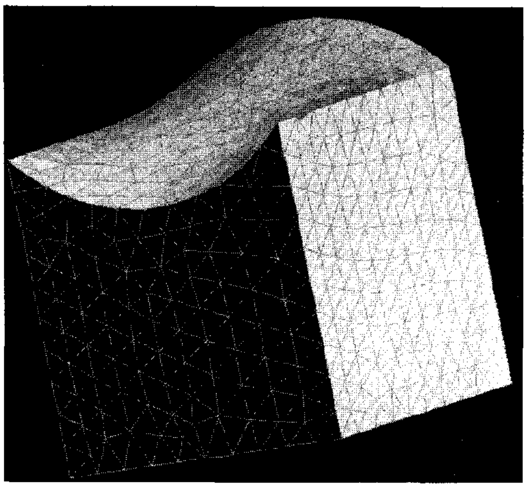

Fig. 9. Mesh for block with bezier loop

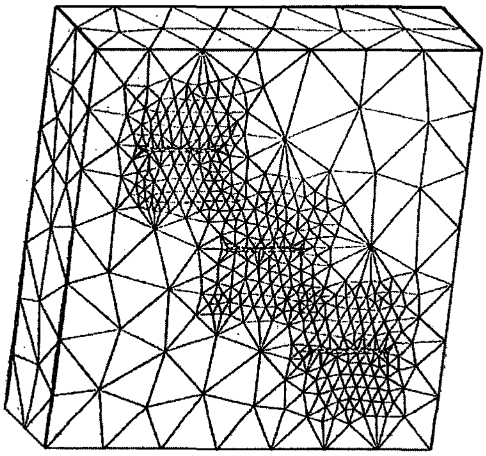

Fig. 9. Mesh for a plate with cracks

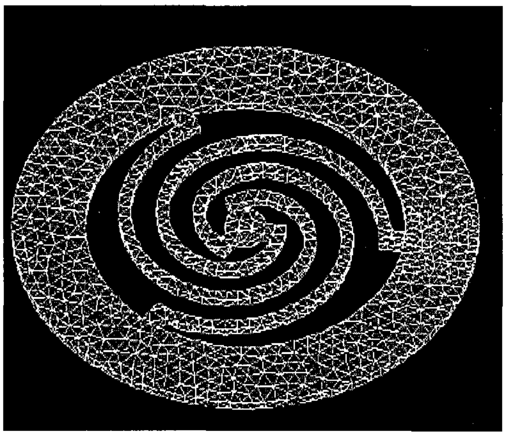

Fig. 10. Mesh for a wobble actuator

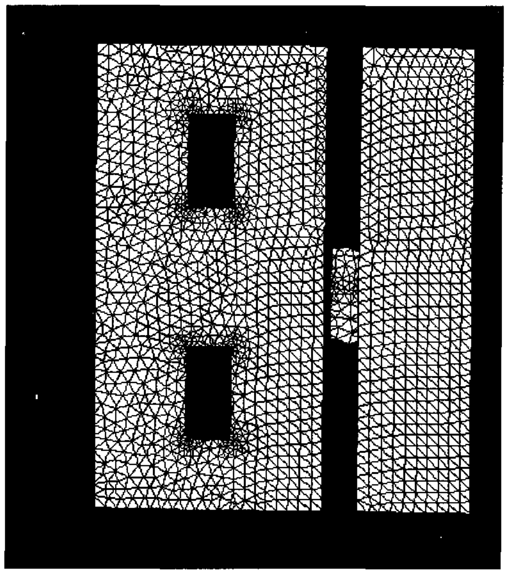

Fig. 11. Mesh for a first-wall

## 4. Conclusions

An automatic mesh generation system for three-dimensional structures consisting of free-form surfaces has been presented. Here several local node patterns are selected and are automatically superposed based on the fuzzy knowledge processing technique. In addition, several computational geometry techniques were successfully applied to node and element generation. The developed system was utilized to generate meshes of three-dimensional complex geometries. The key features of the present algorithm are an easy control of complex three-dimensional node density distribution with a fewer input data by means of the fuzzy knowledge processing technique, and fast node and element generation owing to some computa- tional geometry techniques. The effectiveness of the present system is demonstrated through several mesh generations for three-dimensional complex geometry.

## 5. References

[1] Lee, D. T. and Schachter, B. J., "Two Algorithms for Constructing a Delaunay Triangulation", *Int. J. of Computer and Information Sciences*, Vol. 9, No.3, pp. 219-242, 1980.

[2] Shephard, M. S., Yerry, M. A. and Baehmann, L., "Automatic Mesh Generation Allowing for Efficient a Priori and a Posteriori Mesh Refinement", *Computer Methods in Applied Mechanics and Engineering*, Vol. 55, pp. 161-180, 1986.

[3] Watson, D. F., "Computing The N-Dimensional Delaunay Tessellation with Application to Voronoi Polytopes", *The Computer Journal*, Vol. 24, No. 2,

pp. 167-172, 1981.
[4] Bowyer, A., "Computing Dirichlet Tessellations", *The Computer Journal*, Vol. 24, No. 2, pp. 162-172, 1981.
[5] Schroeder, W. J. and Shephard, M. S., "Combined Octree/Delaunay Method for Fully Automatic 3-D Mesh Generation", *International Journal for Numerical Methods in Engineering*, Vol. 29, pp. 37-55, 1990.
[6] Pourazady, M. and Radhakrishnan, M., "Optimization of a Triangular Mesh", *Computers and Structures*, Vol. 40, No. 3, pp. 795-804, 1991.
[7] Shephard, M. S. and Georges, M. K., "Automatic Three-Dimensional Mesh Generation by the Finite Octree Technique" *International Journal for Numerical Methods in Engineering*, Vol. 32, pp. 709-749, 1991.
[8] Buratynski, E. K., "Fully Automatic Three-Dimensional Mesh Generator for Complex Geometries", *Int. Journal for Numerical Methods in Engineering*, Vol. 30, pp. 931-952, 1990.
[9] Al-Nasra, M. and Nguyen, D. T., "An Algorithm for Domain Decomposition in Finite Element Analysis", *Computers and Structures*, Vol. 39, No. 3/4, pp. 277- 289, 1981.
[10] Sezer, L. and Zeid, I., "Automatic Quadrilateral/Triangular Free-Form Mesh Generation For Planar Regions", *Int. Journal for Numerical Methods in Engineering*, Vol. 32, pp. 1441-1483, 1991.
[11] Green, P. J. and Sibson, R., "Computing Dirichlet Tessellations in The Planes", *The Computer Journal*, Vol. 21, No. 2, pp. 168-173, 1977.
[12] Taniguchi, T. and Ohta, C., "Delaunay-Based Grid Generation for 3D Body with Complex Boundary Geometry", *Grid Generation Conference*, 1991.
[13] Sloan, S. W., "Fast Algorithm for Constructing Delaunay Triangulation in the Plane", *Advanced Engineering Software*, Vol. 9, No. 1, pp. 34-55, 1987.
[14] Buratynski, E. K., "Fully Automatic Three-Dimensional Mesh Generator for Complex Geometries", *Int. Journal for Numerical Methods in Engineering*, Vol. 30, pp. 931-952, 1990.
[15] Sezer, L. and Zeid, I., "Automatic Quadrilateral/Triangular Free-Form Mesh Generation For Planar Regions", *International Journal for Numerical Methods in Engineering*, Vol. 32, pp. 1441-1483, 1991.
[16] Chiyokura, H., *Solid Modeling with Designbase: Theory and Implementation*, Addition-Wesley, 1992.
[17] Cavendish, J. C., "Automatic Triangulation of Arbitrary Planar Domains for the Finite Element Method", *International Journal of Numerical Method in Engineering*, Vol. 8, pp. 679-696, 1974.
[18] Zadeh, L. A., "Fuzzy algorithms", *Information and Control*, Vol. 12, pp. 94-102, 1968.
[19] Lee, J.-S., "Automated CAE System for Three-Dimensional Complex Geometry", *Doctoral Thesis*, The University of Tokyo, 1995.

## 저 자 소 개

**이양창 (Yang-Chang Lee)**
2005년 제15권 제2호 참조

**이준성 (Joon-Seong Lee)**
2002년 제12권 제3호 참조

**최윤종(Yoon-Jong Choi)**
2000년 : 경기대학교 기계공학과 졸업 (공학사)
2002년 : 숭실대학교 기계공학과 석사과정 졸업(공학석사)
2003년～현재 경기대학교 대학원 기계공학과 박사과정

관심분야 : 퍼지 이론, 지능형자동차, 신뢰성평가

Phone : +82-31-249-9810
Fax : +82-31-249-9796
E-mail : nakhwa7@hanmail.net

**김남용(Nam-Yong Kim)**
2005년 : 경기대학교 기계공학과 졸업 (공학사)
2005년～ 현재 경기대학교 대학원 기계공학과 석사과정

관심분야 : 계산기하학적 수법, 신뢰성평가

Phone : +82-31-249-9818
E-mail : cdr400rr@hanmail.net