

# 2차 퍼지수에 대한 정규 퍼지 확률 Normal Fuzzy Probability for Quadratic Fuzzy Number

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## 요 약

2차 곡선들에 의해 정의되는 두 개의 2차 퍼지수의 연산들에 대하여 연구하였다. 그리고 그 연산들에 의하여 생성되는 퍼지수들에 대한 정규 퍼지 확률을 계산하였다.

## Abstract

We study some operations of two quadratic fuzzy numbers defined by quadratic curves. And we calculate the normal fuzzy probability for fuzzy numbers generated by the operations.

**Key words** : normal fuzzy probability, quadratic fuzzy number

## 1. 서 론

The operations of two fuzzy numbers  $(A, \mu_A)$  and  $(B, \mu_B)$  are based on the Zadeh's extension principle([4]). We consider four operations addition  $A(+ )B$ , subtraction  $A(- )B$ , multiplication  $A(\cdot )B$  and division  $A(/ )B$  defined in Definition 3.2.

Let  $(\Omega, \mathcal{F}, P)$  be a probability space, where  $\Omega$  denotes the sample space,  $\mathcal{F}$  the  $\sigma$ -algebra on  $\Omega$ , and  $P$  a probability measure. A fuzzy set  $A$  on  $\Omega$  is called a fuzzy event. Let  $\mu_A(\cdot)$  be the membership function of the fuzzy event  $A$ . Then the probability of the fuzzy event  $A$  is defined by Zadeh([1,3]) as

$$P(A) = \int_{\Omega} \mu_A(\omega) dP(\omega), \quad \mu_A(\cdot) : \Omega \rightarrow [0, 1]$$

In this paper, we define the normal fuzzy probability using the normal distribution, and we derive the explicit formula for the normal fuzzy probability for a quadratic fuzzy number and give some examples.

## 2. Normal fuzzy probability

Let  $(\Omega, \mathcal{F}, P)$  be a probability space, and  $X$  be a

random variable defined on it. Let  $g$  be a real-valued Borel-measurable function on  $\mathbb{R}$ . Then  $g(X)$  is also a random variable.

**Definition 2.1.** We say that the mathematical expectation of  $g(X)$  exists if  $E[g(X)]$  of the random variable  $g(X)$  defined by

$$E[g(X)] = \int_{\Omega} g(X(\omega)) dP(\omega) = \int_{\mathbb{R}} g(x) dP$$

is finite.

We note that a random variable  $X$  defined on  $(\Omega, \mathcal{F}, P)$  induces a measure  $P_X$  on a Borel set  $B \in \mathcal{B}$  defined by the relation  $P_X(B) = P\{X^{-1}(B)\}$ . Then  $P_X$  becomes a probability measure on  $\mathcal{B}$  and is called the probability distribution of  $X$ . It is known that if  $E[g(X)]$  exists, then  $g$  is also integrable over  $\mathbb{R}$  with respect to  $P_X$ . Moreover, the relation

$$(2.1) \quad \int_{\Omega} g(X) dP = \int_{\mathbb{R}} g(t) dP_X(t)$$

holds. We note that the integral on the right-hand side of (2.1) is the Lebesgue-Stieltjes integral of  $g$  with respect to  $P_X$ . In particular, if  $g$  is continuous on  $\mathbb{R}$  and  $E[g(X)]$  exists, we can rewrite (2.1) as follows

$$\int_{\Omega} g(X) dP = \int_{\mathbb{R}} g dP_X = \int_{-\infty}^{\infty} g(x) dF(x),$$

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where  $F$  is the distribution function corresponding to  $P_X$ , and the last integral is a Riemann-Stieltjes integral.

Let  $F$  be absolutely continuous on  $\mathbb{R}$  with probability density function  $f(x) = F'(x)$ . Then  $E[g(X)]$  exists if and only if the integral  $\int_{-\infty}^{\infty} |g(x)|f(x)dx$  is finite and in that case we have

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)dx.$$

**Example 2.2.** Let the random variable  $X$  (denoted  $X \sim N(m, \sigma^2)$ ) have the normal distribution with the probability density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-m)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$$

where  $\sigma^2 > 0$  and  $m \in \mathbb{R}$ . Then  $E|X|^\gamma < \infty$  for every  $\gamma > 0$ , and we have

$$EX = m \text{ and } E(X - m)^2 = \sigma^2.$$

The induced measure  $P_X$  is called the normal distribution.

A fuzzy set  $A$  on  $\Omega$  is called a fuzzy event. Let  $\mu_A(\cdot)$  be the membership function of the fuzzy event  $A$ . Then the probability of the fuzzy event  $A$  is defined by Zadeh ([1,3]) as

$$\mathcal{P}(A) = \int_{\Omega} \mu_A(\omega) dP(\omega), \quad \mu_A(\cdot) : \Omega \rightarrow [0, 1].$$

**Definition 2.3.** The normal fuzzy probability  $\mathcal{P}(A)$  of a fuzzy set  $A$  on  $\mathbb{R}$  is defined by

$$\mathcal{P}(A) = \int_{\mathbb{R}} \mu_A(x) dP_X,$$

where  $P_X$  is the normal distribution of  $X$ .

For a triangular fuzzy number  $A = (a_1, a_2, a_3)$  having membership function

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x < a_2, \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x < a_3, \\ 0, & a_3 \leq x, \end{cases}$$

the normal fuzzy probability  $\mathcal{P}(A)$  ([5]) is

$$\begin{aligned} \mathcal{P}(A) &= \frac{m - a_1}{a_2 - a_1} \left( F_N\left(\frac{a_2 - m}{\sigma}\right) - F_N\left(\frac{a_1 - m}{\sigma}\right) \right) \\ &+ \frac{\sigma}{\sqrt{2\pi}(a_2 - a_1)} \left( e^{-\frac{(a_1 - m)^2}{2\sigma^2}} - e^{-\frac{(a_2 - m)^2}{2\sigma^2}} \right) \\ &+ \frac{m - a_3}{a_2 - a_3} \left( F_N\left(\frac{a_3 - m}{\sigma}\right) - F_N\left(\frac{a_2 - m}{\sigma}\right) \right) \\ &+ \frac{\sigma}{\sqrt{2\pi}(a_2 - a_3)} \left( e^{-\frac{(a_2 - m)^2}{2\sigma^2}} - e^{-\frac{(a_3 - m)^2}{2\sigma^2}} \right) \end{aligned}$$

where  $F_N(a)$  is the standard normal distribution, that is,

$$F_N(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^a e^{-\frac{t^2}{2}} dt.$$

### 3. Quadratic fuzzy number

Similar to triangular fuzzy number, the quadratic fuzzy number is defined by quadratic curve.

**Definition 3.1.** A quadratic fuzzy number is a fuzzy number  $A$  having membership function

$$\mu_A(x) = \begin{cases} 0, & x < \alpha, \\ a(x - k)^2 + 1, & \alpha \leq x < \beta, \\ 0, & \beta \leq x, \end{cases}$$

where  $a < 0$ .

The above quadratic fuzzy number is denoted by  $A = [\alpha, k, \beta]$ .

**Definition 3.2.** The addition, subtraction, multiplication, and division of two fuzzy sets are defined as, for all  $x \in A$  and  $y \in B$ ,

1. Addition  $A(+)B$  :  
 $\mu_{A(+)B}(z) = \sup_{z=x+y} \min\{\mu_A(x), \mu_B(y)\}.$
2. Subtraction  $A(-)B$  :  
 $\mu_{A(-)B}(z) = \sup_{z=x-y} \min\{\mu_A(x), \mu_B(y)\}.$
3. Multiplication  $A(\cdot)B$  :  
 $\mu_{A(\cdot)B}(z) = \sup_{z=x \cdot y} \min\{\mu_A(x), \mu_B(y)\}.$
4. Division  $A(/)B$  :  
 $\mu_{A(/)B}(z) = \sup_{z=x/y} \min\{\mu_A(x), \mu_B(y)\}.$

In this section, we study the above four operations for two quadratic fuzzy numbers.

**Theorem 3.3.** For two quadratic fuzzy number  $A = [x_1, k, x_2]$  and  $B = [x_3, m, x_4]$ , we have

1.  $A(+)B = [x_1 + x_3, k + m, x_2 + x_4].$
2.  $A(-)B = [x_1 - x_4, k - m, x_2 - x_3].$
3.  $\mu_{A(\cdot)B}(x) = 0$  on the interval  $[x_1 x_3, x_2 x_4]^c$  and  $\mu_{A(\cdot)B}(x) = 1$  at  $x = km$ . Note that  $A(\cdot)B$  may be not a quadratic fuzzy number.
4.  $\mu_{A(/)B}(x) = 0$  on the interval  $\left[\frac{x_1}{x_4}, \frac{x_2}{x_3}\right]^c$  and  $\mu_{A(/)B}(x) = 1$  at  $x = \frac{k}{m}$ . Note that  $A(/)B$  may be not a quadratic fuzzy number.

*Proof.* Note that

$$\mu_A(x) = \begin{cases} 0, & x < x_1, \\ -a(x - k)^2 + 1 = -a(x - x_1)(x - x_2), & x_1 \leq x < x_2, \\ 0, & x_2 \leq x, \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x < x_3, \\ -b(x-m)^2+1 \\ = -b(x-x_3)(x-x_4), & x_3 \leq x < x_4, \\ 0, & x_4 \leq x. \end{cases}$$

We calculate four operations using  $\alpha$ -cuts. Let  $A_\alpha = [a_1, a_2]$  and  $B_\alpha = [b_1, b_2]$  be the  $\alpha$ -cuts of  $A$  and  $B$ , respectively. Since  $\alpha = -a(a_1-k)^2+1$  and  $\alpha = -a(a_2-k)^2+1$ , we have  $A_\alpha = [a_1, a_2] = \left[ k - \sqrt{\frac{1-\alpha}{a}}, k + \sqrt{\frac{1-\alpha}{a}} \right]$ .

Similarly,  $B_\alpha = [b_1, b_2] = \left[ m - \sqrt{\frac{1-\alpha}{b}}, m + \sqrt{\frac{1-\alpha}{b}} \right]$ .

1. Addition : By the above facts,

$$A_\alpha(+ )B_\alpha = [a_1 + b_1, a_2 + b_2] \\ = \left[ k + m - \sqrt{\frac{1-\alpha}{a}} - \sqrt{\frac{1-\alpha}{b}}, k + m + \sqrt{\frac{1-\alpha}{a}} + \sqrt{\frac{1-\alpha}{b}} \right].$$

Thus  $\mu_{A(+ )B}(x) = 0$  on the interval

$$\left[ k + m - \frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}}, k + m + \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} \right]^c \\ = [x_1 + x_3, x_2 + x_4]^c \text{ and } \mu_{A(+ )B}(x) = 1 \text{ at } x = k + m. \\ \text{Therefore}$$

$$\mu_{A(+ )B}(x) = \begin{cases} 0, & x < x_1 + x_3, \\ -\frac{ab}{(\sqrt{a} + \sqrt{b})^2} (x - (k + m))^2 + 1, & x_1 + x_3 \leq x < x_2 + x_4, \\ 0, & x_2 + x_4 \leq x, \end{cases}$$

i.e.,  $A(+ )B = [x_1 + x_3, k + m, x_2 + x_4]$ .

2. Subtraction :

Since  $A_\alpha(- )B_\alpha = [a_1 - b_2, a_2 - b_1]$

$$= \left[ k - m - \frac{\sqrt{1-\alpha}}{a} - \sqrt{\frac{1-\alpha}{b}}, k - m + \sqrt{\frac{1-\alpha}{a}} + \sqrt{\frac{1-\alpha}{b}} \right],$$

we have  $\mu_{A(- )B}(x) = 0$  on the interval

$$\left[ k - m - \frac{1}{\sqrt{a}} - \frac{1}{\sqrt{b}}, k - m + \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} \right]^c \\ = [x_1 - x_4, x_2 - x_3]^c \text{ and } \mu_{A(- )B}(x) = 1 \text{ at } x = k - m. \\ \text{Therefore}$$

$$\mu_{A(- )B}(x) = \begin{cases} 0, & x < x_1 - x_4, \\ -\frac{ab}{(\sqrt{a} + \sqrt{b})^2} \{x - (k - m)\}^2 + 1, & x_1 - x_4 \leq x < x_2 - x_3, \\ 0, & x_2 - x_3 \leq x, \end{cases}$$

i.e.,  $A(- )B = [x_1 - x_4, k - m, x_2 - x_3]$ .

3. Multiplication :

Since  $A_\alpha(\cdot )B_\alpha = [a_1 b_1, a_2 b_2]$

$$= \left[ \left( k - \sqrt{\frac{1-\alpha}{a}} \right) \left( m - \sqrt{\frac{1-\alpha}{b}} \right), \left( k + \sqrt{\frac{1-\alpha}{a}} \right) \left( m + \sqrt{\frac{1-\alpha}{b}} \right) \right],$$

$\mu_{A(\cdot )B}(x) = 0$  on the interval

$$\left[ \left( k - \frac{1}{\sqrt{a}} \right) \left( m - \frac{1}{\sqrt{b}} \right), \left( k + \frac{1}{\sqrt{a}} \right) \left( m + \frac{1}{\sqrt{b}} \right) \right]^c \\ = [x_1 x_3, x_2 x_4]^c \text{ and } \mu_{A(\cdot )B}(x) = 1 \text{ at } x = km. \text{ Therefore}$$

$$\mu_{A(\cdot )B}(x) = \begin{cases} 0, & x < x_1 x_3, \\ \frac{1}{4} (4 - 2k^2 a - 2m^2 b - 4\sqrt{ab}x) \\ + 2(k\sqrt{a} + m\sqrt{b})\sqrt{(k\sqrt{a} - m\sqrt{b})^2 + 4\sqrt{ab}x} \\ 0, & x_1 x_3 \leq x < x_2 x_4, \\ 0, & x_2 x_4 \leq x. \end{cases}$$

4. Division : Since

$$A_\alpha(/ )B_\alpha = \left[ \frac{a_1}{b_2}, \frac{a_2}{b_1} \right] \\ = \left[ \frac{k - \sqrt{\frac{1-\alpha}{a}}}{m + \sqrt{\frac{1-\alpha}{b}}}, \frac{k + \sqrt{\frac{1-\alpha}{a}}}{m - \sqrt{\frac{1-\alpha}{b}}} \right],$$

$\mu_{A(/ )B}(x) = 0$  on the interval

$$\left[ \frac{\sqrt{b}(k\sqrt{a}-1)}{\sqrt{a}(m\sqrt{b}+1)}, \frac{\sqrt{b}(k\sqrt{a}+1)}{\sqrt{a}(m\sqrt{b}-1)} \right]^c = \left[ \frac{x_1}{x_4}, \frac{x_2}{x_3} \right]^c$$

and  $\mu_{A(/ )B}(x) = 1$  at  $x = \frac{k}{m}$ . Therefore  $\mu_{A(/ )B}(x)$

$$= \begin{cases} 0, & x < \frac{x_1}{x_4}, \\ \frac{a(1-bm^2)x^2 + 2\sqrt{ab}(1+\sqrt{ab}km)x + b(1-ak^2)}{(\sqrt{ax} + \sqrt{b})^2}, & \frac{x_1}{x_4} \leq x < \frac{x_2}{x_3}, \\ 0, & \frac{x_2}{x_3} \leq x. \end{cases}$$

**Example 3.4.** For two quadratic fuzzy numbers  $A = [1, 2, 3]$  and  $B = [2, 5, 8]$ , we calculate exactly the above four operations using  $\alpha$ -cuts.

Note that

$$\mu_A(x) = \begin{cases} 0, & x < 1, \\ -(x-2)^2+1, & 1 \leq x < 3, \\ 0, & 3 \leq x, \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x < 2, \\ -\frac{1}{9}(x-5)^2+1, & 2 \leq x < 8, \\ 0, & 8 \leq x. \end{cases}$$

Put  $A_\alpha = [a_1, a_2]$  and  $B_\alpha = [b_1, b_2]$ . Since  $\alpha = -(a_1-2)^2+1$  and  $\alpha = -(a_2-2)^2+1$ , we have  $A_\alpha = [a_1, a_2] = [2 - \sqrt{1-\alpha}, 2 + \sqrt{1-\alpha}]$ .

Similarly,  $B_\alpha = [b_1, b_2] = [5 - 3\sqrt{1-\alpha}, 5 + 3\sqrt{1-\alpha}]$ .

1. Addition : By the above facts,

$$A_\alpha(+ )B_\alpha = [a_1 + b_1, a_2 + b_2] \\ = [7 - 4\sqrt{1-\alpha}, 7 + 4\sqrt{1-\alpha}].$$

Thus  $\mu_{A(+ )B}(x) = 0$  on the interval  $[3, 11]^c$  and  $\mu_{A(+ )B}(x) = 1$  at  $x = 7$ .

Therefore

$$\mu_{A(+ )B}(x) = \begin{cases} 0, & x < 3, \\ -\frac{1}{16}(x-7)^2+1, & 3 \leq x < 11, \\ 0, & 11 \leq x, \end{cases}$$

i.e.,  $\mu_{A(+ )B}(x) = [3, 7, 11]$ .

2. Subtraction : Since

$$A_\alpha(- )B_\alpha = [a_1 - b_2, a_2 - b_1] \\ = [-3 - 4\sqrt{1-\alpha}, -3 + 4\sqrt{1-\alpha}],$$

we have  $\mu_{A(-)B}(x)=0$  on the interval  $[-7,1]^c$  and  $\mu_{A(-)B}(x)=1$  at  $x=-3$ . Therefore

$$\mu_{A(-)B}(x) = \begin{cases} 0, & x < -7, \\ -\frac{1}{16}(x+3)^2 + 1, & -7 \leq x < 1, \\ 0, & 1 \leq x, \end{cases}$$

i.e.,  $\mu_{A(-)B}(x) = [-7, -3, 1]$ .

3. Multiplication : Since

$$A_a(\cdot)B_a = [a_1 \cdot b_1, a_2 \cdot b_2] = [13 - 3a - 11\sqrt{1-a}, 13 - 3a + 11\sqrt{1-a}],$$

$\mu_{A(\cdot)B}(x)=0$  on the interval  $[2,24]^c$  and  $\mu_{A(\cdot)B}(x)=1$  at  $x=10$ . Therefore

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < 2, \\ -\frac{1}{18}(6x+43) - 11\sqrt{12x+1}, & 2 \leq x < 24, \\ 0, & 24 \leq x. \end{cases}$$

Note that  $A(\cdot)B$  is not a quadratic fuzzy number.

4. Division : Since

$$A_a(/)B_a = \left[ \frac{a_1}{b_2}, \frac{a_2}{b_1} \right] = \left[ \frac{2 - \sqrt{1-a}}{5 + 3\sqrt{1-a}}, \frac{2 + \sqrt{1-a}}{5 - 3\sqrt{1-a}} \right],$$

$\mu_{A(/)B}(x)=0$  on the interval  $[\frac{1}{8}, \frac{2}{3}]^c$  and

$\mu_{A(/)B}(x)=1$  at  $x = \frac{2}{5}$ . Therefore

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{1}{8}, \\ \frac{-(8x-1)(2x-3)}{(3x+1)^2}, & \frac{1}{8} \leq x < \frac{3}{2}, \\ 0, & \frac{3}{2} \leq x. \end{cases}$$

Note that  $A(/)B$  is not a quadratic fuzzy number.

### 4. Main results

In this section, we derive the explicit formula for the normal fuzzy probability for a quadratic fuzzy number and give some examples.

**Theorem 4.1.** Let  $X \sim N(m, \sigma^2)$  and  $A = [\alpha, k, \beta]$  be a quadratic fuzzy number. Then the normal fuzzy probability is

$$\begin{aligned} \mathcal{P}(A) &= \frac{\sigma}{\sqrt{2\pi}} \left( (a\alpha + am + b)e^{-\frac{(a-m)^2}{2\sigma^2}} \right. \\ &\quad \left. - (a\beta + am + b)e^{-\frac{(\beta-m)^2}{2\sigma^2}} \right) \\ &\quad + (a\sigma^2 + am^2 + bm + c) \\ &\quad \cdot \left[ F_N\left(\frac{\beta-m}{\sigma}\right) - F_N\left(\frac{\alpha-m}{\sigma}\right) \right]. \end{aligned}$$

*Proof.* Note that

$$\mu_A(x) = \begin{cases} 0, & x < \alpha, \\ ax^2 + bx + c, & \alpha \leq x < \beta, \\ 0, & \beta \leq x. \end{cases}$$

Putting  $\frac{x-m}{\sigma} = t$ ,

$$\begin{aligned} \mathcal{P}(A) &= \frac{1}{\sqrt{2\pi}} \int_{\frac{\alpha-m}{\sigma}}^{\frac{\beta-m}{\sigma}} \{ a(\sigma t + m)^2 + b(\sigma t + m) \\ &\quad + c \} e^{-\frac{t^2}{2}} dt \\ &= \frac{a\sigma^2}{\sqrt{2\pi}} \int_{\frac{\alpha-m}{\sigma}}^{\frac{\beta-m}{\sigma}} t^2 e^{-\frac{t^2}{2}} dt \\ &\quad + \frac{2a\sigma m + b\sigma}{\sqrt{2\pi}} \int_{\frac{\alpha-m}{\sigma}}^{\frac{\beta-m}{\sigma}} t e^{-\frac{t^2}{2}} dt \\ &\quad + \frac{(am^2 + bm + c)}{\sqrt{2\pi}} \int_{\frac{\alpha-m}{\sigma}}^{\frac{\beta-m}{\sigma}} e^{-\frac{t^2}{2}} dt \\ &= \frac{\sigma}{\sqrt{2\pi}} \left( (a\alpha + am + b)e^{-\frac{(a-m)^2}{2\sigma^2}} \right. \\ &\quad \left. - (a\beta + am + b)e^{-\frac{(\beta-m)^2}{2\sigma^2}} \right) \\ &\quad + (a\sigma^2 + am^2 + bm + c) \\ &\quad \cdot \left[ F_N\left(\frac{\beta-m}{\sigma}\right) - F_N\left(\frac{\alpha-m}{\sigma}\right) \right]. \end{aligned}$$

**Remark** If the quadratic fuzzy number  $A = [\alpha, k, \beta]$  is represented by

$$\mu_A(x) = \begin{cases} 0, & x < \alpha, \\ a(x-k)^2 + 1, & \alpha \leq x < \beta, \\ 0, & \beta \leq x. \end{cases}$$

then

$$\begin{aligned} \mathcal{P}(A) &= \frac{\sigma}{\sqrt{2\pi}} \left( a(\alpha + m - 2k)e^{-\frac{(a-m)^2}{2\sigma^2}} \right. \\ &\quad \left. - a(\beta + m - 2k)e^{-\frac{(\beta-m)^2}{2\sigma^2}} \right) \\ &\quad + (a(\sigma^2 + (m-k)^2) + 1) \\ &\quad \cdot \left[ F_N\left(\frac{\beta-m}{\sigma}\right) - F_N\left(\frac{\alpha-m}{\sigma}\right) \right]. \end{aligned}$$

**Example 4.2.** In the case of the quadratic number  $A = [1, 2, 3]$ , the normal fuzzy probability with respect to  $X \sim N[3, 2^2]$  is 0.2304.

$$\begin{aligned} \mathcal{P}(A) &= \int_{-\infty}^{\infty} \frac{-x^2 + 4x - 3}{2\sqrt{2\pi}} e^{-\frac{(x-3)^2}{8}} dx \\ &= \frac{0}{\sqrt{2\pi}} e^{-\frac{1}{2}} - \frac{4}{\sqrt{2\pi}} e^0 \\ &\quad - 4(F_N(0) - F_N(-1)) \\ &= 0.2304 \end{aligned}$$

Since the results of two operations (addition, subtraction) of two quadratic fuzzy numbers are quadratic fuzzy numbers, the normal fuzzy probability can be calculated by Theorem 4.1. But multiplication and division of two quadratic fuzzy numbers may be not quadratic fuzzy numbers, so we have to calculate by the definition.

**Example 4.3.** Let  $X \sim N(5, 2^2)$  and consider the fuzzy numbers in Example 3.4.

1. Multiplication

$$\begin{aligned}
 \mathcal{P} &= \int_2^{24} \frac{-(6x+43)+11\sqrt{12x+1}}{18} \\
 &\quad \cdot \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-5)^2}{8}} dx \\
 &= -\frac{1}{6\sqrt{2\pi}} \int_2^{24} x e^{-\frac{(x-5)^2}{8}} dx \\
 &\quad + \frac{43}{36\sqrt{2\pi}} \int_2^{24} e^{-\frac{(x-5)^2}{8}} dx \\
 &\quad + \frac{1}{2\sqrt{2\pi}} \int_2^{24} 11\sqrt{12x+1} e^{-\frac{(x-5)^2}{8}} dx \\
 &= 0.6402
 \end{aligned}$$

2. Division

$$\begin{aligned}
 \mathcal{P} &= \int_{\frac{1}{8}}^{\frac{3}{2}} \frac{-(8x-1)(2x-3)}{(3x+1)^2} \\
 &\quad \cdot \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-5)^2}{8}} dx \\
 &= \frac{-1}{2\sqrt{2\pi}} \int_{\frac{1}{8}}^{\frac{3}{2}} \frac{16x^2-26x+3}{9x^2+6x+1} e^{-\frac{(x-5)^2}{8}} dx \\
 &= 0.0145
 \end{aligned}$$

References

- [1] M. Bhattacharyya, Fuzzy Markovian decision process, *Fuzzy Sets and Systems*, Vol. 99, pp. 273-282, 1998.
- [2] D. Dubois and H. Prade, *Fuzzy Sets and Systems : Theory and Applications*, Academic Press, London 1980.
- [3] L. A. Zadeh, Probability measures of fuzzy events, *J. Math. Anal. Appl.*, Vol. 23, pp. 421-427, 1968.
- [4] L. A. Zadeh, *The concept of a linguistic variable and its application to approximate reasoning - I*, Information Sciences, Vol. 8, pp. 199-249, 1975.
- [5] J. C. Song and Y. S. Yun, On the normal fuzzy probability, *Journal of Basic Sciences*, Vol. 17, No. 2, pp.17-27, 2004.

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