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Reverse-optimization Alignment Algorithm using Zernike Sensitivity

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When aligning catoptric or catadioptric telescopes for space cameras, it is difficult to align precisely if the field of view is large or there are several reflective surfaces. The quantitative knowledge of mirror misalignments greatly helps align a misaligned telescope precisely, and also reduce the alignment time. This paper describes a generalized reverse-optimization alignment solution algorithm using Zernike sensitivity, and proposes the minimum number of fields to take interferograms. This method was successfully applied on a Cassegrain telescope design for Earth observation from space with arbitrary misalignments and a model including some primary mirror deformation.

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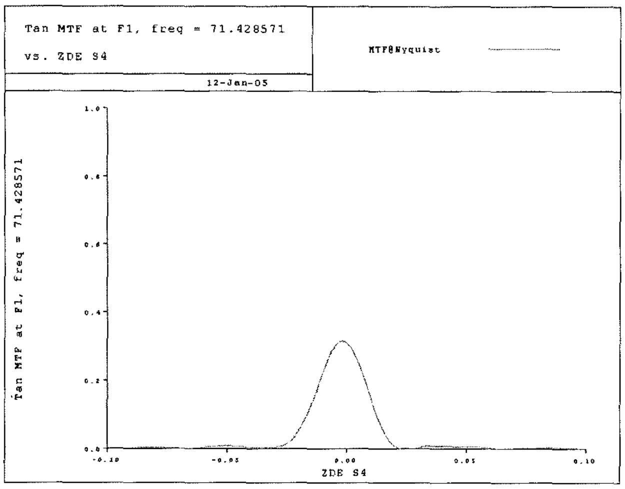
I. INTRODUCTION

Space telescopes often use catoptric or catadioptric design such as a Cassegrain-variant due to the advantages in the size and mass. A Cassegrain telescope can be aligned by inspecting the interferograms from the double-pass setup in the laboratory. When the Cassegrain telescope has relatively small field of view (e.g. <+/-0.1 deg), it can be aligned easily by using the on-axis interferogram. For example, a 450 mm diameter Cassegrain collimator [1] with +/-0.09 deg field of view was reported to be easily aligned by inspecting the on-axis interferogram only. However, it becomes more difficult when the field of view is relatively large. This was observed while aligning a +/-1 deg field of view Cassegrain-variant telescope[2]. Even though the on-axis interferogram showed sufficiently low RMS wavefront error (WFE), the RMS WFEs of the opposite off-axis fields were unacceptably unbalanced. The reverse-optimization function (ALI) in Code V had been used to calculate the as-is misalignment parameters to solve this problem. This paper describes a similar reverse-optimization algorithm independently, by using the Zernike sensitivities on misalignments. Understanding of this algorithm can help determine the minimum number of interferograms needed for the reverse-optimization alignment for more general cases. The algorithm was successfully applied for a Cassegrain- variant, and the range of misalignments for accurate alignment solution is also discussed.

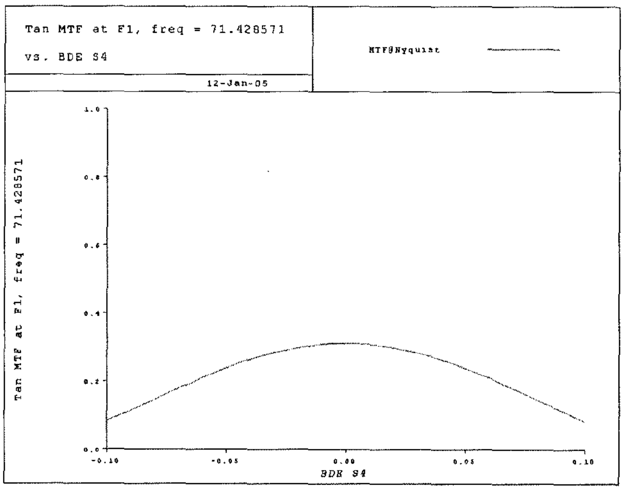
II. ZERNIKE SENSITIVITY OF MISALIGNMENT PARAMETERS

RMS WFE or modulation transfer function (MTF) are usually used for calculating sensitivity tables in commercial optics design softwares. However, Zernike sensitivity analysis is known to be useful in determining the alignment compensators and setting up an optimized alignment logic of an optical system [3]. Unlike MTF sensitivities, low-order Zernike coefficients obtained from the wavefront at the exit pupil of a telescope are usually linearly varying to misalignments if the ranges of the misalignments are sufficiently small. Fig. 1 shows typical MTF profiles versus varying misalignment parameters of a Cassegrain-variant telescope. The secondary mirror is despaced or tilted, and the MTF response is not linear to the misalignments over the typical ranges of misalignments. On the contrary, Zernike sensitivities on misalignments are approximately linear over the same misalignment ranges.

There are various kinds of Zernike polynomials depending on the expansion order and the shape of the



(a) MTF vs. Despace of M2



(b) MTF vs. Tilt of M2

FIG. 1. MTF sensitivity of a Cassegrain telescope.

area over which the polynomials are defined. Standard Zernike polynomials were used in [3], but Fringe Zernike polynomials as shown in Table 1 [4] were used for this paper. Fringe Zernike polynomials were preferred because the key low-order Zernike terms related to the alignment are expressed in a series unlike in the standard Zernike polynomials. Fringe Zernike coefficients are also often the only option for commercial interferometers.

Zernike sensitivity analysis can also be used to reversely calculate unknown misalignments. For a Cassegrain-variant like the Medium-sized Aperture Camera (MAC) [5] (Fig. 2), an earth observation camera for small satellites, the Zernike sensitivities of the key components can be plotted as in Fig. 3. The secondary mirror (M2) and correction lens barrel (CLB) were used to generate misalignments. The fringe Zernike coefficients from Z_3 to Z_8 (defocus to SA3) were plotted, as Z_0 to Z_2 (piston to tilt) are irrelevant to the image quality. Conspicuous linearity is shown within the misalignment range. The translational misalignment error range was +/-0.1 mm, and the rotational misalignment error range was +/-0.06 deg (~1000 μrad). It was evident that the secondary mirror is more sensitive than the correction lens barrel per same misalignment range, and hence five alignment compensators - despace, X/Y-tilts, tilts around X/Y axes - on the secondary mirror were used.

TABLE 1. Low-order Fringe Zernike polynomial terms.

No.	Polynomial	Description
0	1	Piston
1	$\rho\cos\theta$	Tilt about x-axis
2	$\rho\sin\theta$	Tilt about y-axis
3	$2\rho^2-1$	Focus shift
4	$\rho^2\cos 2\theta$	Astigmatism, axis at 0 or 90 deg
5	$\rho^2\sin 2\theta$	Astigmatism, axis at 45 deg
6	$(3\rho^2-2)\rho\cos\theta$	3rd order coma along y-axis
7	$(3\rho^2-2)\rho\sin\theta$	3rd order coma along x-axis
8	$6\rho^4-6\rho^2+1$	3rd order spherical aberration

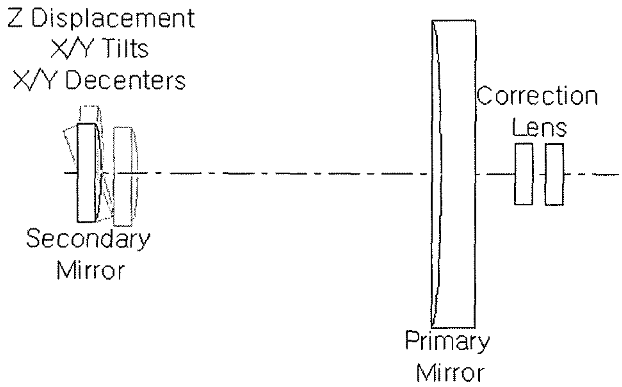
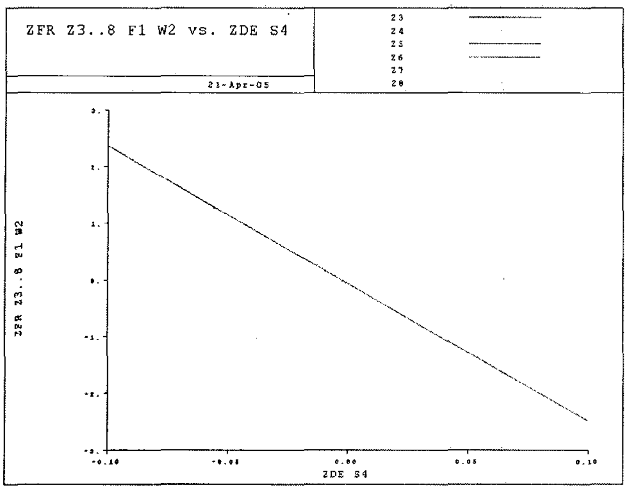


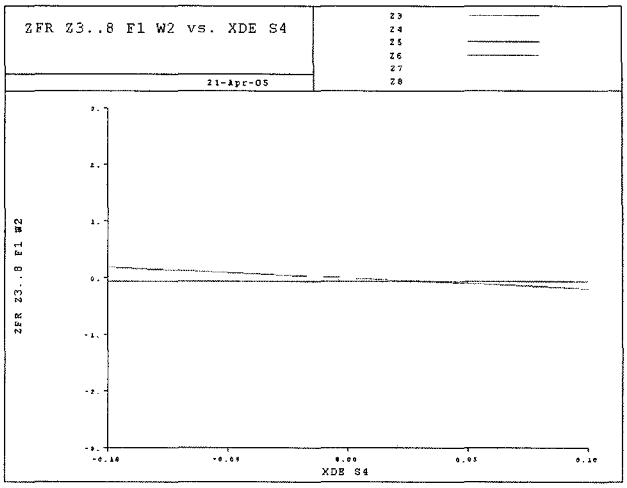
FIG. 2. Alignment compensators for MAC.

III. REVERSE-OPTIMIZATION ALGORITHM

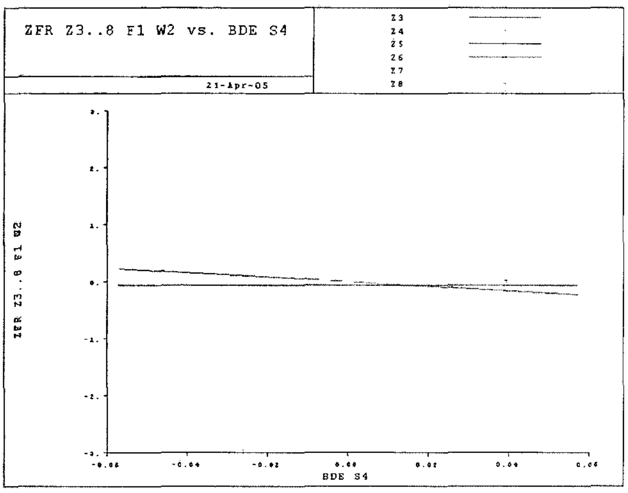
With the Zernike sensitivities and the Zernike coefficients taken from a misaligned telescope, we can reversely calculate the misalignment parameters. The slope of each straight line in Fig. 3 is the Zernike sensitivity on the misalignment parameter. As the Zernike coefficients are linear to misalignments, each Zernike coefficient extracted from the misaligned telescope can be expressed as a linear combination of different misalignment parameters with relevant Zernike sensitivities. At m th field, n th fringe Zernike coefficient ${}^{m}Z_{n}$ can be



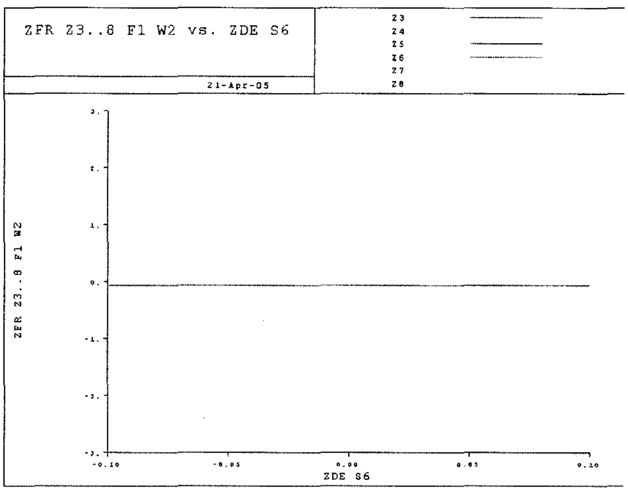
(a) M2 despace



(b) M2 X-decenter



(c) M2 Beta-tilt



(d) CLB despace

FIG. 3. Zernike sensitivities on misalignment parameters of a Cassegrain-variant telescope.

expressed as follows.

$$^{m}Z_n = {}^{m}c_n + \sum_i \left[\frac{d(^{m}Z_n)}{dx_i} \right] \Delta x_i$$

The constant $^{m}c_n$ is the Zernike coefficient of the perfectly aligned telescope from the design, when all Δx_i are zeros. $\frac{d^m Z_n}{dx_i}$ is the Zernike sensitivity on the misalignment parameter x_i, where it can be despace, X/Y decenter, and tilt around X/Y.

With i number of unknowns (misalignment parameters), we can solve the simultaneous equations for the unknowns provided there are minimum i number of unique equations. Table 2 shows the Zernike sensitivities of the Cassegrain-variant of Fig. 2 at on-axis (field 1) and off-axis (field 2) fields. Equations as above can be set up for different Zernike coefficients. However, for the field 1, some sensitivities are virtually zeros and there remain only four equations for Z_3, Z_6, Z_7, and Z_8. As there are five unknowns, at least five equations are needed to solve for the five unknowns. Table 2 also shows the Zernike sensitivities for the field 2, which gives six unique equations for Z_3, Z_4, Z_5, Z_6, Z_7 and Z_8. With any five from the six equations can be used to solve for the five unknown misalignment errors.

Table 3 shows an alignment solution example. When the secondary mirror was arbitrarily decentered, despaced, and tilted as in Table 3, one can solve for the unknown misalignment errors by solving pertinent five Zernike coefficient equations from field 2 only. (Z_3, Z_4, Z_5, Z_6 and Z_7 were used for this example.) It should be noticed that the 1st alignment solution does not give perfect solution but leaves some residual error. The residual error is removed extremely well at an iterated (2nd) alignment solution. This is because the

TABLE 2. Zernike sensitivities on misalignment parameters of M2 (in wv/mm or wv/deg).

Field 1 on-axis (0.0, 0.0)

		Z3	Z4	Z5	Z6	Z7	Z8
M2	X-Dec 0.1mm	2.16E-09	-7.35E-12	-5.85E-11	-1.89E+00	-9.99E-11	-2.67E-09
	Y-Dec 0.1mm	2.18E-09	-2.95E-11	-5.85E-11	-9.99E-11	-1.89E+00	-2.67E-09
	Z-Des 0.1mm	-2.37E+01	7.87E-15	-2.80E-11	-1.77E-10	-1.77E-10	1.60E-01
	Tilt around X 0.06deg	-3.01E-09	4.59E-11	-9.94E-11	1.19E-12	3.94E+00	3.33E-09
	Tilt around Y 0.06deg	2.51E-09	-4.85E-10	9.94E-11	-3.94E+00	-1.19E-12	-3.59E-09

Field 2 off-axis (1.0, 0.0)

		Z3	Z4	Z5	Z6	Z7	Z8
M2	X-Dec 0.1mm	-4.37E-01	7.48E-03	3.88E-11	-1.90E+00	-5.74E-10	-9.90E-04
	Y-Dec 0.1mm	1.91E-09	-6.37E-11	-7.43E-03	2.55E-10	-1.90E+00	-2.37E-09
	Z-Des 0.1mm	-2.38E+01	-3.65E-02	5.96E-12	-1.46E-01	1.94E-10	1.60E-01
	Tilt around X 0.06deg	-8.41E-09	3.11E-10	-6.67E+00	-4.76E-10	3.93E+00	1.01E-08
	Tilt around Y 0.06deg	2.19E+00	6.83E+00	-1.93E-11	-3.87E+00	-2.35E-10	-1.61E-03

TABLE 3. Reverse-optimization alignment solution to arbitrary misalignments.

M2 Misalignment		1st Alignment (mm, deg)		2nd Alignment (mm, deg)	
Type	mm, deg	Solution	Error	Solution	Error
X-Decenter	0.1	0.1027	0.0027	0.0027	6.07E-18
Y-Decenter	-0.1	-0.0865	0.0135	0.0135	0
Z-Despace	0.1	0.1009	0.0009	0.0009	1.95E-18
Tilt around X	-0.1	-0.093	0.007	0.007	0
Tilt around Y	0.1	0.0982	-0.0018	-0.0018	9.97E-18

Zernike coefficient to misalignment parameter may not be sufficiently linear or one or more misalignment parameters may be coupled to one another. Fig. 4 shows a Zernike sensitivity to secondary mirror tilt with larger misalignment range, and non-linearity can be seen more clearly. We can determine approximately how much misalignment we can reversely calculate. In order to accurately calculate the unknown misalignment, the error needs to be within sufficiently linear range. The example in Table 3 gives practically accurate alignment solution at the 1st run, but the number of reverse-optimization run should be determined by the RMS WFE requirement after the alignment. The alignment solution is also limited by the alignment tool (e.g. micrometer) resolution.

In real alignment situations, the mounted mirrors of the telescope are not as ideal as in the design, but often deformed during the assembly procedure and/or due to the gravity. With a deformed mirror, the Zernike sensitivity would not be as the same as an ideal case, and the measured wavefront and hence the Zernike coefficients would also be different. Another calculation was carried out to check whether this reverse-optimization algorithm can be used for a Cassegrain-variant telescope with some deformation on the primary mirror.

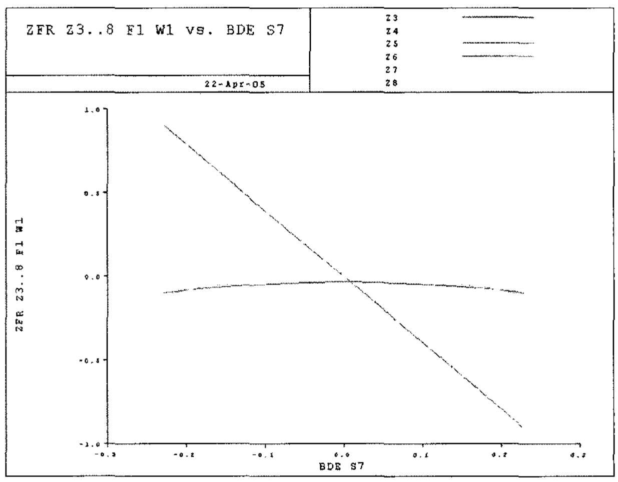


FIG. 4. Non-linearity example of Zernike sensitivity for a wider misalignment range.

TABLE 4. Zernike sensitivities with M1 deformation (in wv/mm or wv/deg).

Field 2 off-axis (1.0, 0.0)							
		Z3	Z4	Z5	Z6	Z7	Z8
M2	X-Dec 0.1mm	−4.37E-01	7.49E-03	1.00E−05	−1.90E+00	4.40E−05	−1.31E−03
	Y-Dec 0.1mm	5.40E-05	−4.60E-05	−7.42E−03	−3.30E−05	−1.90E+00	−9.00E−06
	Z-Des 0.1mm	−2.38E+01	−3.65E−02	3.10E−05	−1.46E−01	1.26E−04	1.61E−01
	Tilt around X 0.06deg	−1.54E−03	1.68E−03	−6.67E+00	6.60E−05	3.93E+00	−1.89E−03
	Tilt around Y 0.06deg	2.20E+00	6.84E+00	2.47E−03	−3.86E+00	3.53E−03	−2.14E−02

TABLE 5. Reverse-optimization alignment solution to arbitrary misalignments with M1 deformation.

M2 Misalignment		1st Alignment (mm, deg)		2nd Alignment (mm, deg)	
Type	mm, deg	F2	Error	F2	Error
X-Decenter	0.1	0.1028	0.0028	0.0028	0
Y-Decenter	−0.1	−0.0863	0.0137	0.0137	0
Z-Despace	0.1	0.1009	0.0009	0.0009	1.95E−18
Tilt around X	−0.1	−0.093	0.007	0.007	0
Tilt around Y	0.1	0.0982	−0.0018	−0.0018	9.97E−18

Fig. 5 shows a primary mirror deformation example occurred by mounting. The interferogram file from the deformed primary mirror was inserted on the primary mirror of the design to calculate the Zernike sensitivity with the primary mirror deformation. The Zernike sensitivities from the telescope with the deformed primary mirror are shown in Table 4. Some arbitrary misalignments were given to the telescope with the primary mirror deformation, and the as-is Zernike coefficients were extracted. The simultaneous equations were solved similarly, and the alignment solution was still very accurate. The alignment solution with the primary mirror deformation included is shown in Table 5. Other manufacturing error such as the radii of curvature and conic constants was also considered before calculating the sensitivities by including the measured values supplied by the mirror manufacturer.

The reverse-optimization algorithm can be also useful for a system with more alignment degrees of freedom such as a three-mirror anastigmat (TMA) camera for hyperspectral imaging applications[6]. Once the number of alignment compensators is determined as above, the amount of misalignments can be calculated as long as the Zernike coefficients are linearly varying to the misalignments likewise. When there are i number of alignment compensators (unkowns), the m number of needed fields, from which the interferograms are taken can be expressed as $6m \leqq i$. However, it should be noticed that the number of non-zero unique Zernike sensitivity equations should also be greater than or equal to, and this is why field 2 was chosen rather than field 1 in the case of Cassegrain-variant telescope above. For example, when there are seven alignment compensators for a TMA telescope, minimum two fields are needed to take interferograms. In general, the unknown misalignments can be solved by solving the matrix equation as follows including the case when there are more equations than the number of unknown misalignments. For such an overdetermined system, the matrix equation can be solved by the least squares fitting method.

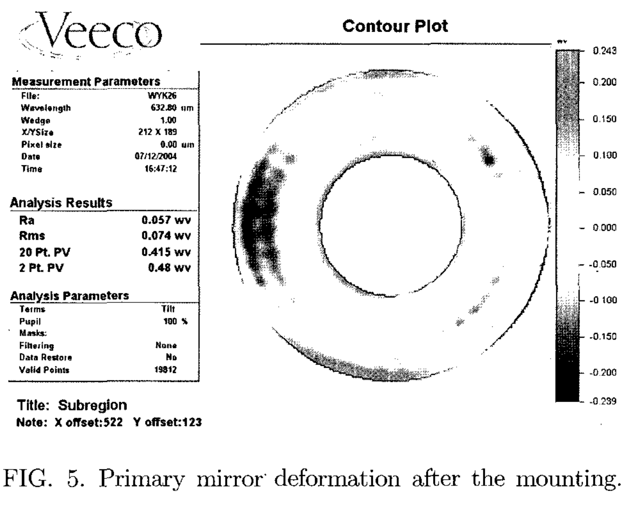


FIG. 5. Primary mirror deformation after the mounting.

$$\begin{pmatrix} \frac{d(^1Z_3)}{dx_1} \cdots \frac{d(^1Z_3)}{dx_i} \\ \vdots \quad \ddots \quad \vdots \\ \frac{d(^1Z_8)}{dx_1} \cdots \frac{d(^1Z_8)}{dx_i} \\ \vdots \\ \frac{d(^mZ_3)}{dx_1} \cdots \frac{d(^mZ_3)}{dx_i} \\ \vdots \quad \ddots \\ \frac{d(^mZ_8}{dx_1} \ \frac{d(^mZ_8)}{dx_i} \end{pmatrix} \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_i \end{pmatrix} = \begin{pmatrix} ^1Z_3 - {^1c_3} \\ ^1Z_4 - {^1c_4} \\ \vdots \\ ^1Z_8 - {^1c_8} \\ \vdots \\ ^mZ_3 - {^mc_3} \\ ^mZ_4 - {^mc_4} \\ \vdots \\ ^mZ_8 - {^mc_8} \end{pmatrix}$$

IV. CONSLUSION

A generalized reverse-optimization algorithm using Zernike sensitivity was reviewed and the minimum number of fields to take the interferograms was proposed. The proposed minimum number of fields and pertinent selection of the field can save time in measuring the interferograms at different fields of view. The algorithm was applied to a Cassegrain-variant telescope design with arbitrary misalignments on the secondary mirror, and to the same telescope with some primary mirror deformation due to mounting. It was shown that the proposed algorithm was sufficiently robust to calculate the alignment solutions very accurately for both cases. This method can be applied to more complicated system such as a TMA system, and is to be explored further in future alignment experiments.

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