Predicting cutting forces in face milling with the orthogonal machining theory

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Abstract This paper presents an effective cutting force model that enables us to predict the instantaneous cutting force in face milling from knowledge of the work material properties and the cutting conditions. The development of the model is based on the orthogonal machining theory with the effective rake angle, which is defined in the plane containing the cutting velocity vector and the chip flow vector. Face milling tests are performed at different feeds and, a fairly good agreement is shown between the predicted cutting forces and the test results.

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1. Introduction

Predicting cutting forces accurately is one of the most important objectives for researching machining processes. In planning machining process, it is very important to know cutting forces in advance. Forces exerted by the tool on the work material can cause deflections that lead to geometric errors or difficulty in meeting tolerance specifications. Large reaction forces of the work material on the tool can cause failure of the tool. The product of the force and velocity vectors is used to predict power requirements or production rates. These considerations can be used to reduce time and cost for designing a new tool or for planning machining processes of new work materials.

Up to now, many researchers have investigated the machining process. However, primarily due to the difficulty in work material deformation with large plastic strain, strain rate, high temperature and their coupling effects, a general predictive cutting model is not developed yet.

In both experimental and analytical investigations of machining processes, it has been usual to consider the relatively simple case of orthogonal machining. The reason is that many practical machining processes can be approximated quite closely to orthogonal machining conditions and in this case, the plane strain conditions for analysis can be applied. For the orthogonal machining, up to the present, the shear plane model has been effectively used. The goal of this method is to predict the shear angle, the chip thickness and the force generated, with many useful experimental equations being proposed. The slip line method, which has widely been used to analyze metal forming problems, has also been considered for the analysis of machining processes after 1960s. Some headway has been made in considering the cutting temperature with the flow stress of the work material allowed to very with strain and strain rate. Oxley [1] developed an orthogonal machining theory that the cutting forces, temperatures etc can be determined from the work material properties (flow stress and thermal properties) and cutting conditions, and good agreement was shown between experiment and theory. Oxley's machining theory is more inclusive and powerful than any other one.

To date most machining theories have mainly been applied to processes in which the undeformed chip thickness and the cutting velocity remain constant during cutting and chip formation takes place under approximately steady state conditions. Young et al. [2] predicted forces in a simple face milling operation in which the undeformed chip geometry changes during cutting, but their research was only effective in case that both axial and radial rake angles of a face milling cutter were zero, in other words, an orthogonal cutting process was assumed.

In this paper, a method for predicting cutting forces in face milling considering axial and radial rake angles of a face milling cutter is presented. From knowledge of the work material properties and the cutting conditions, without any machining test, the method enables us to predict cutting forces accurately.

2. Orthogonal Machining Theory [1]

In the model on which the theory is based (Fig. 1) the tool cutting edge is assumed to be perfectly sharp and the plane AB, near the center of the primary shear deformation zone, and the tool-chip interface are both assumed to be in the direction of maximum shear stress and maximum shear strain rate. The basis of the theory is to analyze the stress distributions along AB and the tool-chip interface in terms of the shear angle ϕ, work material properties and cutting conditions, and then to select ϕ so that the resultant forces transmitted by AB and the interface are in equilibrium.

Once ϕ is known then the various components of force can be determined from the following relations:

$$F_c = R\cos(\lambda - \alpha)$$

$$F_t = R\sin(\lambda - \alpha)$$

$$F = R\sin\lambda \tag{1}$$

$$N = R\cos\lambda$$

$$R = \frac{F_s}{\cos\theta} = \frac{k_{AB} t_1 w}{\sin\phi\cos\theta}$$

where t_1 is the undeformed chip thickness, w is the width of cut and k_{AB} is the shear flow stress along AB. By applying the appropriate stress equilibrium equation along AB, it can be shown that the angle θ made by the resultant force R with AB is given by:

$$\tan\theta = 1 + 2(\frac{1}{4}\pi - \phi) - Cn \tag{2}$$

where C is the constant in the empirical relation:

$$\dot{\gamma}_{AB} = CV_S / \ell \tag{3}$$

in which $\dot{\gamma}_{AB}$ is the maximum shear strain rate at AB, V_S is the shear velocity ℓ is the length of AB, and n is the strain-hardening index in the empirical stress/strain relation:

$$\sigma = \sigma_1 \varepsilon^n \tag{4}$$

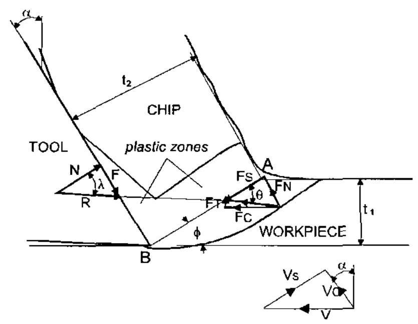


Fig.1 Model of orthogonal chip formation

The angle θ can also be expressed as:

$$\theta = \phi + \lambda - \alpha \tag{5}$$

To determine k_{AB} and n, the temperature at AB, T_{AB}, is needed together with the strain rate and strain at AB. T_{AB} can be found from the equation:

$$T_{AB} = T_O + n_f\left[\frac{1-\beta}{\rho C_p t_1 w}\frac{F_s\cos\alpha}{\cos(\phi-\alpha)}\right] \tag{6}$$

where T_O is the initial work temperature, F_s is the shear force along AB, n_f is a factor which allows for the fact that not all of the plastic work of chip formation has occurred at AB and β is the proportion of heat conducted into the work. β is estimated from the empirical equations [4]:

$$\left.\begin{aligned} &\beta = 0.5 - 0.35\log(R_T\tan\phi) \;\; for \\ &\qquad\qquad 0.04 \le R_T\tan\phi \le 10.0 \\ &\beta = 0.5 - 0.15\log(R_T\tan\phi) \;\; for \\ &\qquad\qquad R_T\tan\phi > 10.0 \end{aligned}\right\} \tag{7}$$

β ($0 \le \beta \le 1$) is expressed only as the function of $R_T\tan\phi$. R_T is a non-dimensional Peclet number given by:

$$R_T = \rho C_p V t_1 / k_c \tag{8}$$

where ρ and k_c are the density and the thermal conductivity of the work material, respectively.

The strain at AB is given by:

$$\gamma_{AB} = \frac{1}{2}\frac{\cos\alpha}{\sin\phi\cos(\phi-\alpha)} \tag{9}$$

The average temperature at the tool-chip interface from which the average shear flow stress at the interface is determined is taken as:

$$T_{int} = T_O + \frac{1-\beta}{\rho C_p t_1 w}\frac{F_s\cos\alpha}{\cos(\phi-\alpha)} + \Psi\Delta\theta_m \tag{10}$$

where $\Delta\theta_m$ is the maximum temperature rise in the chip and the factor Ψ ($0 < \Psi \le 1$) allows for possible variations of temperature along the interface. The temperature rise is found from the empirical relation [4]:

$$\log\left(\frac{\Delta\theta_m}{\Delta\theta_c}\right) = 0.06 - 0.195\delta\left(\frac{R_T t_2}{l_n}\right)^{\frac{1}{2}} + 0.5\log\left(\frac{R_T t_2}{l_n}\right) \tag{11}$$

where δ is the ratio of the thickness of the rectangular plastic zone used to represent the deformation at the interface to the chip thickness, $\Delta\theta_c$ is the average temperature rise in the chip given by:

$$\Delta\theta_c = F\sin\phi / \rho C_p t_1 w\cos(\phi-\alpha) \tag{12}$$

The tool-chip contact length is given by:

$$l_n = \frac{t_1\sin\theta}{\cos\lambda\sin\phi}\left\{1 + \frac{Cn}{3\left[1 + 2\left(\frac{1}{4}\pi - \phi\right) - Cn\right]}\right\} \tag{13}$$

The shear strain rate at the tool-chip interface can be found from the equation:

$$\dot{\gamma}_{int} = V_c / \delta t_2 \tag{14}$$

The cutting forces, temperatures, etc. can be obtained from the above equations for given cutting conditions if the appropriate work material properties and the value of C in equations (2), (3) and (13) and in equations (11) and (14) are known. Briefly, the method used is to calculate for a range of values of ϕ the resolved shear stress at the tool-chip interface ($\tau_{int} = F/l_n w$) from the resultant cutting force obtained from the stress on AB and then for the same range of values to calculate the temperature and strain rate at the tool-chip interface and the corresponding values of shear flow stress. The influence of strain on flow stress above a strain of $\varepsilon_{int} = 1$ is assumed to have negligible effect on flow stress. The solution is taken as the value of ϕ which gives $\tau_{int} = k_{chip}$.

To determine C, the stress boundary condition is used. For a uniform normal stress at the interface the average normal stress is given by:

$$\sigma_N = N / l_n w \tag{15}$$

This stress can also be found from the stress boundary condition at B by working from A along AB (Fig. 1). If AB turns through the angle ($\phi - \alpha$) to meet the interface at right angle, as it must do if the interface is assumed to be a direction of maximum shear stress, then it can be shown that:

$$\sigma'_N / k_{AB} = 1 + \frac{1}{2}\pi - 2\alpha - 2Cn \qquad (16)$$

C can now be determined from the condition that σ_N and σ'_N must be equal.

The value δ is found from minimum work considerations. For given cutting conditions, a value of δ exists which gives a combination of strain rate and temperature that minimizes the shear flow stress. It is assumed that δ will take up value satisfying this minimum work condition.

3. Predicting Forces in Oblique Machining

Although many practical machining processes are approximately orthogonal, they could often be represented more accurately by oblique machining with the inclination angle i. In oblique machining, it is important to know the direction of chip flow angle and to determine which is more important angle between normal and effective rake angle in analyzing machining process. The effective rake angle, as shown in Fig. 2, is measured in the plane containing the cutting velocity and the chip velocity.

In this paper, the effective rake angle is utilized to calculate forces in oblique machining. Once orthogonal force components in the plane containing the cutting velocity and the chip velocity is calculated by Oxley's machining theory, these force components are transformed by rotation transformation matrix [6] into 3-dimensional force components, and then the force components in face milling process are determined. Here, the method to be transformed into 3-dimensional force components by the effective rake angle is briefly introduced.

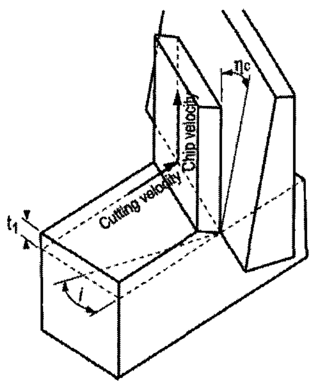


Fig. 2 Oblique machining

Figure 3 shows the geometry of the cutting configuration at a cutter edge of a face milling cutter. In relation to the practical cutting condition, the positive direction of the z-axis corresponds to that of the depth of cut, the positive direction of y-axis, to the tangential force, and the negative direction of x-axis, to the chip thickness. On plane ACH in Fig. 3 a cutter edge contacts the work material, and the chip flows in the $\overline{AS}$ direction. Here, α_a is the approach angle, α_r the radial rake angle, α_x the axial rake angle, and i the inclination angle. Fig. 4 shows the cross section of the work material and cutting tool in the plane where cutting and chip flow occur. In this figure, α_e is the effective rake angle and is given by:

$$\sin\alpha_e = \sin\eta_c \sin i + \cos\eta_c \cos i \sin\alpha_n \qquad (17)$$

where η_c is the chip flow angle and is assumed to be equivalent to i according to Stabler's flow law [7]. In Fig. 4, h_e is the effective undeformed chip thickness defined in the plane where cutting and chip flow occur.

The normal rake angle α_n can be given by the following equation from the geometric relation of the tool:

$$\tan\alpha_n = \cos i(\tan\alpha_r \cos\alpha_n + \tan\alpha_x \sin\alpha_a) \qquad (18)$$

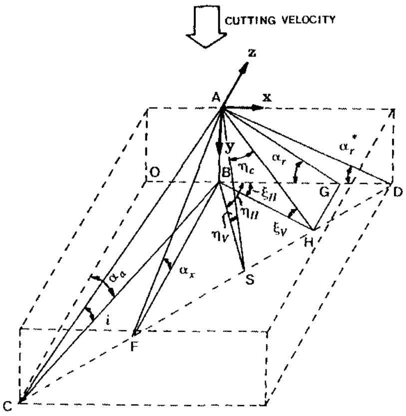


Fig. 3 Geometrical representation of cutter edge

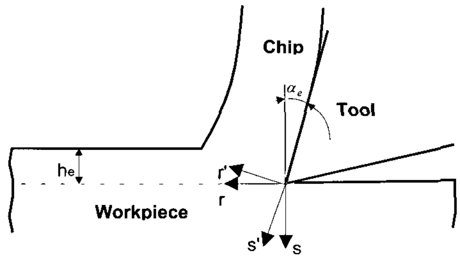


Fig. 4 Cross section of the work material and cutting tool in the plane where cutting takes place and chip flow occurs

By geometric transformation, the forces in the primed coordinate system can be expressed as:

$$\begin{bmatrix} F_{r'} \\ F_{s'} \end{bmatrix} = \begin{bmatrix} \cos\alpha_e & -\sin\alpha_e \\ \sin\alpha_e & \cos\alpha_e \end{bmatrix} \begin{bmatrix} F_r \\ F_s \end{bmatrix} \qquad (19)$$

Here, F_r and F_s can be considered as the orthogonal forces with respect to the effective rake angle, and the width and the depth of cut, and can be predicted from Oxley's orthogonal machining theory. Then, the cutting force components in x, y, and z directions are given by:

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = [T] \begin{bmatrix} F_r \\ F_s \end{bmatrix} \qquad (20)$$

where the transformation matrix $[T]$ can be calculated from the geometric relation of the tool edge configuration including the approach angle, the radial rake and axial rake angles, as shown in Fig. 3, and Stabler's flow law.

$$[T] = \begin{bmatrix} \sin\alpha_r^* \cos\alpha_x / \mathrm{N} & -\cos\eta_V \cos\eta_H \\ -\cos\alpha_r^* \cos\alpha_x / \mathrm{N} & -\sin\eta_V \sin\eta_H \\ -\cos\alpha_r^* \sin\alpha_x / \mathrm{N} & \cos\eta_V \cos\eta_H \end{bmatrix} \qquad (21)$$

where, $\mathrm{N} = \sqrt{\cos^2\alpha_x + \cos^2\alpha_r^* \sin^2\alpha_x}$.

4. Analysis of Face Milling Process

For practical process, a face mill rotates, whereas the work material is translates, being driven independently by the machine table. From the viewpoint of relative motion this system can be replaced by an equivalent system in which the work is treated as stationary while the cutter rotates and translates at the same time. Fig. 5 shows a face milling process.

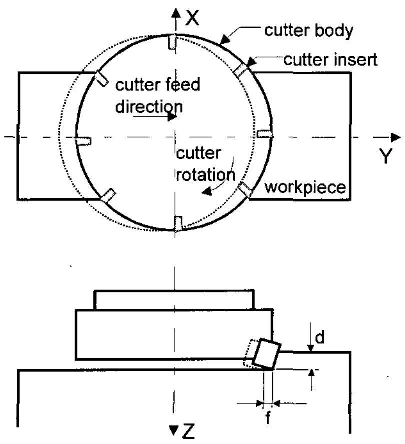


Fig. 5 Face milling cutter and geometry

The undeformed chip thickness considering a circular path of a milling cutter is given by:

$$t_1(\theta) = R_c + f_t \sin\theta - (R_c^{\,2} - f_t^{\,2}\cos^2\theta)^{1/2}$$

$$\approx f_t \sin\theta \qquad (22)$$

where f_t is the feed per revolution per teeth and R_c is the radius of milling cutter.
In case where the approach angle exists, the undeformed chip thickness and the width of cut can be represented by:

$$t_1 = f_t \sin\theta\cos\alpha_a, \qquad w = d/\cos\alpha_a \qquad (23)$$

where d is depth of cut.

The global coordinate system XYZ shown in Fig. 5 can be expressed in terms of the local coordinate system xyz in equation (20) by the relation:

$$\begin{bmatrix} F_X \\ F_Y \\ F_Z \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \qquad (24)$$

5. Predicted and Experimental Results

To validate the theoretical analysis, experiments are performed and the force components in face milling process are measured. The work material used in the experiment is 0.2% carbon steel and its flow stress property is expressed by stress/strain relation in equation (4). The flow stress σ_1 and strain hardening index n are the functions of temperature and strain rate, and can be expressed by velocity modified temperature T_{mod}, with T_{mod} given by:

$$T_{\text{mod}} = T\left\{1 - v_c \log\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o}\right)\right\} \qquad (25)$$

where T is the temperature, $\dot{\varepsilon}$ is the strain rate and v_c and $\dot{\varepsilon}_o$ are constants taken to be 0.09 and 1, respectively. Fig. 6 shows curves of σ_1 and n plotted with respect to the velocity modified temperature. These curves indicate a clear dynamic strain-ageing (blue brittle) range where flow stress increases with the increase in temperature, which is a typical phenomenon for plain carbon steel.

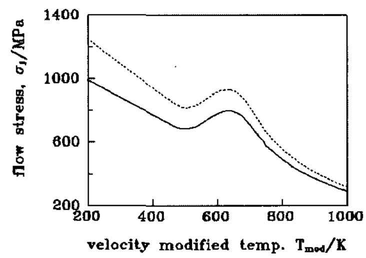
The influence of carbon content on specific heat is found to be small and the following equation can be used for all kinds of the steel. [1]

$$C_p/(Jkg^{-1}K^{-1}) = 420 + 0.504T/{}^oC \qquad (26)$$

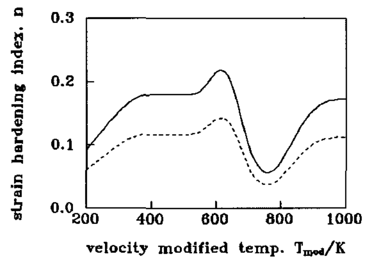
There is marked influence of carbon content on thermal conductivity and for 0.2% carbon steel it is given by: [1]

$$k_c/(Wm^{-1}K^{-1}) = 54.17 - 0.0298T/{}^oC \qquad (27)$$

In the analysis and experiments, the system consists of a fly cutter which is a single-toothed rotating face mill. The diameter of the cutter is 100 mm, the approach angle 15°, the radial rake angle 0° and axial rake angle 7°. The rotation speed is 320 rpm and the depth of cut is 2 mm. Two feed rates are performed as 48 mm/min and 64 mm/min. Table 1 shows the experimental equipment. Table 2 shows the cutting conditions for experiments. Tool force signals measured by the tool dynamometer for milling are amplified by a strain gauge type amplifier and passed through a low pass filter (band width: 2.5kHz). Then, these signals are sent to the A/D convert and stored in a computer.



(a) flow stress, σ_1



(b) strain hardening index, n

Fig. 6 Flow stress results plotted against velocity modified temperature: solid line – 0.2% carbon steel; dot line ___ 0.38% carbon steel

Figure 7 shows the predicted and experimental results for cutting conditions in CASE 1. The dot line and the solid line represent the experimental and predicted value, respectively. FX, FY and FZ represent the principal, the feed and the thrust force component, respectively. The results in Fig. 7 show that the experimental force components and the corresponding theoretical curves are in good agreement in terms of shape and amplitude.

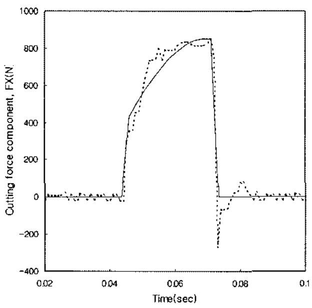
Figure 8 is the predicted and experimental results from CASE 2, the feed rate of which is 1.33 times larger than that of CASE 1. Compared with CASE 1, the principal and the feed force component increase, whereas the thrust force component is negligible. In the same manner as in CASE 1, the experimental and the predicted results show close resemblance in terms of shape and amplitude. These results enable us to predict the instantaneous cutting force in face milling process, in which cutting conditions vary continuously, from Oxley's orthogonal machining theory and the effective rake angle.

Table 1 Experimental equipment

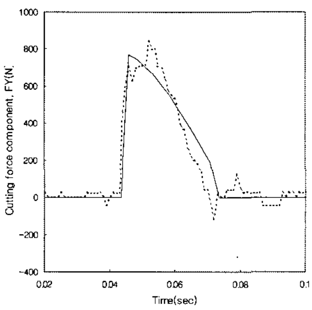
Milling machine	HITACHI numerical control milling machine (VA-40)
Dynamometer	KISTLER AST 1207(for milling, strain gauge type)
Amplifier	KYOWA DPM-310B(strain gauge type)
Oscilloscope	T912(TEKTRONIX)
A/D converter	Lab-master
Datacorder	KYOWA RTP-501AL (max. freq. 2.5kHz)
Computer	IBM-PC

Table 2 Cutting conditions

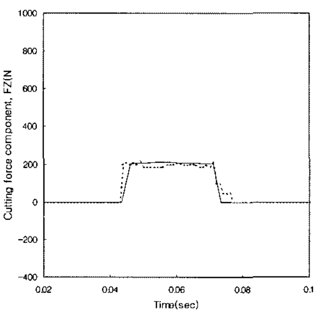
Cutter body	V (rpm)	d (mm)	f (mm/min)	CASE
Diameter 10mm Approach angle 15° Axial rake angle 7° Radial rake angle 0°	320	2	48	1
			64	2



(a) FX

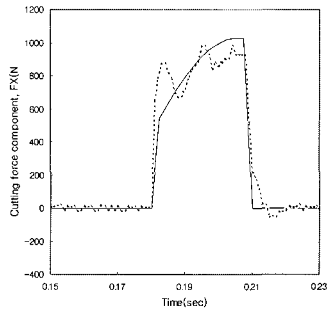


(b) FY

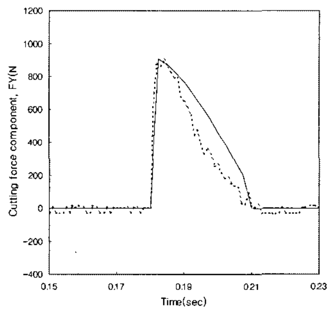


(c) FZ

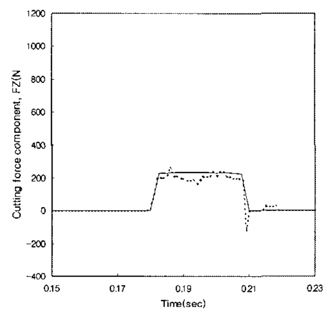
Fig. 7 Predicted(solid line) and experimental(dot line) cutting force components for CASE 1



(a) FX



(b) FY



(c) FZ

Fig. 8 Predicted(solid line) and experimental(dot line) cutting force components for CASE 2

6. Conclusions

In this research, an effective method is presented to predict the cutting forces in face milling process in which cutting conditions vary continuously. From the knowledge of the work material properties and the cutting conditions, without any machining test, the method enables us to calculate accurately magnitude and shape of cutting force pulses. To validate the theoretical analysis, experiments are performed and the force components are measured. The measured results and the corresponding theoretical results show close resemblance, which can be effectively utilized to design a new tool or to plan a process in face mill.

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