

# Comparison between Direct and Indirect Implementation of Generalized Hoek and Brown Failure Criterion in Numerical Analysis Procedure

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## 범용 Hoek-Brown 파괴기준식의 직접 및 간접적 적용에 관한 수치해석과정의 비교 분석

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**Abstract** Friction angle and cohesion of rock masses can be estimated from Hoek and Brown failure criterion and then plastic corrections can be applied using Mohr-Coulomb yield function. This study finds that this estimation procedure would not be appropriate for weak rock masses and for cases where low confining stress is expected to develop. A procedure is outlined in this paper for estimating plastic corrections directly from Hoek and Brown material model. Comparative study shows that direct procedure would simulate non-linear failure surface better than indirect procedure especially in the low confining stress regime.

**KeyWords** Hoek and Brown failure criterion, Mohr-Coulomb yield function, plastic correction, direct procedure, indirect procedure

**초 록** Hoek-Brown의 파괴기준식으로부터 암반의 내부마찰각 및 점착강도를 계산한 후, Mohr-Coulomb의 항복함수를 이용하여 소성 보정이 적용될 수 있는 것으로 알려져 있다. 하지만 본 연구에서는 이러한 계산 과정이 연약 암반이나 낮은 봉압 조건의 암반에 대해서는 적합하지 않다는 사실을 보여주하고자 한다. 즉, Hoek-Brown 재료 모델로부터 직접 및 간접적 적용에 의해 소성 보정을 수행하는 과정을 제시하였으며, 이를 통해 직접적 적용이 간접적 적용에 비해, 비선형 파괴면을 더욱 효과적으로 모사할 수 있고, 특히 봉압이 낮은 응력 조건에서 효과적임을 보여주하고자 한다.

**핵심어** Hoek-Brown 파괴기준식, Mohr-Coulomb 항복함수, 소성 보정, 직접적 적용, 간접적 적용

## 1. Introduction

The Hoek and Brown failure criterion was first published in 1980 and was originally developed for hard rock masses. The strengths and limitations of this criterion were again proposed in an update in 1988<sup>1)</sup>. It was then proposed that the correct use of this criterion should be decided based on the nature of rock mass which does not contain any structural discontinuities and the volume of

rock under consideration might contain four or more closely spaced and almost uniform discontinuity sets. In 1997, a generalized Hoek and Brown failure criterion was published for accommodating hard and soft rock masses<sup>2)</sup>. In that paper, the use of Geological Strength Index (GSI) was proposed instead of Rock Mass Rating (RMR) as in the case of earlier failure criterion. The applicability of this failure criterion was also enumerated in details with some field applications and post-failure behaviour of rock masses was conceptualized as numerical analysis tool. The generalized criterion was again revised in 2002 to do away with some limitations in the earlier version<sup>3)</sup>.

Hoek and Brown rock mass parameters i.e.  $m$  and  $s$  are used to estimate equivalent Mohr-Coulomb friction angle ( $\phi$ ) and cohesion ( $c$ ) for plastic corrections in elasto-plastic numerical analysis procedure<sup>4)</sup>. Yield function

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can be derived from the Mohr-Coulomb or Drucker-Prager criteria for geological material like soil and rock mass once the friction angle and cohesion are known<sup>5,6,7</sup>). Thus, the idea was to estimate equivalent or instantaneous  $c$  and  $\phi$  of rock mass from  $m$ ,  $s$  and uniaxial compressive strength of intact rock,  $\sigma_c$  and then apply Mohr-Coulomb yield function to correct stresses in plastic regime. From now onwards, estimation of equivalent or instantaneous  $c$  and  $\phi$  from Hoek and Brown rock mass parameters and their subsequent application to plastic corrections using Mohr-Coulomb yield function will be termed as "indirect procedure". This indirect procedure is applied in commercial numerical software package like FLAC -2D ver. 4.0 or older if Hoek and Brown material model has to be incorporated in the numerical procedure<sup>7</sup>). In the new FLAC version 5.0, the indirect procedure is replaced by a direct method based on reference<sup>8</sup>).

However, direct formulation of Hoek and Brown rock mass failure criterion was developed to simulate stress and displacement patterns around tunnel opening based on theory of visco-plasticity<sup>9</sup>). In this paper, a procedure is developed to apply plastic corrections directly using generalized Hoek and Brown failure criterion as yield function. This procedure considers non-linear shear yield function with non-associative flow rule and models tensile yield function with associative flow rule. From now onwards, direct implementation of generalized Hoek and Brown yield criterion in elasto-plastic finite element analysis will be termed as "direct procedure". This study has mainly focused on two major objectives. Firstly, this study intend to show that the estimation of equivalent  $c$  and  $\phi$  from Hoek and Brown rock mass parameters may not be appropriate for cases where low compressive or tensile confining stress ( $\sigma_3$ ) is expected to develop. Examples of such cases will be mine roofs or sidewalls at shallow depth coal mines and/or excavations made in weak rock strata. In these conditions, it is better to apply direct procedure for the analysis of stresses using elasto-plastic finite element procedure. This paper provides a step-by-step description of direct procedure for plane strain condition. Secondly, a comparative study between direct and indirect procedures is also portrayed using a numerical example for weak rock mass. This example evaluates that plastic corrections made by direct procedure simulates non-linear yield surface more precisely

than the indirect procedure especially in the low confining stress regime. Lastly, recommendations are given for the proper use of direct and indirect procedures in applications of numerical analysis.

## 2. Generalized hoek and brown rock mass failure driterion

The generalized Hoek and Brown rock mass yield criterion for drained condition is expressed as follows<sup>3</sup>):

$$f^{HB} = \sigma_1 - \sigma_3 - \sigma_c \left( m_b \frac{\sigma_3}{\sigma_c} + s \right)^a \quad (1)$$

where  $\sigma_1$  and  $G_1$  are principal stresses and should be arranged as  $\sigma_1 \leq \sigma_2 \leq \sigma_3$  (considering compressive stress is negative)

$\sigma_c$  = uniaxial compressive strength of intact rock and negative value will be considered.

$m_b$  is reduced material constant based on intact property of rock,  $m_i$  and Geological Strength Index (GSI) and  $s$  and  $a$  are rock mass properties based on GSI<sup>3</sup>).

## 3. Estimation of cohesion and friction angle from Hoek and Brown failure criterion

Adaptation of Mohr-Coulomb model by approximating non-linear failure surface of Hoek and Brown material model (Eq. 1) can be done in two ways:

- i) Estimation of instantaneous  $c$  and  $\phi$  depending on confining stress,  $\sigma_3$ <sup>4</sup>)
- ii) Estimation of average  $c$  and  $\phi$  depending on maximum confining stress which is related to insitu stress<sup>3</sup>)

In general, both of these techniques provide good estimates of  $c$  and  $\phi$  for very strong rock mass type having high  $\sigma_c$ ,  $m_i$  and GSI values. However, for weak rock mass having low compressive strength and low GSI value, equivalent  $c$  and  $\phi$  may not provide satisfactory estimates of rock mass parameters. The estimated value of  $c$  and  $\phi$  will be much worse for low compressive or tensile confining stress which may be developed around an excavation at a shallow depth of cover.

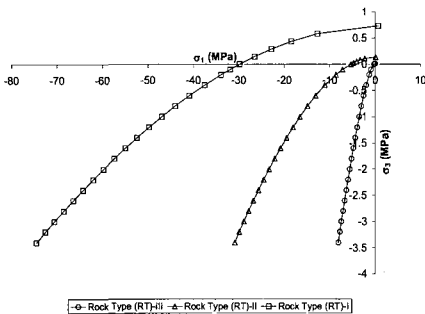
**3.1 Estimation of instantaneous cohesion and friction angle depending on confining stress**

Mohr-Coulomb friction and cohesion for rock masses were estimated from Hoek and Brown failure criterion and were adopted in FLAC 2D version 4.0 or older software<sup>4)</sup>. Estimation of these rock parameters is also postulated for a failure criterion represented by  $\tau = \alpha_1 (\alpha_0 = \alpha_N)^\beta$  where  $\tau$  and  $\alpha_N$  are shear and normal stresses and  $\beta$  and  $\alpha_i$  ( $i = 0, 1$ ) are rock parameters, which relate to friction angle and cohesion<sup>10)</sup>. After estimating equivalent  $c$  and  $\phi$  of rock masses, Mohr-Coulomb failure criterion is applied for plastic corrections. The original concept of estimating  $c$  and  $\phi$  is reformulated for generalized failure criterion as given in Eq. 1. In this study, estimation of instantaneous friction angle and cohesion are conducted based on three rock mass types viz. very strong (RT-I), strong (RT-II), and weak (RT-III). The properties of rock masses are given in Table 1. These material properties are chosen such a way that three different rock mass types can be distinguished easily. Fig. 1 depicts Hoek and Brown failure surfaces in  $\sigma_3 - \alpha_1$  plane of these three rock mass types.

Estimation of Mohr-Coulomb  $c$  and  $\phi$  of rock masses begins with the fundamental assumption that for a given confining stress  $\sigma_3$ , the tangent of Eq. 1 will represent the equivalent Mohr-Coulomb surface

**Table 1.** Hoek and Brown parameters of three rock mass types

Rock Type $s$	$\sigma_c$ (MPa)	$m_i$	GSI
Very Strong (RT-I)	100	25	75
Strong (RT-II)	50	18	60
Weak (RT-III)	10	7	35



**Fig. 1.** Hoek and Brown failure surfaces of three rock types

as given below :

$$\sigma_1 - \sigma_3 N_\phi + \sigma_c^M = 0 \tag{2}$$

where  $N_\phi = \tan^2\left(\frac{\phi}{2} + \frac{\pi}{4}\right)$ . The tangent of both Eq. 1 and Eq. 2 can be written as

$$\frac{\partial \sigma_3}{\partial \sigma_1} = \frac{1}{N_\phi} = \frac{1}{1 + a m_b (m_b \sigma_3 / \sigma_c + S)^{a-1}} \tag{3}$$

Substituting  $\sigma_1$  in Eq. 1,  $\sigma_c^M$  can be estimated as

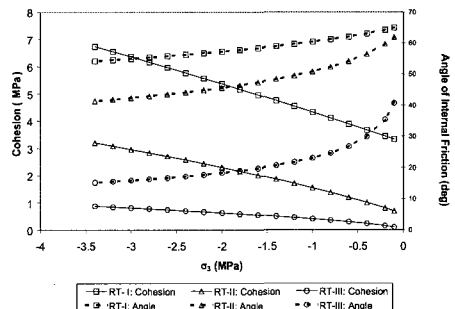
$$\sigma_c^M = -\sigma_3(1 - N_\phi) - \sigma_c (m_b \sigma_3 / \sigma_c + S)^a \tag{4}$$

From Eq. 3 and Eq. 4, the equivalent  $c$  and  $\phi$  can be estimated for a given  $\sigma_3$  as

$$\phi = 2 \tan^{-1} \sqrt{N_\phi} - \frac{\pi}{2}$$

$$c = \frac{\sigma_c^M}{2\sqrt{N_\phi}} \tag{5}$$

It can be seen that both  $c$  and  $\phi$  are functions of confining stress,  $\sigma_3$ . Fig. 2 shows the variations of  $c$  and  $\phi$  for these three rock masses with  $\sigma_3$  in the compressive regime. It is clear that as  $\sigma_3$  decreases, the value of  $c$  and  $\phi$  increases. For weak rock mass, angle  $\phi$  drops over 20 degrees as the confining stress changes from -0.2 MPa to -3.4 MPa. Moreover, for positive (tensile) confining stress the friction angle may be projected as high as 80 to 90 degrees for all three rock mass types. In addition, for the similar range of  $\sigma_3$ , cohesion increases over 3 MPa for very



**Fig. 2.** Relationship between confining stress and equivalent cohesion and friction angle

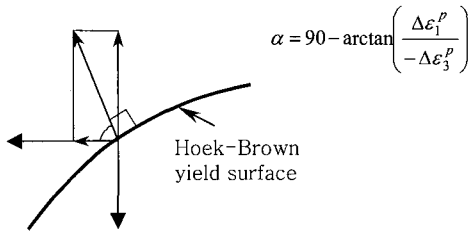


Fig. 3. Direction of plastic strain increment vector

strong rock mass type. This high variability leads to inaccurate stress correction in plastic regime and will be discussed later.

However, high value of friction angle for low level of confining stress can be viewed as a special case in rock mechanics problems. It was shown that for low level of confinement in non-linear yield criterion along with associated flow rule the direction of plastic strain increment vector represents either axial splitting of rock sample in unconfined compression test<sup>8)</sup> or formation of tension cracks in back of slopes according to the limit analysis method of plasticity<sup>11)</sup>.

In the incremental plasticity theory, the plastic strain increment is estimated by differentiating the plastic potential function, Q with respect to stresses as given below :

$$\{\epsilon^p\} = \frac{\partial Q}{\partial \sigma} \lambda^s \tag{6}$$

where  $\lambda^s$  is the plastic multiplier. For associative flow rule,  $Q = f^{HB}$ . Considering Hoek-Brown yield criterion with associated flow rule the principal plastic strain increment can be obtained as

$$\begin{aligned} \Delta \epsilon_1^p &= \frac{\partial f^{HB}}{\partial \sigma_1} \lambda^s \\ \Delta \epsilon_3^p &= \frac{\partial f^{HB}}{\partial \sigma_3} = -\lambda^s \left( 1 + am_b \left( m_b \frac{\sigma_3}{\sigma_c} + s \right)^{a-1} \right) \end{aligned} \tag{7}$$

From above expressions, the direction of plastic strain increment vector can be obtained as shown in Fig. 3.

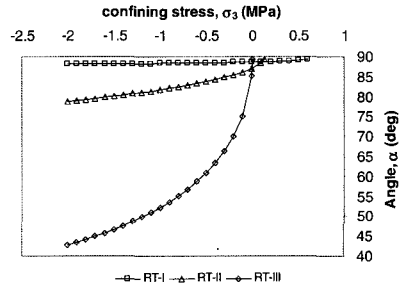


Fig. 4. Directional angle of plastic strain increment vector for low level of confining stress

The directional angle  $\alpha$ , a of the plastic strain increment vector from the major principal axis direction is calculated for 3 rock types as shown in Fig. 4. A value of close to  $90^\circ$  would mean that the formation of crack in the rock sample is almost parallel to the direction of major principal stress signifying the axial splitting on an unconfined compression test. It can be seen that for stronger rock, RT-I, axial splitting is much more evident for low level of confining stress as compared to weak rock type, RT-III.

### 3.2 Estimation of average cohesion and friction angle depending on the maximum confining stress

Hoek *et al.* (2002) have estimated Mohr-Coulomb friction angle and cohesion based on equivalent characteristics of both the failure criteria<sup>3)</sup>. In this context, a value of c and  $\phi$  are calculated depending on the maximum confining stress,  $\sigma_{3max}$  which has to be determined for each individual case. The maximum confining stress depends on the depth of excavation or tunnel and also related to “rock mass strength” which is defined in their paper.

However, this technique is not applicable for shallow depth mining conditions e.g. block caving method or shallow depth coal mines and for weak rock mass where failure zone may extend up to the surface. Hence, this procedure as well as comparative studies will not be discussed in this paper.

### 4. Implementation of Hoek and Brown material model in finite element analysis

Mohr-Coulomb failure criterion has been implemented

in numerical analysis procedure and was published in many literatures<sup>5,6,7</sup>). Using the similar implementation methodology of Mohr-Coulomb failure criterion as given in “theory and background manual” of FLAC 2D version 4.0 software, author has developed a procedure for the implementation of Hoek and Brown material model in finite element analysis procedure as discussed below. The similar procedure is maintained to compare results obtained using both direct and indirect methods.

Fig. 5 shows four different zones, namely shear yield region, tensile yield region, safe region and infeasible region in  $\sigma_3 - \sigma_1$  plane. The shear yield region is designated where Hoek and Brown yield criterion has been violated while tensile yield zone is assumed where confining stress exceeds user-defined or the maximum tensile strength of rock mass. The safe region implies the stress point in  $\sigma_3 - \sigma_1$  plane where both tensile and shear yield criteria are not satisfied. The infeasible region signifies that  $\sigma_1$  cannot exceed  $\sigma_3$  or the value of  $\sigma_3$  cannot be lower than the value of  $\sigma_1$ .

In direct procedure, both shear and tensile yield criteria have also been considered. Shear yield criterion is applied using Eq. 1 with non-associative flow rule. Mohr-Coulomb form of plastic potential function with a variable plastic parameter (a function of plastic shear strain) was applied with Hoek and Brown yield function for the analysis of ground reaction curve of rock masses<sup>12</sup>). Analytical findings of that study were also compared with the results obtained from FLAC 2D. In this study, similar potential function is adopted as given in Eq. 8 with constant parameter,  $N_\psi$  for deriving

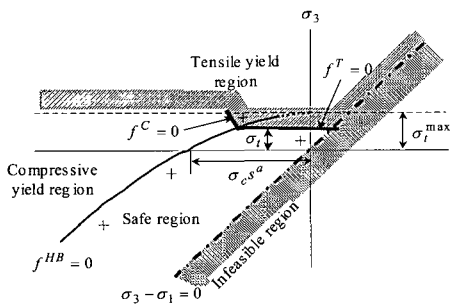


Fig. 5. Different regions in  $\sigma_3 - \sigma_1$  plane with Hoek and Brown yield surface

plastic corrections with Hoek and Brown yield criterion.

$$Q^S = \sigma_1 - \sigma_3 N_\psi \tag{8}$$

where  $N_\psi = \tan^2\left(\frac{\psi}{2} + \frac{\pi}{4}\right)$  and  $\psi$  = dilation angle.

Tensile yield criterion is given in Eq. 9 and plastic correction is applied with associative flow rule using plastic potential function as given in Eq. 10.

$$f^T = \sigma_t - \sigma_3 = 0 \tag{9}$$

$$Q^T = -\sigma_3 \tag{10}$$

In this formulation, it is considered that if the value of  $\sigma_3$  exceeds the maximum tensile strength estimated using Eq. 11, tensile failure will be declared irrespective of  $s_1$  value.

$$\sigma_t^{\max} = -\frac{s \sigma_c}{m} \tag{11}$$

A composite yield criterion is developed to demark between shear and tensile yield regions by bisecting the tangent at point C as shown in Fig. 6. The equation of composite yield criterion is given in Eq. 12.

$$f^C = \sigma_3 - \sigma_t + l_p(\sigma_1 - n_p) = 0 \tag{12}$$

where

$$l_p = \sqrt{1 + K^2} + K$$

$$K = 1 + am_b(m_b \sigma_t / \sigma_c + s)^{a-1}$$

$$n_p = \sigma_t + \sigma_c(m_b \sigma_t / \sigma_c + s)^a$$

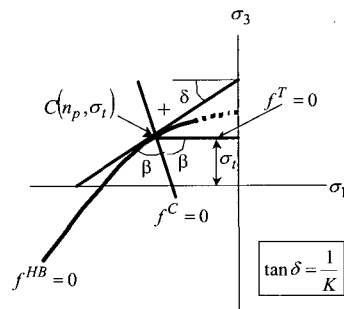


Fig. 6. Composite yield criterion for Hoek and Brown material model

#### 4.1 Plastic corrections

Plastic corrections are applied in the similar fashion as performed in FLAC 2D ver. 4 for Mohr- Coulomb criterion<sup>4)</sup>. The procedure is briefly discussed below.

For shear yielding case, plastic strain increments are obtained using Eq. 13 as given below :

$$\Delta \epsilon_i^p = \lambda^s \frac{\partial \mathbf{Q}^s}{\partial \sigma_i} \quad i = 1, 3 \quad (13)$$

where  $\lambda^s$  is plastic multiplier and has to be determined for new stresses that lie on the yield surface. Considering the total strain increment can be divided into elastic and plastic strain increments and stress increment can only occur due to change in elastic strain increment, new state of stress can be written as :

$$\begin{aligned} \sigma_1^N &= \sigma_1^I - \lambda^s (\alpha_1 - \alpha_2 N_\psi) \\ \sigma_2^N &= \sigma_2^I - \lambda^s \alpha_2 (1 - N_\psi) \\ \sigma_3^N &= \sigma_3^I - \lambda^s (-\alpha_1 N_\psi + \alpha_2) \end{aligned} \quad (14)$$

where  $\alpha_1 = K + 4/3G$  and  $\alpha_2 = K - 2/3G$ . The parameters  $K$  and  $G$  are bulk and shear modulus of the material. The stresses expressed by  $\sigma_i^I$  ( $i = 1, 3$ ) are trial stresses obtained from total stress increment prior to plastic correction.

For tensile failure case, the flow rule is obtained from Eq. 14 with plastic multiplier  $\lambda^T$ .

$$\Delta \epsilon_i^p = \lambda^T \frac{\partial \mathbf{Q}^T}{\partial \sigma_i} \quad (15)$$

With the same reasoning as given above, the new state of stress can be obtained as

$$\begin{aligned} \sigma_1^N &= \sigma_1^I + \lambda^T \alpha_2 \\ \sigma_2^N &= \sigma_2^I + \lambda^T \alpha_2 \\ \sigma_3^N &= \sigma_3^I + \lambda^T \alpha_1 \end{aligned} \quad (16)$$

#### 4.2 Finite element procedure with plane strain condition

Step 1. For each load step and for every iteration within a load step, compute stress increment for each gauss point of an element using following calculations.

$$\{\Delta \sigma\} = [\mathbf{D}]\{\Delta \epsilon\} \quad (17)$$

where  $\{\Delta \epsilon\}^T = \{\Delta \epsilon_x \quad \Delta \epsilon_y \quad \Delta \gamma_{xy}\}$  and

$$\{\Delta \sigma\}^T = \{\Delta \sigma_x \quad \Delta \sigma_y \quad \Delta \tau_{xy}\}$$

$[\mathbf{D}]$  is the constitutive matrix of the element

Step 2. Update trial stress vector as  $\{a\}^I = \{a\}^O + \{\Delta \sigma\}$  where  $\{a\}^O$  is the stress vector of last iteration. The out-of-plane stress,  $\sigma_z$  is calculated as  $\sigma_z = \nu(\sigma_x + \sigma_y)$ ,  $\nu$  being Poisson's ratio.

Step 3. Calculate principle stresses and their direction cosines from trial stress vector. Arrange principle stresses in the order  $\sigma_1 \leq \sigma_2 \leq \sigma_3$  recognizing  $\sigma_z$  will be one of the principle stresses.

Step 4. Calculate the functions  $f^T$  and  $f^C$  using Eq. 9 and Eq. 12. If the point designated by  $\sigma_1$  and  $\sigma_3$  falls inside the tensile failure region (Fig. 5) i.e  $\sigma_3$  exceeds  $\sigma_t$  or  $f^C$  is positive and simultaneously  $f^T$  is negative, tensile yield is declared. Plastic multiplier  $\lambda^T$  is then calculated from Eq. 18 and plastic corrections are applied to obtain new principle stresses using Eq. 16.

$$\lambda^T = \frac{f^T(\sigma_3^I)}{\alpha_1} \quad (18)$$

Step 5. If trial stress falls inside compressive failure region (Fig. 5), plastic multiplier  $\lambda^S$  is obtained by solving Eq. 19 such that new state of stress lies on the yield surface given by Eq. 1.

$$\begin{aligned} \sigma_1^N - \sigma_3^N - [m_b \sigma_3^N / \sigma_c + s]^2 &= 0 \\ \sigma_1^N - \sigma_3^N + \mathcal{L}(1 + N_\psi)(\alpha_2 - \alpha_1) - \sigma_c [m_b (\sigma_3^N - \mathcal{L}(\alpha_2 - \alpha_1 N_\psi)) / \sigma_c + s]^2 &= 0 \end{aligned} \quad (19)$$

The above equation is solved using Newton-Rapson method with the following constraint :

$$\sigma_3^I - \lambda^S (\alpha_2 - \alpha_1 N_\psi) \leq \sigma_t^{\max} \quad (20)$$

After solving  $\lambda^S$ , new principle stress can be obtained using Eq. 14. If the trial stress falls inside

the safe region, no plastic correction is applied and new principle stresses are updated with trial stresses.

Step 6. Assuming the direction of principle stresses would not change due to plastic corrections, stresses at Cartesian coordinate system can be obtained using the direction cosines obtained in Step 3.

## 5. Performance of direct and indirect procedures for stress corrections

The direct and indirect implementation procedures are compared using an example of weak rock mass (Rock Type-III) having material properties as shown in Table 2. Since it is found that weak rock mass is more sensitive to indirect procedure, this example considered for weak rock mass only. For indirect procedure, Mohr-Coulomb criterion is used as yield function. For this example, all calculations are performed using Excel spreadsheet software.

Three trial stress points, A, B and C in  $\sigma_3 - \sigma_1$  plane are assumed such that they fall in compressive failure region as given in Table 3. Note that any point in that region could have been considered for the analysis. These three points are chosen having low to high confining stresses to show the performance of direct and indirect procedures. Fig. 7 shows the location of points and their new positions based on plastic correction applied with both direct and indirect procedures. It is clear that new stress points obtained using direct procedure lie on the failure surface as desired. However, with indirect procedure new stress points still lie inside the compressive yield region

even after plastic correction is applied. Moreover, the location of new stress point is further away from the yield surface for lower  $\sigma_3$  value (Point A) causing more error in estimation. For higher confining stress, the difference between direct and indirect estimation of new stress point seems to diminish (Point C).

Similar performance is also expected for stronger rock mass having less difference in the results obtained from direct and indirect procedures.

## 6. Conclusions

This paper has attempted to identify the problems associated with the numerical application of using Mohr-Coulomb instantaneous cohesion intercept and friction angle estimated from Hoek and Brown material model. A numerical procedure has been explained in detail to apply Hoek and Brown material model directly in finite element analysis. Comparative studies between direct and indirect

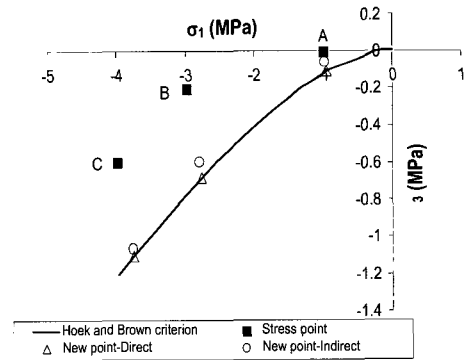


Fig. 7. Comparison between direct and indirect procedures for stress correction

Table 2. Properties of weak rock mass

Modulus of elasticity (GPa)	Poisson's ratio	Dilation angle (deg)	$\alpha_c$ (MPa)	mi	GSI
1.0	0.3	8.0	10.0	7.0	35

Table 3. Results of direct and indirect procedures for three stress points

Point Name	$\sigma_3^I$ (MPa)	$\sigma_1^I$ (MPa)	$\lambda^s$ Direct ( $\times 10^{-4}$ )	$\sigma_3^N$ Direct (MPa)	$\sigma_1^N$ Direct (MPa)	$\lambda^s$ Indirect ( $\times 10^{-4}$ )	$\sigma_3^N$ Indirect (MPa)	$\sigma_1^N$ Indirect (MPa)
A	0.0	-1.0	-0.9026	-0.1087	-0.9474	-0.4785	-0.0576	-0.9721
B	-0.2	-3.0	-4.0208	-0.6843	-2.7657	-3.2829	-0.5954	-2.8087
C	-0.6	-4.0	-4.1727	-1.1026	-3.7569	-3.8552	-1.0644	-3.7754

procedures showed that indirect application of Hoek and Brown material model in numerical analysis will not provide expected results as in the case of direct application. Based on the studies outlined in this paper, following conclusions can be drawn :

1. Estimation of instantaneous  $c$  and  $\phi$  from Hoek and Brown material model should be avoided if low confining stress is expected. Low confining stress may develop at shallow depth excavations, both in tunnels and mines roofs or sidewalls. The estimated values of  $f$  will be unrealistic for low confining stress regime.
2. If the rock mass under investigation is weak in nature, no attempt should be made to estimate instantaneous  $c$  and  $\phi$  indirectly.
3. For the above two conditions, either Mohr-Coulomb or Hoek and Brown material model should be employed in numerical analysis procedure. Indirect procedure as defined in this paper should be avoided under such conditions.
4. For stronger rock type, indirect estimation of cohesion is more sensitive than that of friction angle. However, for higher confining stress regime, application of indirect procedure as given in FLAC 2D ver. 4.0 or older may provide sufficiently close results as in the case of direct applications. Thus, indirect procedure may be applicable in hard rock tunneling and deep mining conditions.

## References

- [1] Hoek E. and Brown E.T., 1988, The Hoek and Brown Failure Criterion-A 1988 Update, 15<sup>th</sup> Canadian Symposium of Rock Mechanics, Toronto.
- [2] Hoek E. and Brown E.T., 1997, Practical Estimate of Rock Mass Strength, Int. J. of Rock Mech. and Mining Sci. & Geomech. Abs., Vol. 34, No. 8, pp. 1165-1186.
- [3] Hoek E., Torres C.C., and Corkum B., 2002, Hoek and Brown Failure Criterion-2002 Edition, 5<sup>th</sup> North American Rock Mechanics Symposium and 17<sup>th</sup> Tunneling Association of Canada Conference (NARMS-TAC), pp. 267-271.
- [4] Hoek E., 1990, Estimating Mohr-Coulomb Friction and Cohesion Values from the Hoek and Brown Failure Criterion, Int. J. of Rock Mech. and Mining Sci. & Geomech. Abs., Vol 27, No. 3, pp. 227-229.
- [5] Zienkiewicz O.C and Corneau I.C., 1974, Visco-Plasticity-Plasticity and Creep in Elastic Solids-A Unified Numerical Solution Approach, Int. J. of Numerical Methods in Engineering, Vol. 8, pp. 821-845.
- [6] Bathe K.J., 1982, Finite Element Procedures in Engineering Analysis, Prentice Hall Publication, 735p.
- [7] FLAC 2D, version 4.0 : Theory and Background Manual, 2002, Itasca Consulting Group, Inc., Minneapolis, Minnesota, USA.
- [8] Cundall P., Carranza-Torres C. and Hart R., 2003, A New Constitutive Model Based on the Hoek-Brown Failure Criterion, Proceedings of the 3<sup>rd</sup> International FLAC Symposium, *FLAC and Numerical Modeling in Geomechanics*, Sudbury, Canada, October 21-24, Balkema.
- [9] Srivastava R.K., Sharma K.G. and Varadarajan A., 1995, Finite Element Analysis of Circular, Horse-Shoe and Interacting Circular Tunnel, J. of Rock Mechanics and Tunneling Technology, Vol. 1, No. 2, pp. 109-133.
- [10] Mostafa M.E., 2003, General Rock Failure Criterion, Mining Technology, Vol. 112, pp. A64-A68.
- [11] Chen W.F. and Liu X.L., 1990, Limit Analysis in Soil Mechanics, Developments in Geotechnical Engineering, Vol. 52.
- [12] Alonoso E, Alejano L.R., Varas F., Fdez-Manin G. and Carranza-Torres C., 2003, Ground Response Curves for Rock Masses Exhibiting Strain-Softening Behaviour, Int. Journal for Numerical and Analytical Methods in Geomechanics, Vol. 27, pp. 1153-1185.



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