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FUZZY CONVERGENCE THEORY - II

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ABSTRACT. In this paper convergence of fuzzy filters and graded fuzzy filters have been studied in graded L-fuzzy topological spaces.

0. INTRODUCTION

This paper is the continuation of our earlier paper (Mondal & Samanta [10]) where convergence of fuzzy nets has been studied. In this paper we deal with the convergence of fuzzy filters. In 1979 a theory of convergence of fuzzy filters was developed by Lowen [9] for laminated spaces and afterwards it was extended to arbitrary fuzzy (Chang) spaces by Warren [13]. In 1995 Gahler [6, 7] introduced an idea of graded fuzzy filter in lattice valued setting (which he called L-fuzzy filter) and studied its convergence in Chang fuzzy topological spaces. Later on in the year of 1999 Burton, Muraleetharan & Garcia [1, 2] considered another type of graded fuzzy filter named as generalized filter (g-filter) by relaxing a condition imposed by Gahlar [6, 7] but restricted themselves in I-fuzzy setting where $I = [0, 1]$ and studied relations among prime prefilters, prime g-filters and ultrafilters.

In this paper we study the convergence of both crisp fuzzy filters and graded fuzzy filters in L-fuzzy setting, where the underlying fuzzy topological space is a graded L-fuzzy topological space of the type as considered in Chattopadhyay, Hazra & Samanta [4], Höhle [8], and Šostak [12].

In Section 2 we study the graded convergence of Warren type fuzzy filters (*cf.* Warren [13]) and investigate its relation with the graded convergence of associated fuzzy nets.

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In Section 3 we deal with the convergence of g-filters. In doing so we have established decomposition theorem involving the convergence of a g-filter with the convergence of a family of Warren type fuzzy filters. Relationship between the convergence of g-filters and gp-mappings has been studied.

1. NOTATION AND PRELIMINARIES

In this paper X denotes a nonempty set; unless otherwise mentioned, L denotes a completely distributive order dense complete lattice with an order reversing involution $'$ whereas $L_0 = L \setminus \{0\}$. Let 0 and 1 denote respectively the least and the greatest elements of L. Let L^X be the collection of all L-fuzzy subsets of X and $\mathrm{Pt}(L^X)$ the set of all L-fuzzy points of X. $M(L)$ denotes the set of all molecules of L whereas $M(L^X)$ denotes the set of all molecule points of L^X. By $\tilde{0}$ and $\tilde{1}$ we denote the constant L-fuzzy subsets of X taking values 0 and 1 respectively. For $p_x \in \mathrm{Pt}(L^X)$ and $A, B \in L^X$ we say $p_x \,\mathrm{q}\, A$ if $p_x \not\leq A'$ and $A \,\mathrm{q}\, B$ if $A \not\subseteq B'$. For other notations we follow Liu [14].

Definition 1.1 (Šostak [12]). A function $\tau : L^X \to L$ is called an *L-fuzzy topology on X* if it satisfies the following conditions:
(O1) $\tau(\tilde{0}) = \tau(\tilde{1}) = 1$,
(O2) $\tau(A_1 \wedge A_2) \geq \tau(A_1) \wedge \tau(A_2)$, for $A_1, A_2 \in L^X$, and
(O3) $\tau(\bigvee_{i\in\Delta} A_i) \geq \bigwedge_{i\in\Delta} \tau(A_i)$ for any $\{A_i\}_{i\in\Delta} \subset L^X$.
The pair (X, τ) is called an *L-fuzzy topological space* and τ is also called a *gradation of openness* on X.

Definition 1.2 (Šostak [12]). A function $\mathcal{F} : L^X \to L$ is called an *L-fuzzy co-topology of X* if it satisfies the following conditions:
(C1) $\mathcal{F}(\tilde{0}) = \mathcal{F}(\tilde{1}) = 1$,
(C2) $\mathcal{F}(A_1 \vee A_2) \geq \mathcal{F}(A_1) \wedge \mathcal{F}(A_2)$, for $A_1, A_2 \in L^X$, and
(C3) $\mathcal{F}\Big(\bigwedge_{i\in\Delta} A_i\Big) \geq \bigwedge_{i\in\Delta} \mathcal{F}(A_i)$ for any $\{A_i\}_{i\in\Delta} \subset L^X$.
The pair $(X, \mathcal{F})$ is called an *L-fuzzy co-topological space* and $\mathcal{F}$ is also called a *gradation of closedness* on X.

Definition 1.3 (Mondal & Samanta [10]). Let (X, τ) be an L-fuzzy topological space and let $Q : \mathrm{Pt}(L^X) \times L^X \to L$ be a mapping defined by

$$Q(p_x, A) = \bigvee\{\tau(U);\ p_x \,\mathrm{q}\, U \subseteq A\}.$$

Then Q is said to be a *gradation* of q-neighborhoodness in (X, τ).

Definition 1.4 (Mondal & Samanta [10]). Let (X, τ) be an L-fuzzy topological space and let $Q : \mathrm{Pt}(L^X) \times L^X \to L$ be a mapping defined by

$$Q(p_x, A) = \bigvee \{\tau(U);\ p_x \,\mathrm{q}\, U \subset A\}.$$

Then Q is said to be a *gradation* of q-neighborhoodness.

Proposition 1.5 (Mondal & Samanta [10]). *Let Q be a gradation of q-neighbourhoodness in an L-fuzzy topological space (X, τ). Then*

(QN1): $\forall\, p_x \in \mathrm{Pt}(L^X),\ Q(p_x, \tilde{1}) = 1,\ Q(p_x, \tilde{0}) = 0.$

(QN2): $Q(p_x, A) \le Q(p_x, B)$ *if* $A, B \in L^X,\ A \subseteq B.$

(QN3): $\forall\, p_x \in \mathrm{Pt}(L^X)$ *and* $\forall\, A, B \in L^X,\ Q(p_x, A \wedge B) = Q(p_x, A) \wedge Q(p_x, B).$

(QN4): $Q(p_x, A) \not\le k$ *implies that there exists a* $B_p \in L^X$ *such that* $p_x \,\mathrm{q}\, B_p \subseteq A$ *and* $\bigwedge_{(r_y \,\mathrm{q}\, B_p)} Q(r_y, B_p) \not\le k.$

Proposition 1.6 (Mondal & Samanta [10]). *Let $Q : \mathrm{Pt}(L^X) \times L^X \to L$ be a mapping satisfying* (QN1)–(QN3) *of* Proposition 1.5. *Let $\bar{\tau} : L^X \to L$ be defined by $\bar{\tau}(A) = \bigwedge_{(p_x \, q \, A)} Q(p_x, A)$. Then $(X, \bar{\tau})$ forms an L-fuzzy topological space. If further the condition* (QN4) *of* Proposition 2.4 *is satisfied by Q then the mapping $\bar{Q} : \mathrm{Pt}(L^X) \times L^X \to L$ defined by*

$$\bar{Q}(p_x, A) = \bigvee \{\bar{\tau}(U);\ p_x \,\mathrm{q}\, U \subset A\}$$

is identical with Q.

Proposition 1.7 (Mondal & Samanta [10]). *Let Q be a gradation of q-neighbourhoodness in an L-fuzzy topological space (X, τ) and $\bar{\tau} : L^X \to L$ be defined by $\bar{\tau}(A) = \bigvee_{(p_x \,\mathrm{q}\, A)} Q(p_x, A)$ then $\bar{\tau}$ is an L-fuzzy topology on X and $\bar{\tau} = \tau$.*

Definition 1.8 (Mondal & Samanta [10]). Let (X, τ) be an L-fuzzy topological space and $e \in \mathrm{Pt}(L^X)$. The q-neighborhood system of the fuzzy point e with respect to the Chang fuzzy topology τ_r, denoted by $\tilde{Q}_r(e)$, is defined by $\tilde{Q}_r(e) = \{U \in L^X;\ \exists\, V \in \tau_r \text{ satisfying } e \,\mathrm{q}\, V \subseteq U\}$.

Definition 1.9 (Mondal & Samanta [10]). Let (X, τ) be an L-fuzzy topological space and $N : \mathrm{Pt}(L^X) \times L^X \to L$ be a mapping defined by

$$N(p_x, A) = \bigvee \{\tau(U);\ p_x \in U \subseteq A\}.$$

Then N is said to be a *gradation of neighborhoodness* in (X, τ).

Definition 1.10. Let (X,τ) be an L-fuzzy topological space and $e \in \mathrm{Pt}(L^X)$. The neighborhood system of the fuzzy point e with respect to the Chang fuzzy topology τ_r, denoted by $\tilde{N}_r(e)$, is defined by

$$\tilde{N}_r(e) = \{U \in L^X;\ \exists\ V \in \tau_r \text{ satisfying } e \in V \subseteq U\}.$$

Definition 1.11 (Liu [14]). Let L be a complete lattice. Define a relation '$<<$' in L as follows: $\forall\ a, b \in L$, $a << b$ if and only if $\forall\ S \subset L$, $\bigvee S \geq b \Rightarrow \exists\ s \in S$ such that $s \geq a$, $\forall\ a \in L$, denote $\beta(a) = \{b \in L; b << a\}$, $\beta^0(a) = M(\beta(a))$.

Definition 1.12 (Chattopadhyay, Hazra & Samanta [4]). Let (X,τ) and (Y,δ) be two L-fuzzy topologies and $f : X \to Y$ be a mapping. Then f is called a *gradation preserving map* (gp-map) if for each $B \in L^Y, \delta(B) \leq \tau(f^{-1}(B))$.

2. Fuzzy Filter and its Convergence

Definition 2.1. Let X be a nonempty crisp set. A *fuzzy filter* on L^X is a non-empty family $\mathcal{G}$ of L-fuzzy subsets of X such that

(i) $\tilde{0} \notin \mathcal{G}$,
(ii) $\mathcal{G}$ is closed under finite intersection, and
(iii) $\forall\ A, B \in L^X$ if $B \in \mathcal{G}$ and $B \subset A$ then $A \in \mathcal{G}$.

Example 2.2. Let (X,τ) be an L-fuzzy topological space with τ as a gradation of openness on X, $e \in M(L^X)$. Then, for every $r \in L_0$, $\tilde{Q}_r(e)$ and $\tilde{N}_r(e)$ are fuzzy filters on L^X.

Example 2.3. Let X be an infinite crisp set then for each $r \in L_0$ the collection $\{A \in L^X;\ A'_{r'}$ is finite$\}$ is a fuzzy filter on L^X where $A'_{r'}$ is the r'-cut of A'.

Definition 2.4. Let (X,τ) be an L-fuzzy topological space $\mathcal{G} \subset L^X$ be a fuzzy filter on L^X, $e \in \mathrm{Pt}(L^X)$. Then e is called a *cluster point* of $\mathcal{G}$ of *upper grade* l (respectively, *lower grade* k), denoted by $\mathcal{G} \propto^l e$ (respectively, $\mathcal{G} \propto_k e$), if

$$l' = \bigwedge\{r \in L_0;\ U \cap A \neq \tilde{0},\ \forall\ U \in \tilde{Q}_r(e) \text{ and } A \in \mathcal{G}\}$$

(respectively, if

$$k' = \bigvee\{r \in L_0;\ \exists\ U \in \tilde{Q}_r(e) \text{ and } \exists\ A \in \mathcal{G} \text{ such that } A \cap U = \tilde{0}\}).$$

And e is called a *limit point* of $\mathcal{G}$ of *upper grade* l (respectively, *lower grade* k), denoted by $\mathcal{G} \rightarrow^{l} e$ (respectively, $\mathcal{G} \rightarrow_{k} e$), if $l' = \bigwedge\{r \in L_0;\ \tilde{Q}_r(e) \subset \mathcal{G}\}$ (respectively, $k' = \bigvee\{r \in L_0;\ \tilde{Q}_r(e) \not\subseteq \mathcal{G}\}$).

Proposition 2.5. *For any fuzzy filter $\mathcal{G}$ in an L-fuzzy topological space (X, τ), we have the following properties.*

(i) $\mathcal{G} \propto^{l} e$ *and* $\mathcal{G} \propto_{k} e \;\Rightarrow\; k \not> l$.
(ii) $\mathcal{G} \rightarrow^{l} e$ *and* $\mathcal{G} \rightarrow_{k} e \;\Rightarrow\; k \not> l$.

Proof. (i) Let $\mathcal{U} = \{r \in L_0;\ \forall\ U \in \tilde{Q}_r(e)$ and $\forall\ V \in \mathcal{G}, U \cap V \neq \tilde{0}\}$ and $\mathcal{V} = \{r \in L_0;\ \exists\ U \in \tilde{Q}_r(e),\ V \in \mathcal{G};\ U \cap V = \tilde{0}\}$. Then obviously $\mathcal{U} \cap \mathcal{V} = \varnothing$ and $\mathcal{U} \cup \mathcal{V} = L_0$. Also from the definition of limit points of upper grade and lower grade of a fuzzy filter we have $l' = \bigwedge \mathcal{U}$ and $k' = \bigvee \mathcal{V}$. If $\bigwedge \mathcal{U} > \bigvee \mathcal{V}$ then there exists $m \in L_0$ such that $\bigwedge \mathcal{U} > m > \bigvee \mathcal{V} \;\Rightarrow\; m \notin \mathcal{U}$ and $m \notin \mathcal{V}$, which is contradictory to the fact that $\mathcal{U} \cup \mathcal{V} = L_0$. So, $l' = \bigwedge \mathcal{U} > \bigvee \mathcal{V} = k'$ is not possible. This implies $k \not> l$.

(ii) Similar to (i). □

Proposition 2.6. *If L be an order dense chain then, in an L-fuzzy topological space (X, τ), we have the following properties.*

(i) $\mathcal{G} \propto^{l} e$ *and* $\mathcal{G} \propto_{k} e \;\Rightarrow\; k = l$.
(ii) $\mathcal{G} \rightarrow^{l} e$ *and* $\mathcal{G} \rightarrow_{k} e \;\Rightarrow\; k = l$.

Proof. (i) As in Proposition 2.5, if we consider the partitions $\mathcal{U}$ and $\mathcal{V}$ of L_0 and $l' = \bigwedge \mathcal{U}$, $k' = \bigvee \mathcal{V}$ then we have $k \leq l$. If possible let $k < l$ then $k' > l' \;\Rightarrow\; \exists\, m \in L_0$ such that $k' > m > l' \;\Rightarrow\; \bigvee \mathcal{V} > m > \bigwedge \mathcal{U} \;\Rightarrow\; m \in \mathcal{V}$ and $m \in \mathcal{U}$, which is contradictory to the fact that $\mathcal{U} \cap \mathcal{V} = \varnothing$. Hence $k \not< l$.

(ii) Similar to (i) □

Note 2.7. If in addition L is a chain then in the L-fuzzy topological space (X, τ), as there is no difference between $\mathcal{G} \propto^{l} e$ and $\mathcal{G} \propto_{l} e$ so they will be commonly denoted by $\mathcal{G} \propto(l)\ e$. Similarly, $\mathcal{G} \rightarrow^{l} e$ and $\mathcal{G} \rightarrow_{l} e$ will be commonly denoted by $\mathcal{G} \rightarrow(l)\ e$.

Proposition 2.8. *Let (X, τ) be an L-fuzzy topological space with τ as a gradation of openness on X, $\mathcal{G} \subseteq L^X$ be a fuzzy filter on L^X, $e \in \mathrm{Pt}(L^X)$. Then, for $k \in L$, we have*

(i) $\mathcal{G} \rightarrow^{k} e \;\Rightarrow\; \mathcal{G} \propto^{l} e$ *for some* $l \geq k$,
(ii) $\mathcal{G} \propto^{k} e \geq f \;\Rightarrow\; \mathcal{G} \propto^{l} f$ *for some* $l \geq k$,
(iii) $\mathcal{G} \rightarrow^{k} e \geq f \;\Rightarrow\; \mathcal{G} \rightarrow^{l} f$ *for some* $l \geq k$,

(iv) $\mathcal{G} \infty_k e \ \Rightarrow\ \mathcal{G} \rightarrow_l e$ *for some* $l \leq k$,

(v) $\mathcal{G} \infty_k e \leq f \ \Rightarrow\ \mathcal{G} \infty_l f$ *for some* $l \leq k$, *and*

(vi) $\mathcal{G} \rightarrow_k e \leq f \ \Rightarrow\ \mathcal{G} \rightarrow_l f$ *for some* $l \leq k$.

The proof is straightforward.

Definition 2.9. Let (X, τ) be an L-fuzzy topological space and $\mathcal{G}, \mathcal{H}$ be any two fuzzy filters on L^X. Say $\mathcal{H}$ is finer than $\mathcal{G}$ or subfilter of $\mathcal{G}$, or say $\mathcal{G}$ is coarser than $\mathcal{H}$ if $\mathcal{G} \subseteq \mathcal{H}$.

Proposition 2.10. *Let* (X, τ) *be an L-fuzzy topological space and* $\mathcal{G}, H$ *be fuzzy filters on* L^X, $\mathcal{H}$ *be coarser than* $\mathcal{G}$, $e \in \mathrm{Pt}(L^X)$. *Then, for* $k \in L$, *we have*

(i) $\mathcal{H} \rightarrow^k e \ \Rightarrow\ \mathcal{G} \rightarrow^l e$ *for some* $l \geq k$,

(ii) $\mathcal{G} \infty^k e \ \Rightarrow\ \mathcal{H} \infty^l e$ *for some* $l \geq k$,

(iii) $\mathcal{H} \rightarrow_k e \ \Rightarrow\ \mathcal{G} \rightarrow_l e$ *for some* $l \leq k$, *and*

(iv) $\mathcal{G} \infty_k e \ \Rightarrow\ \mathcal{H} \infty_l e$ *for some* $l \leq k$.

Proposition 2.11. *Let* (X, τ) *be an L-fuzzy topological space,* $\mathcal{G}$ *be a fuzzy filter on* L^X, Δ *be the collection of all subfilters of* $\mathcal{G}$, $e \in \mathrm{Pt}(L^X)$. *Then we have*

(i) $\mathcal{G} \rightarrow^l e \ \Rightarrow\ l = \bigwedge_{\mathcal{H} \in \Delta}\{r \in L;\ \mathcal{H} \rightarrow^r e\}$,

(ii) $\mathcal{G} \infty^l e \ \Rightarrow\ l = \bigvee_{\mathcal{H} \in \Delta}\{r \in L;\ \mathcal{H} \infty^r e\}$,

(iii) $\mathcal{G} \infty(l)\, e \ \Rightarrow\ l = \bigvee_{\mathcal{H} \in \Delta}\{r \in L;\ \mathcal{H} \rightarrow (r)e\}$, *if L is a chain,*

(iv) $\mathcal{G} \infty(l)\, e \ \Rightarrow\ \exists$ *a subfilter* $\mathcal{H}$ *of* $\mathcal{G}$ *such that* $\mathcal{H} \rightarrow (l)e$ *if L is a chain,*

(v) $\mathcal{G} \rightarrow_l e \ \Rightarrow\ l = \bigwedge_{\mathcal{H} \in \Delta}\{r \in L;\ \mathcal{H} \rightarrow_r e\}$, *and*

(vi) $\mathcal{G} \infty_l e \ \Rightarrow\ l = \bigvee_{\mathcal{H} \in \Delta}\{r \in L;\ \mathcal{H} \infty_r e\}$.

Proof. (i) For, any $\mathcal{H} \in \Delta$, $\mathcal{H} \rightarrow^r e$ and $\mathcal{G} \rightarrow^l e$ implies $r \geq l$, so

$$l \leq \bigwedge_{\mathcal{H} \in \Delta} \{r \in L;\ \mathcal{H} \rightarrow^r e\}.$$

Again as a particular case taking $\mathcal{H} = \mathcal{G}$ we get $l \geq \bigwedge_{\mathcal{H} \in \Delta}\{r \in L;\ \mathcal{H} \rightarrow^r e\}$. Hence the proof follows.

(ii) Similar to (i).

(iii) Let $\mathcal{H}$ be a subfilter of $\mathcal{G}$ such that $\mathcal{H} \rightarrow (r)e$. Then, for every $s > r'$, $\tilde{Q}_s(e) \subseteq \mathcal{H}$. So, $U \in \tilde{Q}_s(e)$ and $V \in \mathcal{G}$ implies $U, V \in \mathcal{H}$ since $\tilde{Q}_s(e), \mathcal{G} \subseteq \mathcal{H}$. This implies $U \cap V \neq \tilde{0}$.

So, for some $l \geq r$, $\mathcal{G} \infty(l)\, e$. Again as $\mathcal{H}$ is any subfilter of $\mathcal{G}$, so $\mathcal{G} \infty(l)\, e \Rightarrow$

$$l \geq \bigvee_{\mathcal{H} \in \Delta} \{r \in L;\ \mathcal{H} \rightarrow (r)e\}.$$

Next let $\mathcal{G}$ $\infty(l)$ e in (X,τ) and let $\mathcal{B} = \mathcal{G} \cup \left(\bigcup_{m>l'} \tilde{Q}_m(e)\right)$. Then $U_1, U_2 \in \bigcup_{m>l'} \tilde{Q}_m(e)$ implies that there exists a $m_1, m_2 \in L_0$ such that $m_1, m_2 > l'$ and $U_1 \in \tilde{Q}_{m_1}(e)$ and $U_2 \in \tilde{Q}_{m_2}(e)$.

Without loss of generality let $m_1 > m_2$ then $U_1, U_2 \in \tilde{Q}_{m_2}$ (as $\tilde{Q}_{m_1}(e) \subseteq \tilde{Q}_{m_2}(e)$). So,

$$U_1 \cap U_2 \in \tilde{Q}_{m_2}(e) \Rightarrow U_1 \cap U_2 \in \bigcup_{m>l'} \tilde{Q}_m(e),$$

i. e., $\bigcup_{m>l'} \tilde{Q}_m(e)$ has the finite intersection property. $\mathcal{G}$ being a fuzzy filter, also has the finite intersection property.

Again $\mathcal{G}$ $\infty(l)$ e implies that, for all $m > l'$, $U \in \mathcal{G}$ and $V \in \tilde{Q}_m(e)$ means $U \cap V \neq \tilde{0}$. Therefore $\mathcal{B} = \mathcal{G} \cup \left(\bigcup_{m>l'} \tilde{Q}_m(e)\right)$ has the finite intersection property. As $\tilde{0} \notin \mathcal{G}$ and $\tilde{0} \notin \bigcup_{m>l'} \tilde{Q}_m(e)$, so $\tilde{0} \notin \mathcal{B}$. Denote the filter generated by $\mathcal{B}$ as $\Uparrow\mathcal{B}$.

So, $\Uparrow\mathcal{B}$ is a subfilter of $\mathcal{G}$. Let $\mathcal{H} = \Uparrow\mathcal{B}$ then, for all $m > l'$, $\tilde{Q}_m(e) \subseteq \mathcal{H}$. This implies that, for some $r \geq l, \mathcal{H} \to (r)e$.

The proofs of (iv)–(vi) are straightforward. □

Definition 2.12. Let (X,τ) be an L-fuzzy topological space, S be a molecule net on $L^X, \mathcal{G}$ be a fuzzy filter on L^X. For S we define the fuzzy filter associated with the net S as the family $\mathcal{G}(S)$ of all fuzzy subsets of X with which the net S eventually quasi-coincides. For $\mathcal{G}$, let $\mathcal{D}(\mathcal{G}) = \{(e,A) \in M(L^X) \times \mathcal{G};\ e \,\mathrm{q}\, A\}$ and equip $\mathcal{D}(\mathcal{G})$ with the relation $\leq$ on it as $\forall\ (e,A),\ (d,B) \in \mathcal{D}(\mathcal{G}),\ (e,A) \leq (d,B) \iff A \supseteq B$. Define the *molecule net associated with the fuzzy filter* $\mathcal{G}$ as the mapping $S(\mathcal{G}) : \mathcal{D}(G) \to M(L^X)$, defined by $S(\mathcal{G})(e,A) = e\ \forall\ (e,A) \in \mathcal{D}(G)$.

Definition 2.13 (Mondal & Samanta [10]). Let (X,τ) be an L-fuzzy topological space and $e \in \mathrm{Pt}(L^X)$. Let D be any directed set and $S : D \to \mathrm{Pt}(L^X)$ be any fuzzy net. For $U \in L^X$ if $\exists\ m \in D$ such that $S(n) \,\mathrm{q}\, U\ \forall\ n \geq m$ holds then we say

$$S \,\mathrm{q}\, U \text{ eventually;}$$

if, for every $m \in D$, there exists $n \in D$ such that $n \geq m$ and $S(n) \,\mathrm{q}\, U$ then we say $S \,\mathrm{q}\, U$ frequently. Call e a *cluster point with upper grade* l, denoted by $S \infty^l e$ (respectively, a *cluster point with lower grade* k, denoted by $S \infty_k e$) of a fuzzy net $S : D \to \mathrm{Pt}(L^X)$, if

$$l' = \bigwedge \{r \in L_0;\ \forall\ U \in \tilde{Q}_r(e),\ U \,\mathrm{q}\, S \text{ frequently}\}$$

(respectively, if $k' = \bigvee \{r \in L_0;\ \exists\ V \in \tilde{Q}_r(e)$ such that $V \not{\mathrm{q}}\, S$ eventually$\}$). Call e a *limit point of upper grade* l of S, denoted by $S \to^l e$ (respectively, a *limit point of* *lower grade* k of S, denoted by $S \rightarrow_k e$) if

$$l' = \bigwedge\{r \in L_0;\ \forall\, U \in \tilde{Q}_r(e),\ U \,\mathrm{q}\, S \text{ eventually}\}$$

(respectively, $k' = \bigvee\{r \in L_0;\ \exists\, V \in \tilde{Q}_r(e)$ such that $V \,\not\mathrm{q}\, S$ frequently$\}$).

Proposition 2.14. *Let (X,τ) be an L-fuzzy topological space, $\mathcal{G}$ be a fuzzy filter on L^X, S be a molecule net in L^X, $e \in \mathrm{Pt}(L^X)$. Then, for $k \in L$, we have the following properties.*

(i) $S \rightarrow^k e \iff \mathcal{G}(S) \rightarrow^k e.$
(ii) $\mathcal{G} \rightarrow^k e \iff S(\mathcal{G}) \rightarrow^k e.$
(iii) $\mathcal{G} \infty^k e \iff S(\mathcal{G}) \infty^k e.$
(iv) $S \infty^k e \Rightarrow \mathcal{G}(S) \infty^l e$ *for some* $l \geq k$.

Proof. (i) $S \rightarrow^k e \iff k' = \bigwedge\{r \in L_0;\ \forall\, U \in \tilde{Q}_r(e),\ S \,\mathrm{q}\, U \text{ eventually}\}$
$\iff k' = \bigwedge\{r \in L_0;\ \forall\, U \in \tilde{Q}_r(e),\ U \in \mathcal{G}(S)\}$
$\iff k' = \bigwedge\{r \in L_0;\ \tilde{Q}_r(e) \subseteq \mathcal{G}(S)\}$
$\iff \mathcal{G}(S) \rightarrow^k e.$

Similarly, we can prove the other results. □

Proposition 2.15. *Let (X,τ) be an L-fuzzy topological space, $\mathcal{G}$ be a fuzzy filter on L^X, S be a molecule net in L^X, $e \in \mathrm{Pt}(L^X)$. Then, for $k \in L$, we have the following properties.*

(i) $S \rightarrow_k e \iff \mathcal{G}(S) \rightarrow_k e.$
(ii) $\mathcal{G} \rightarrow_k e \iff S(\mathcal{G}) \rightarrow_k e.$
(iii) $\mathcal{G} \infty_k e \iff S(\mathcal{G}) \infty_k e.$
(iv) $S \infty_k e \Rightarrow \mathcal{G}(S) \infty_l e$ *for some* $l \geq k$.

Proof. (i) $S \rightarrow_k e \iff k' = \bigvee\{r \in L_0;\ \exists\, U \in \tilde{Q}_r(e)$ such that $S \,\not\mathrm{q}\, U$ frequently$\}$ $\iff k' = \bigvee\{r \in L_0,\ \exists\, U \in \tilde{Q}_r(e)$ such that $U \notin \mathcal{G}(S)\} \iff k' = \bigvee\{r \in L_0;\ \tilde{Q}_r(e) \not\subseteq \mathcal{G}(S)\} \iff \mathcal{G}(S) \rightarrow_k e.$

(ii) $\mathcal{G} \rightarrow_k e \Rightarrow k' = \bigvee\{r \in L_0;\ \tilde{Q}_r(e) \not\subseteq \mathcal{G}\}$.

Now $\tilde{Q}_r(e) \not\subseteq \mathcal{G} \Rightarrow \exists\, U \in \tilde{Q}_r(e)$ such that $U \notin \mathcal{G}$. Then, for every $(f,V) \in D(\mathcal{G})$, $U \not\supseteq V$.

Now $U \not\supseteq V \Rightarrow U' \not\subseteq V' \Rightarrow \exists\, x \in X$ such that $U'(x) \not\leq V'(x)$. As $M(L)$ is a join generating subset of L so there exists $k \in M(L)$ such that $U'(x) \geq k \not\leq V'(x)$ $\Rightarrow k_x \in M(L^X)$ and $k_x \in U'$ but $k_x \notin V' \Rightarrow k_x \,\mathrm{q}\, V$ but $k_x \,\not\mathrm{q}\, U \Rightarrow (k_x, V) \in D(\mathcal{G})$. Again $(k_x, V) \geq (f, V)$ but $S(\mathcal{G})(k_x, V) = k_x \,\not\mathrm{q}\, U \Rightarrow S(\mathcal{G}) \,\not\mathrm{q}\, U$ frequently.

Conversely, if $U \in \mathcal{G}$ then for all $(f,V),(g,U) \in D(\mathcal{G})$ with $(f,V) \geq (g,U)$ we have $V \subseteq U$. Now $f \,q\, V$ and hence $f \,q\, U$. So, $[S(\mathcal{G})(f,V)] \,q\, U$ *i. e.,* $S(\mathcal{G}) \,q\, U$ eventually. Hence $S(\mathcal{G}) \,\not{q}\, U$ frequently $\Rightarrow U \notin \mathcal{G} \Rightarrow \tilde{Q}_r(e) \not\subseteq \mathcal{G}$. So, $U \in \tilde{Q}_r(e)$ and $S(\mathcal{G}) \,\not{q}\, U$ frequently $\Rightarrow \tilde{Q}_r(e) \not\subseteq \mathcal{G}$. So, we can say now that

$$\tilde{Q}_r(e) \not\subseteq \mathcal{G} \iff \exists\, U \in \tilde{Q}_r(e) \text{ such that } S(\mathcal{G}) \,\not{q}\, U \text{ frequently.}$$

Hence,

$$\bigvee\{r \in L_0;\ \tilde{Q}_r(e) \not\subseteq \mathcal{G}\} = \bigvee\{r \in L_0;\ \exists\, U \in \tilde{Q}_r(e) \text{ such that } S(\mathcal{G}) \,\not{q}\, U \text{ frequently}\},$$

i. e., $\mathcal{G} \rightarrow_k e \iff S(\mathcal{G}) \rightarrow_k e$.

(iii) Let, for some $r \in L_0$ there exists $U \in \tilde{Q}_r(e)$ and$V \in \mathcal{G}$ such that $U \cap V = \tilde{0}$. Take $(f,V) \in D(\mathcal{G})$. We shall show that for all $(g,W) \in D(\mathcal{G})$ if $(g,W) \geq (f,V)$ then $S(\mathcal{G})(g,W) \,\not{q}\, U$. Suppose $S(\mathcal{G})(g,W) \,q\, U$, then $g \,q\, U$. Again

$$(g,W) \geq (f,V) \Rightarrow g \,q\, W \subseteq V,$$

so $g \,q\, V$. Therefore $g \in M(L^X)$, $g \,q\, U$ and $g \,q\, V \Rightarrow g \,q\, (U \cap V) \Rightarrow U \cap V \neq \tilde{0}$, a contradiction.

Next let for some $r \in L_0$ $\exists\, U \in \tilde{Q}_r(e)$ such that $S(\mathcal{G}) \,\not{q}\, U$ eventually. Then, there exists $(f,V) \in D(\mathcal{G})$ such that, for all $(g,W) \in D(\mathcal{G})$, $(g,W) \geq (f,V)$. This implies $S(\mathcal{G})(g,W) \,\not{q}\, U$, *i. e.,* $g \,\not{q}\, U$.

Therefore for all $g \,q\, V$ as $(g,V) \geq (f,V)$ so $S(\mathcal{G})(g,V) \,\not{q}\, U$, *i. e.,* $g \,\not{q}\, U$, *i. e.,* $\forall\, g \,q\, V$, $g \,\not{q}\, U$. So, $U \cap V = \tilde{0}$. Thus (iii) is proved.

(iv) Let $U \in \tilde{Q}_r(e)$ and $V \in \mathcal{G}(S)$ be such that $U \cap V = \tilde{0}$. Now $V \in \mathcal{G}(S) \Rightarrow S \,q\, V$ eventually $\Rightarrow \exists\, m \in D$ such that $\forall\, n \geq m$, $S(n) \,q\, V$. We shall show that $S \,\not{q}\, U$ eventually. Suppose $S \,q\, U$ frequently, then $\exists\, p \in D$ such that $p \geq m$ and $S(p) \,q\, U$. Now $S(p) \,q\, V$, $S(p) \,q\, U$ and $S(p) \in M(L^X) \Rightarrow S(p) \,q (U \cap V) \Rightarrow U \cap V \neq \tilde{0}$, a contradiction.

Thus for $U \in \tilde{Q}_r(e)$, $V \in \mathcal{G}(S)$ if $U \cap V = \tilde{0}$ then $S \,\not{q}\, U$ eventually. So,

$$\begin{aligned} l' &= \bigvee\{r \in L_0;\ \exists\, U \in \tilde{Q}_r(e),\ \exists\, V \in \mathcal{G}(S);\ U \cap V = \tilde{0}\} \\ &\leq \bigvee\{r \in L_0;\ \exists\, U \in \tilde{Q}_r(e) \text{ such that } S \,\not{q}\, U \text{ eventually}\} = k', \end{aligned}$$

i. e., $\mathcal{G}(S) \infty_l\, e \Rightarrow S \infty_k\, e$ for some $k \leq l$. □

Definition 2.16 (Mondal & Samanta [10]). Let $(X, \mathcal{F})$ be an L-fuzzy co-topological space with $\mathcal{F}$ as a GC on X. For each $A \in L^X$ we define

$$\mathrm{cl}(A, r) = \bigwedge\{D \in L^X; A \subseteq D;\ D \in \mathcal{F}_r\}$$

where $\mathcal{F}_r = \{C \in L^X; \mathcal{F}(C) \geq r\}$. The operator cl is said to be *L-fuzzy closure operator* in $(X, \mathcal{F})$.

Definition 2.17 (Mondal & Samanta [10]). In an L-fuzzy topological space (X, τ),

$$p_x \in \mathrm{cl}(A, m)$$

if and only if, for all $U \in \tau_m$, $p_x \mathrel{q} U \Rightarrow\ U \mathrel{q} A$.

Proposition 2.18. *Let (X, τ) be an L-fuzzy topological space and $A \in L^X$; $e \in M(L^X)$. Then $e \in \mathrm{cl}(A, k')$ implies that there exists a fuzzy filter $\mathcal{G}$ on L^X such that $A' \notin \mathcal{G}$ and for some $l \geq k$, $\mathcal{G} \to^l e\}$.*

Proof. Let $e \in \mathrm{cl}(A, k')$. Then for every $U \in \tilde{Q}_{k'}(e), U \mathrel{q} A$ (by Proposition 2.17), *i. e.*, for every $U \in \tilde{Q}_{k'}(e)\ \exists\ x^u \in X$ such that $U(x^u) \not\leq A'(x^u)\ \Rightarrow\ A(x^u) \not\leq U'(x^u)$. As $M(L)$ is a join generating subset of L so $\exists\ p^u \in M(L)$ such that $A(x^u) \geq p^u \not\leq U'(x^u)\ \Rightarrow\ p^u_{x^u} \in M(L^X)$ and $p^u_{x^u} \mathrel{q} U$ and $p^u_{x^u} \in A$.

As $e \in M(L^X)$ so $\tilde{Q}_{k'}(e)$ is a directed set with respect to the relation ' $\geq$' defined by $\forall\ U, V \in \tilde{Q}_{k'}(e),\ U \geq V \iff U \subseteq V$. So we define a molecule net $S : \tilde{Q}_{k'}(e) \to M(L^X)$ by $S(U) = p^u_{x^u}$. Then S is a molecule net in A and as $\forall\ U \in \tilde{Q}_{k'}(e),\ U \mathrel{q} A$ so $\forall\ U \in \tilde{Q}_{k'}(e),\ U \mathrel{q} S$ eventually, which implies $\bigwedge\{s \in L_0;\ \forall\ U \in \tilde{Q}_s(e),\ U \mathrel{q} S$ eventually$\} \leq k' \Rightarrow S \to^l e$ for some $l \geq k$.

Now, for the associated filter $\mathcal{G}(S)$, by (i) of Proposition 2.14, $\mathcal{G}(S) \to^l e$. If $A' \in \mathcal{G}(S)$ then S eventually quasi-coincides with A' (*i. e.*, S is eventually not in A), this contradicts the fact that S is a fuzzy net in A. So, $A' \notin \mathcal{G}(S)$. □

Definition 2.19. Let X be nonempty crisp set. A nonempty subfamily $\mathcal{A} \subseteq L^X$ is called a *filter base* on L^X, if $\tilde{0} \notin \mathcal{A}$ and $\mathcal{A}$ is closed under finite intersection. For a filter base $\mathcal{A}$ on L^X, denote the filter generated by $\mathcal{A}$ as $\Uparrow\!\mathcal{A}$.

Definition 2.20. Let (X, τ) be an L-fuzzy topological space, $\mathcal{A}$ a filter base on L^X. An L-fuzzy point $e \in \mathrm{Pt}(L^X)$ is called a *cluster point of $\mathcal{A}$ with upper grade k*, denoted by $\mathcal{A} \infty^k e$ (respectively, a *cluster point of $\mathcal{A}$ with lower grade l*, denoted by $\mathcal{A} \infty_l e$) if $\Uparrow\!\mathcal{A} \infty^k e$ (respectively, if $\Uparrow\!\mathcal{A} \infty_l e$); e is called a *limit point* of $\mathcal{A}$ with upper and lower grades m and n, denoted by $\mathcal{A} \to^m e$ and $\mathcal{A} \to_n e$ respectively if $\Uparrow\!\mathcal{A} \to^m e$ and $\Uparrow\!\mathcal{A} \to_n e$.

Proposition 2.21. *Let (X,τ) and (Y,δ) be any two L-fuzzy topological spaces and let $f:(X,\tau)\to(Y,\delta)$ be a gp-map then for any filter base $\mathcal{A}$ in (X,τ) and $\forall\ e\in$ $\mathrm{Pt}(L^X)$, $\mathcal{A}\to^l e\ \Rightarrow\ f[\mathcal{A}]\to^k f(e)$ for some $k\geq l$.*

Proof. Let $\tilde{Q}_r(e)$ and $\underline{\tilde{Q}}_r(f(e))$ be the q-neighborhood systems of e and $f(e)$ with respect to the Chang fuzzy topologies τ_r and δ_r respectively. Suppose $\mathcal{A}$ is a filter base in (X,τ), $e\in\mathrm{Pt}(L^X)$ and $\mathcal{A}\to^l e$. Let $\tilde{Q}_r(e)\subseteq\uparrow\mathcal{A}$. Then $\forall\ V\in\underline{\tilde{Q}}_r(f(e))$, since f is a gp-map, $f^{-1}(V)\in\tilde{Q}_r(e)\Rightarrow\exists\ A\in\mathcal{A}$ such that $f^{-1}(V)\supseteq A$.

Therefore $V\supseteq ff^{-1}(V)\supseteq f(A)\in f[\mathcal{A}]\Rightarrow V\in f[\mathcal{A}]$. □

Proposition 2.22. *Let $f:(X,\tau)\to(Y,\delta)$ be a mapping where (X,τ) and (Y,δ) be any two L-Fuzzy topological spaces. If, for any fuzzy filter base $\mathcal{A}$ and for any $e\in M(L^X)$,*

$$\mathcal{A}\to^k e\ \Rightarrow\ f[\mathcal{A}]\to_l f(e)\ \text{ for some } l\geq k,$$

then f is a gp-map.

Proof. Suppose f be not a gp-map, then $\exists\ V\in L^Y$ such that $\tau(f^{-1}(V))\not\geq\delta(V)$. Therefore from the order dense property of L we get $k_1,k_2\in L$ such that

$$\tau(f^{-1}(V))\not\geq k_1<k_2<\delta(V).$$

Now we have

$\tau(f^{-1}(V))\not\geq k_1$

$\Rightarrow\ \bigwedge_{e\,q\,f^{-1}(V)}\{Q(e,f^{-1}(V));\ e\in M(L^X)\}\not\geq k_1$

$\Rightarrow\ \exists\ e^0\in M(L^X)$ such that $e^0\,q\,f^{-1}(V)$ and $Q(e^0,f^{-1}(V))\not\geq k_1$

$\Rightarrow\ \bigvee\{\tau(U);\ e^0\,q\,U\subseteq f^{-1}(V)\}\not\geq k_1$

$\Rightarrow\ \forall\ U\in L^X$ with $\tau(U)\geq k_1$ and $e^0\,q\,U$, $U\not\subseteq f^{-1}(V)$

$\Rightarrow\ f^{-1}(V)\notin\tilde{Q}_{k_1}(e^0)$

$\Rightarrow\ ff^{-1}(V)\notin f(\tilde{Q}_{k_1}(e^0))$.

For, if $f^{-1}(V)\notin\tilde{Q}_{k_1}(e^0)$ but $ff^{-1}(V)\in f(\tilde{Q}_{k_1}(e^0))$ then $\exists\ W\in\tilde{Q}_{k_1}(e^0)$ such that $ff^{-1}(V)=f(W)$,

$$V\supseteq ff^{-1}(V)=f(W)$$

$\Rightarrow\ f^{-1}(V)\supseteq W\Rightarrow\ f^{-1}(V)\in\tilde{Q}_{k_1}(e^0)$ (as $e^0\in M(L^X)\ \Rightarrow\tilde{Q}_{k_1}(e^0)$ is a fuzzy filter), a contradiction. So, $f^{-1}(V)\notin\tilde{Q}_{k_1}(e^0)$.

Hence

$$V\supseteq ff^{-1}(V)\notin f(\tilde{Q}_{k_1}(e^0)) \tag{1}$$

Again $e^0 \,\mathrm{q}\, f^{-1}(V) \;\Rightarrow\; f(e^0)\,\mathrm{q}\, V$ and we have $\delta(V) > k_2$. So,

$$V \in \tilde{Q}'_{k_2}(f(e^0)). \tag{2}$$

So, by (1) and (2), we have $f(\tilde{Q}_{k_1}(e^0)) \not\supseteq \tilde{Q}'_{k_2}(f(e^0))$. This means if

$$f[\tilde{Q}_{k_1}(e^0)] \to_l f(e^0)$$

then $l' \geq k_2$. But from the definition of convergence we have if $\tilde{Q}_{k_1}(e^0) \to^k e^0$ then $k \geq k'_1$. This implies $k' \leq k_1$.

Therefore $l' \geq k_2 > k_1 \geq k' \;\Rightarrow\; l < k$, a contradition to the given condition. Hence f is a gp-map. □

3. Lattice Valued Generalized Filter

Definition 3.1 (Burton, Muraleetharan & Gutiérrez [1]). Let $\mathcal{G} : L^X \to L$ be a mapping satisfying

(GF1) $\mathcal{G}(\tilde{0}) = 0$; $\mathcal{G}(\tilde{1}) = 1$,
(GF2) $\forall\, A_1, A_2 \in L^X$, $\mathcal{G}(A_1 \wedge A_2) \geq \mathcal{G}(A_1) \wedge \mathcal{G}(A_2)$, and
(GF3) $\forall\, A, B \in L^X$, $\mathcal{G}(B) \geq \mathcal{G}(A)$ if $A \subset B$,

then $\mathcal{G}$ is said to be a *generalized filter* (g-filter) on L^X.

Example 3.2. Let Q be the gradation of q-neighborhoodness in an L-fuzzy topological space (X, τ), $e \in M(L^X)$. We define a mapping $Q_e : L^X \to L$ by

$$Q_e(U) = Q(e, U), \;\forall\, U \in L^X.$$

Then Q_e is a g-filter on L^X.

Example 3.3. Similarly the mapping $N_e : L^X \to L$ for a particular $e \in \mathrm{Pt}(L^X)$, defined by $N_e(U) = N(e, U)$, $\forall\, U \in L^X$ is a g-filter on L^X where N is the gradation of neighborhoodness in an L-fuzzy topological space (X, τ).

Example 3.4. Let X be an infinite crisp set and let $\mathcal{G} : L^X \to L$ be defined by $\mathcal{G}(A) = \bigvee\{r \in L_0; A'[r'] = \text{ finite}\}$ where $A'[r']$ is an r'-cut of A', then $\mathcal{G}$ is a g-filter on L^X.

Definition 3.5. Let $\mathcal{G}$ and $\mathcal{H}$ be any two g-filters on L^X. We say $\mathcal{G}$ is *coarser* than $\mathcal{H}$ or $\mathcal{H}$ is finer than $\mathcal{G}$ if $\mathcal{G} \leq \mathcal{H}$. In this case $\mathcal{H}$ is also called a *subfilter* of $\mathcal{G}$.

Definition 3.6. Let $\mathcal{G}$ be a g-filter in an L-fuzzy topological space (X,τ) and $e \in \mathrm{Pt}(L^X)$ Call e a *limit point* of $\mathcal{G}$, denoted by $\mathcal{G} \to e$ if $Q(e,U) \leq \mathcal{G}(U)\ \forall\ U \in L^X$, where Q is the gradation of q-neighborhoodness in (X,τ). Denote the join of all limit points of $\mathcal{G}$ by $\lim \mathcal{G}$. Call e a *cluster point* of $\mathcal{G}$, denoted by $\mathcal{G} \infty e$ if $\mathcal{G}(A) \not\leq Q'(e,U) \Rightarrow A \cap U \neq \tilde{0},\ \forall\ A,U \in L^X$, where Q is the gradation of q-neighborhoodness on (X,τ). Denote the join of all cluster points of $\mathcal{G}$ by $\mathrm{cl}\,u\mathcal{G}$.

Proposition 3.7. *In an L-fuzzy topological space (X,τ) for any g-filter $\mathcal{G}$ and, for $e, f \in \mathrm{Pt}(L^X)$, we have*

(i) $\mathcal{G} \to e \ \Rightarrow\ \mathcal{G} \infty e$; *if L is complemented,*
(ii) $\mathcal{G} \infty e \geq f \ \Rightarrow\ \mathcal{G} \infty f$, *and*
(iii) $\mathcal{G} \to e \geq f \ \Rightarrow\ \mathcal{G} \to f$.

Proof. (i) Let $\mathcal{G} \to e$ and let $\mathcal{G}(A) \not\leq Q'(e,U)$ for some A, $U \in L^X$. Then from the order dense property of L $\exists\ k \in M(L)$ such that $\mathcal{G}(A) \geq k \not\leq Q'(e,U) \Rightarrow \mathcal{G}(A) \geq k$ and $Q(e,U) \not\leq k'$.

Again $Q(e,U) \not\leq k' \ \Rightarrow\ \exists\ l \in M(L)$ such that $Q(e,U) \geq l \not\leq k'$. So,

$$\begin{aligned}\mathcal{G}(A \cap U) &\geq \mathcal{G}(A) \wedge \mathcal{G}(U) \quad \text{by (GF2)}\\ &\geq \mathcal{G}(A) \wedge Q(e,U) \ \big(\text{as } \mathcal{G} \to e\big)\\ &\geq k \wedge l.\end{aligned}$$

Now $l \leq 1 = k \vee k'$ (as L is complemented) $\Rightarrow l \leq k$ or $l \leq k'$ (as $l \in M(L)$) $\Rightarrow l \leq k$ (as $l \not\leq k'$ is assumed). So, $k \wedge l = l > 0$ (as $l \not\leq k' \Rightarrow l > 0$) $\Rightarrow$ $\mathcal{G}(A \cap U) > 0 \ \Rightarrow\ A \cap U \neq \tilde{0}$, by (GF1) $\Rightarrow \mathcal{G} \infty e$.

(ii) Let $\mathcal{G} \infty e \geq f$ and let $\mathcal{G}(A) \not\leq Q'(f,U)$ for some $A,U \in L^X$. Then as $e \geq f \Rightarrow Q(e,U) \geq Q(f,U)$. So, $Q'(f,U) \geq Q'(e,U) \Rightarrow \mathcal{G}(A) \not\leq Q'(e,U) \Rightarrow$ $A \cap U \neq \tilde{0}$ $\big(\text{as } \mathcal{G} \infty e\big) \Rightarrow \mathcal{G} \infty f$.

(iii) The proof is straightforward. □

Proposition 3.8. *In an L-fuzzy topological space (X,τ) if $\mathcal{H}$ is finer than $\mathcal{G}$ and $p_x \in \mathrm{Pt}(L^X)$ then, we have*

(i) $\lim \mathcal{G} \leq \mathrm{cl}\,u\mathcal{G}$ *if L is complemented,*
(ii) $p_x \in \mathrm{clu}\,\mathcal{G} \iff \mathcal{G} \infty p_x$, *if L is a chain,*
(iii) $p_x \in \lim \mathcal{G} \iff \mathcal{G} \to p_x$, *if L is a chain,*
(iv) $\mathcal{H} \infty p_x \ \Rightarrow\ \mathcal{G} \infty p_x$,
(v) $\mathrm{clu}\,\mathcal{G} \geq \mathrm{clu}\,\mathcal{H}$, *and*

(vi) $\lim \mathcal{G} \leq \lim \mathcal{H}$.

Proof. (i) is clear.

(ii) $\mathcal{G} \infty p_x \Rightarrow p_x \in \mathrm{cl}\, u\mathcal{G}$ is clear.

Let $p_x \in \mathrm{clu}\,\mathcal{G}$ and suppose $\mathcal{G} \not\infty p_x$. Then $\exists\ A, U \in L^X$ such that $\mathcal{G}(A) \not\leq Q'(p_x, U)$ but $A \cap U = \tilde{0}$.

Now $\mathcal{G}(A) \not\leq Q'(p_x, U) \Rightarrow \mathcal{G}'(A) \not\geq Q(p_x, U)\ \mathcal{G}'(A) \not\geq \bigvee\{r \in L_0;\ U \in \tilde{Q}_r(p_x)\} \Rightarrow \exists\ s \in L_0$ such that $s \not\leq \mathcal{G}'(A)$ but $U \in \tilde{Q}_s(p_x)$.

Now $U \in \tilde{Q}_s(p_x) \Rightarrow \exists\ V \in \tau_s$ such that $p_x \,q\, V \subseteq U$. Again $p_x \,q\, V \Rightarrow p \not\leq V'(x) \Rightarrow \exists\ t \in L_0$ such that $p > t \not\leq V'(x)$ (since L is order dense), *i. e.*, $t_x \,q\, V \subseteq U \Rightarrow U \in \tilde{Q}_s(t_x) \Rightarrow Q(t_x, U) \geq s$. As L is a chain so $\mathcal{G} \infty t_x$ from the definition of $\mathrm{clu}\,\mathcal{G}$. Now $\mathcal{G}'(A) \not\geq s \Rightarrow \mathcal{G}(A) \not\leq s' \geq Q'(t_x, U) \Rightarrow \mathcal{G}(A) \not\leq Q'(t_x, U) \Rightarrow A \cap U \neq \tilde{0}$, a contradiction.

(iii) Similar to (ii).

Proofs of (iv)–(vi) are straightforward. □

Proposition 3.9. *Let $\mathcal{G}$ be a g-filter on L^X and let $\mathcal{G}_r = \{U \in L^X;\ \mathcal{G}(U) \geq r\}$ then*

(1) *for every $r \in L_0$, $\mathcal{G}_r$ is a fuzzy filter on L^X,*

(2) *$\forall\ r, s \in L_0$, $\mathcal{G}_r \subseteq \mathcal{G}_s$ if $r \geq s$, and*

(3) $\bigcap_{i\in\Delta} \mathcal{G}_{r_i} = \mathcal{G}_{\bigvee_{i\in\Delta} r_i}$.

Proof. (1) (i) We have $\mathcal{G}(\tilde{1}) = 1 \Rightarrow \mathcal{G}_r \neq \phi\ \forall\ r \in L_0$.

(ii) $\mathcal{G}(\tilde{0}) = 0 \Rightarrow \tilde{0} \notin \mathcal{G}_r\ \forall\ r \in L_0$.

(iii) $U_1, U_2 \in \mathcal{G}_r \Rightarrow \mathcal{G}(U_i) \geq r;\ i = 1, 2$

$$\begin{aligned} &\Rightarrow \mathcal{G}(U_1 \cap U_2) \geq \mathcal{G}(U_1) \wedge \mathcal{G}(U_2), \text{ by (GF2)} \\ &\qquad\qquad\qquad\quad \geq r \\ &\Rightarrow U_1 \wedge U_2 \in \mathcal{G}_r,\ \forall\ r \in L_0. \end{aligned}$$

(iv) Let $U \in \mathcal{G}_r$ and $U \subseteq V$ then

$$\begin{aligned} &\mathcal{G}(V) \geq \mathcal{G}(U), \text{ by (GF3)} \\ &\qquad\quad \geq r \\ &\Rightarrow V \in \mathcal{G}_r,\ \forall\ r \in L_0. \end{aligned}$$

Hence $\mathcal{G}_r$ is a fuzzy filter on L^X.

(2) The proof is straightforward.

(3) $A \in \bigcap_{i\in\Delta} \mathcal{G}_{r_i} \iff \forall\ i \in \Delta,\ A \in \mathcal{G}_{r_i} \iff \forall\ i \in \Delta,\ \mathcal{G}(A) \geq r_i \iff \mathcal{G}(A) \geq \bigvee_{i\in\Delta} r_i \iff A \in \mathcal{G}_{\bigvee_{i\in\Delta} r_i}$. So, $\bigcap_{i\in\Delta} \mathcal{G}_{r_i} = \mathcal{G}_{\bigvee_{i\in\Delta} r_i}$. □

Proposition 3.10. *Let for each $r \in L_0$, $\mathcal{G}_r$ be a collection of L-fuzzy subsets of X satisfying conditions*

(1) *$\mathcal{G}_r$ is a fuzzy filter on L^X for each $r \in L_0$, and*

(2) *For all $r, s \in L_0$, $\mathcal{G}_r \subseteq \mathcal{G}_s$ if $r \geq s$,*

then the mapping $\bar{\mathcal{G}} : L^X \to L$, defined by $\bar{\mathcal{G}}(A) = \bigvee\{r \in L_0;\ A \in \mathcal{G}_r\}$ is a g-filter on L^X. If further $\{\mathcal{G}_r\}_{r \in L_0}$ satisfies Condition (3) *of* Proposition 3.9, *then, for all $r \in L_0$, $\mathcal{G}_r = \bar{\mathcal{G}}_r = \{U \in L^X;\ \bar{\mathcal{G}}(U) \geq r\}$.*

Proof. (1) (i) Since $\forall\ r \in L_0$, $\mathcal{G}_r$ is a fuzzy filter on L^X, it follows that $\bar{\mathcal{G}}(0) = 0$ and $\forall\ r \in L_0$, $\mathcal{G}_r \neq \phi$. So, $\bar{\mathcal{G}}(\tilde{1}) = 1$.

(ii) $A_1 \in \mathcal{G}_{r_1}$, $A_2 \in \mathcal{G}_{r_2} \Rightarrow A_1, A_2 \in \mathcal{G}_{r_1 \wedge r_2}$ (by (2)) $\Rightarrow A_1 \cap A_2 \in \mathcal{G}_{r_1 \wedge r_2} \Rightarrow \bar{\mathcal{G}}(A_1 \cap A_2) \geq r_1 \wedge r_2$. As L is completely distributive so

$$\bar{\mathcal{G}}(A_1 \cap A_2) \geq \bar{\mathcal{G}}(A_1) \wedge \bar{\mathcal{G}}(A_2).$$

(iii) Let $A \subseteq B$. Then for $r \in L_0$, $A \in \mathcal{G}_r \Rightarrow B \in \mathcal{G}_r$. So, $\bar{\mathcal{G}}(B) \geq \bar{\mathcal{G}}(A)$.

(2) Now we shall show that $\forall\ r \in L_0$, $\mathcal{G}_r = \bar{\mathcal{G}}_r$. In fact $A \in \mathcal{G}_r \Rightarrow \bigvee\{k;\ A \in \mathcal{G}_k\} \geq r \Rightarrow \bar{\mathcal{G}}(A) \geq r \Rightarrow A \in \bar{\mathcal{G}}_r$. So, for all $r \in L_0$, $\mathcal{G}_r \subseteq \bar{\mathcal{G}}_r$. Again $B \in \bar{\mathcal{G}}_r \Rightarrow \bar{\mathcal{G}}(B) \geq r \Rightarrow \bigvee\{k \in L_0;\ B \in \mathcal{G}_k\} \geq r$. Let $S = \{k \in L_0;\ B \in \mathcal{G}_k\}$ then for all $k \in S$, $B \in \mathcal{G}_k$. So, $B \in \bigcap_{k \in S} \mathcal{G}_k = \mathcal{G}_{\vee_{k \in S} k} = \mathcal{G}_{k'}$, where $k' \geq r \subseteq \mathcal{G}_r$. So, $B \in \mathcal{G}_r \Rightarrow \bar{\mathcal{G}}_r \subseteq \mathcal{G}_r$. □

Proposition 3.11. *Let $\mathcal{G}$ be a g-filter on an L-fuzzy topological space (X, τ) and let $e \in \mathrm{Pt}(L^X)$ then*

$$\forall\ r \in L_0,\ \mathcal{G} \to e \Rightarrow \mathcal{G}_r \to^l e \quad \text{for some } l \geq r'.$$

Proof. $\mathcal{G} \to e \Rightarrow \mathcal{G}(U) \geq Q(e, U)\ \forall\ U \in L^X \Rightarrow \mathcal{G}_r \supseteq \tilde{Q}_r(e) \Rightarrow$

$$l' = \bigwedge\{s \in L_0;\ \tilde{Q}_s(e) \subseteq \mathcal{G}_r\} \leq r.$$

Therefore $\mathcal{G}_r \to^l e$ for some $l \geq r'$. □

Proposition 3.12. *Let $\mathcal{G}$ be a g-filter on an L-fuzzy topological space (X, τ) and $e \in \mathrm{Pt}(L^X)$. If for every $r \in L_0$, $\mathcal{G}_r \to_k e$ for some $k \geq r'$ then $\mathcal{G} \to e$.*

Proof. Let the given condition be satisfied. To show

$$\mathcal{G}(U) \geq Q(e, U),\ \forall\ U \in L^X,$$

suppose, for some $U \in L^X$, $\mathcal{G}(U) \not\geq Q(e, U)$, *i. e.*, $\mathcal{G}(U) \not\geq \bigvee\{\tau(V);\ e\, q\, V \subseteq U\}$.

$\mathcal{G}(U) \not\geq \bigvee\{\tau(V);\ e\, q\, V \subseteq U\}$

$\Rightarrow \exists\ V \in L^X$ such that $e\, q\, V \subseteq U$ and $\tau(V) \not\leq \mathcal{G}(U)$

$\Rightarrow\ \exists\ \alpha,\beta \in L_0$ such that $\mathcal{G}(U) \not\geq \alpha < \beta < \tau(V)$ (since L is order dense)
$\Rightarrow\ U \notin \mathcal{G}_\alpha$ but $\tau(V) > \beta$ means $U \in \tilde{Q}_\beta(e)$.

Therefore

$$\tilde{Q}_\beta(e) \not\subseteq \mathcal{G}_\alpha \qquad (*)$$

Now according to the given condition $\mathcal{G}_\alpha \to_k e$ for some $k \geq \alpha'$ where

$$k' = \bigvee\{s \in L_0;\ \tilde{Q}_s(e) \not\subseteq \mathcal{G}_\alpha\}.$$

$\Rightarrow\ k' \geq \beta$, by $(*)$ $\Rightarrow\ k' \geq \beta > \alpha\ \Rightarrow\ k < \alpha'$, a contradiction. □

Proposition 3.13. *Let $\mathcal{G}$ be a g-filter on an L-fuzzy topological space (X,τ) and let $e \in \mathrm{Pt}(L^X)$ then $\forall\ r \in L_0$, $\mathcal{G} \infty e\ \Rightarrow\ \mathcal{G}_r \infty_k e$ for some $k \geq r$.*

Proof. Let $\mathcal{G} \infty e$ and suppose $\exists\ r \in L_0$ such that for no $k(\geq r)$, $\mathcal{G}_r \infty_k e$. Then $\mathcal{G}_r \infty_k e\ \Rightarrow\ k \not\geq r\ \Rightarrow\ k' \not\leq r'\ \Rightarrow$

$$\bigvee\{s \in L_0;\ \exists\ U \in \tilde{Q}_s(e),\ \exists\ A \in \mathcal{G}_r;\ A \cap U = \tilde{0}\} \not\leq r'$$

$\Rightarrow\ \exists\ s \in L_0$ such that $s \not\leq r'$ and $\exists\ U \in \tilde{Q}_s(e)$, $\exists\ A \in \mathcal{G}_r$ such that $A \cap U = \tilde{0}$.

Now $U \in \tilde{Q}_s(e)$ and $A \in \mathcal{G}_r\ \Rightarrow\ Q(e,U) \geq s$ and $\mathcal{G}(A) \geq r\ \Rightarrow\ Q'(e,U) \leq s'$ and $\mathcal{G}(A) \geq r$. Therefore, $s \not\leq r'\ \Rightarrow\ s' \not\geq r \Rightarrow\ Q'(e,U) \not\geq \mathcal{G}(A)$ but $A \cap U = \tilde{0}$, a contradiction to the fact $\mathcal{G} \infty e$. □

Proposition 3.14. *Let $\mathcal{G}$ be a g-filter on an L-fuzzy topological space (X,τ) and $e \in \mathrm{Pt}(L^X)$. If for every $r \in L_0$, $\mathcal{G}_r \infty_k e$ for some $k \geq r$ then $\mathcal{G} \infty e$.*

Proof. Let the given condition be satisfied. To show $\mathcal{G} \infty e$, suppose $\mathcal{G} \not\infty e$. Then $\exists\ A, U \in L^X$ such that $\mathcal{G}(A) \not\leq Q'(e,U)$ but $A \cap U = \tilde{0}$. Therefore

$\mathcal{G}(A) \not\leq Q'(e,U)$
$\Rightarrow\ \exists\ \alpha\ \in L_0$ such that $\mathcal{G}(A) > \alpha \not\leq Q'(e,U)$ (from the order dense property of L)
$\Rightarrow\ \mathcal{G}(A) > \alpha$ and $Q(e,U) \not\leq \alpha'$.

Now

$$\begin{aligned}\mathcal{G}(A) > \alpha\ &\Rightarrow\ A \in \mathcal{G}_\alpha \text{ and } Q(e,U) \not\leq \alpha' \\ &\Rightarrow\ \bigvee\{r \in L_0;\ U \in \tilde{Q}_r(e)\} \not\leq \alpha' \text{ (from the definition of } Q(e,U)\text{)} \\ &\Rightarrow\ \exists\ \beta \in L_0 \text{ such that } \beta \not\leq \alpha' \textit{ and } U \in \tilde{Q}_\beta(e).\end{aligned}$$

Therefore,

$$A \in \mathcal{G}_\alpha,\ U \in \tilde{Q}_\beta(e) \text{ but } A \cap U = \tilde{0} \text{ and } \beta \not\leq \alpha'$$
$$\Rightarrow \bigvee\{r \in L_0;\ \exists\, U \in \tilde{Q}_r(e),\ \exists\, A \in \mathcal{G}_\alpha;\ A \cap U = \tilde{0}\} \geq \beta \not\leq \alpha'.$$

Let $k' = \bigvee\{r \in L_0;\ \exists\, U \in \tilde{Q}_r(e),\ \exists\, A \in \mathcal{G}_\alpha;\ A \cap U = \tilde{0}\}$ then $\mathcal{G}_\alpha \infty_k e$ where $k' \geq \beta \not\leq \alpha' \Rightarrow \mathcal{G}_\alpha \infty_k e$ where $k' \not\leq \alpha'$ *i. e.*, $k \not\geq \alpha$ which is contradictory to the given condition. □

Lemma 3.15. *Let (X, τ) and (Y, δ) be any two L-fuzzy topological spaces and $f : (X, \tau) \to (Y, \delta)$ be any mapping then $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.*

Proof. For all $x \in X$,

$$\begin{aligned} f^{-1}(B_1 \cap B_2)(x) &= (B_1 \cap B_2)f(x) \\ &= B_1(f(x)) \wedge B_2(f(x)) \\ &= \left[f^{-1}(B_1)(x)\right] \wedge \left[f^{-1}(B_2)(x)\right] \\ &= \left[f^{-1}(B_1) \cap f^{-1}(B_2)\right](x). \end{aligned}$$

Hence the proof. □

Lemma 3.16. *Let (X, τ) and (Y, δ) be any two L-fuzzy topological spaces and let $Q, \hat{Q}$ be the gradation of $\mathfrak{q}$-neighborhoodness in (X, τ) and (Y, δ) respectively. A mapping $f : (X, \tau) \to (Y, \delta)$ is a gp-map if and only if*

$$\forall\, e \in M(L^X) \text{ and } \forall\, V \in L^Y,\ Q(e, f^{-1}(V)) \geq \hat{Q}(f(e), V).$$

Proof. We have $\hat{Q}(f(e), V) = \bigvee\{\delta(W);\ f(e)\,\mathfrak{q}\,W \subseteq V\}$. Now

$$f(e)\,\mathfrak{q}\,W \subseteq V \ \Rightarrow\ e\,\mathfrak{q}\,f^{-1}(W) \subseteq f^{-1}(V) \text{ and } \tau(f^{-1}(W)) \geq \delta(W),$$

as f is a gp-map. So, $\bigvee\{\tau(U);\ e\,\mathfrak{q}\,U \subseteq f^{-1}(V)\} \geq \bigvee\{\delta(W);\ f(e)\,\mathfrak{q}\,W \subseteq V\}$. So,

$$Q(e, f^{-1}(V)) \geq \hat{Q}(f(e), V), \forall\, e \in M(L^X) \text{ and } \forall\, V \in L^Y.$$

Conversely, let $Q(e, f^{-1}(U)) \geq \hat{Q}(f(e), U)$, $\forall\, e \in M(L^X)$ and $\forall\, U \in L^Y$ and suppose f be not a gp-map. Then $\exists$ at least one $U \in L^Y$ such that $\tau(f^{-1}(U)) \not\geq \delta(U)$. Therefore, by Propositions 1.5, 1.6 and 1.7, we have

$$\bigwedge\{Q(p_x, f^{-1}(U);\ p_x \in M(L^X) \text{ and } p_x\,\mathfrak{q}\,f^{-1}(U)\}$$
$$\not\geq \bigwedge\{\hat{Q}(r_y, U);\ r_y \in M(L^Y) \text{ and } r_y\,\mathfrak{q}\,U\}$$

$\Rightarrow\ \exists\, p_x \in M(L^X)$ such that $p_x \,\mathrm{q}\, f^{-1}(U)$ and $Q(p_x, f^{-1}(U))$

$$\not\geq \bigwedge\{\hat{Q}(r_y, U);\ r_y \in M(L^Y) \text{ and } r_y \,\mathrm{q}\, U\}$$

$\Rightarrow\ Q(p_x, f^{-1}(U)) \not\geq \hat{Q}(r_y, U),\ \forall\, r_y \in M(L^Y)$ with $r_y \,\mathrm{q}\, U$.

This implies

$$Q(p_x, f^{-1}(U)) \not\geq \hat{Q}(f(p_x), U)$$

(since $p_x \in M(L^X)$ and $p_x \,\mathrm{q}\, f^{-1}(U) \ \Rightarrow\ f(p_x) \in M(L^Y)$ and $f(p_x) \,\mathrm{q}\, U$), which is a contradiction. Hence the proof. □

Definition 3.17. Let (X, τ) and (Y, δ) be any two L-fuzzy topological spaces and $f : (X, \tau) \to (Y, \delta)$ be any mapping and $\mathcal{G}$ be any g-filter on X, we define

$$f[\mathcal{G}](B) = \mathcal{G}(f^{-1}(B)),\ \forall\, B \in L^Y.$$

Proposition 3.18. *Let (X, τ) and (Y, δ) be any two L-fuzzy topological spaces and $f : (X, \tau) \to (Y, \delta)$ be any mapping then for any g-filter $\mathcal{G}$ on (X, τ), $f[\mathcal{G}]$ is a g-filter on (Y, δ).*

Proof. (1) As we know $f^{-1}(\tilde{0}_Y) = \tilde{0}_X$ and $f^{-1}(\tilde{1}_Y) = \tilde{1}_X$ so,

$$f[\mathcal{G}](\tilde{0}_Y) = \mathcal{G}\big(f^{-1}(\tilde{0}_Y)\big) = \mathcal{G}(\tilde{0}_X) = 0 \text{ and } f[\mathcal{G}](\tilde{1}_Y) = \mathcal{G}\big(f^{-1}(\tilde{1}_Y)\big) = \mathcal{G}(\tilde{1}_X) = 1.$$

(2) $f[\mathcal{G}](B_1 \cap B_2) = \mathcal{G}(f^{-1}(B_1 \cap B_2)) \geq \mathcal{G}\big(f^{-1}(B_1) \cap f^{-1}(B_2)\big)$, by Lemma 3.15 and (GF3). Therefore, by (GF2),

$$f[\mathcal{G}](B_1 \cap B_2) \geq \mathcal{G}\big(f^{-1}(B_1)\big) \wedge \mathcal{G}\big(f^{-1}(B_2)\big) = f[\mathcal{G}](B_1) \wedge f[\mathcal{G}](B_2).$$

(3) $B_1 \subseteq B_2 \ \Rightarrow\ f^{-1}(B_1) \subseteq f^{-1}(B_2),\ \forall\, B_1, B_2 \in L^Y$. Then, by (GF3),

$$f[\mathcal{G}](B_2) = \mathcal{G}(f^{-1}(B_2)) \geq \mathcal{G}(f^{-1}(B_1)) = f[\mathcal{G}](B_1).$$

Hence $f[\mathcal{G}]$ is a g-filter on (Y, δ). □

Proposition 3.19. *Let (X, τ) and (Y, δ) be any two L-fuzzy topological spaces and $f : (X, \tau) \to (Y, \delta)$ be a gp-map then, for any g-filter $\mathcal{G}$ and for any $e \in \mathrm{Pt}(L^X)$,*

$$\mathcal{G} \to e \ \Rightarrow\ f[\mathcal{G}] \to f(e).$$

Proof. Let Q and $\hat{Q}$ be the gradations of q-neighborhoodness in (X, τ) and (Y, δ) respectively and let $B \in L^Y$, then

$$\mathcal{G}(f^{-1}(B)) \geq Q(e, f^{-1}(B)) \quad [\text{as } \mathcal{G} \to e]$$
$$\geq \hat{Q}(f(e), B),$$

by Lemma 3.16. This implies $f[\mathcal{G}](B) \geq \hat{Q}(f(e), B)$. □

Proposition 3.20. *Let $f : (X,\tau) \to (Y,\delta)$ be a mapping where (X,τ) and (Y,δ) be any two L-fuzzy topological spaces. If, for any g-gilter $\mathcal{G}$ and for any $e \in M(L^X)$,*

$$\mathcal{G} \to e \Rightarrow f[\mathcal{G}] \to f(e)$$

then f is a gp-map.

Proof. Let Q and $\hat{Q}$ be the gradations of q-neighborhoodness in (X,τ) and (Y,δ) respectively. As $e \in M(L^X)$ so the mapping $Q_e : L^X \to L$ given by $Q_e(U) = Q(e,U)$ is a g-filter on L^X and $Q_e \to e$. So, according to the given condition $f[Q_e] \to f(e)$. So, $\forall\, V \in L^Y$, $f[Q_e](V) \geq \hat{Q}(f(e),V) \Rightarrow Q_e(f^{-1}(V)) \geq \hat{Q}(f(e),V)$ $\Rightarrow Q(e, f^{-1}(V)) \geq \hat{Q}(f(e),V)$.

Hence, by Lemma 3.16, f is a gp-map. □

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