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CONTROL STRATEGY OF AN ACTIVE SUSPENSION FOR A HALF CAR MODEL WITH PREVIEW INFORMATION

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ABSTRACT−To improve the ride comfort and handling characteristics of a vehicle, an active suspension which is controlled by external actuators can be used. An active suspension can control the vertical acceleration of a vehicle and the tire deflection to achieve the desired suspension goal. For this purpose, Model Predictive Control (MPC) scheme is applied with the assumption that the preview information of the oncoming road disturbance is available. The predictive control approach uses the output prediction to forecast the output over a time horizon and determines the future control over the horizon by minimizing the performance index. The developed method is applied to a half car model of four degrees-of-freedom and numerical simulations show that the MPC controller improves noticeably the ride qualities and handling performance of a vehicle.

KEY WORDS : Active suspension, Preview information, Model predictive control, Vehicle dynamics

1. INTRODUCTION

The ride comfort and handling performance are important aspects in designing a suspension system of a vehicle. For good ride comfort, the suspension should isolate the body from road input, but for good vehicle handling performance, the tires should closely follow the road profile to improve road holding. The design of vehicle suspension systems is a compromise among conflicting requirements between ride comfort and handling performance. The performance of a passive vehicle suspension is based on trade-off between the two aspects mentioned above (Suh *et al.*, 2001; Lee *et al.*, 2003). Active and semi-active suspension were proposed to overcome this trade-off and to obtain high ride performance (Sohn *et al.*, 2003; Nouillant *et al.*, 2002). An active suspension system uses external actuators to achieve the desired suspension goal. It controls vertical acceleration, pitch motion and tire deflection for ride comfort and handling ability. Active suspension control schemes with preview strategies have been studied by numerous researchers to improve the ride qualities of a vehicle over any other suspensions (Sharp and Pilbeam, 1993; Thompson *et al.*, 1980; Tomizuka, 1976). However, additional capability is required for extreme conditions encountered by off-road vehicles driven over rough terrain from these control strategies. Considerations of the physical limits on the suspension travel become significant for these situations, because harsh bumps may cause the suspension to hit the physical stops known as "bump-stopper". The impact causes a significant jerk on the vehicle chassis and introduces undesired accelerations into the system and degrades the ride qualities of the vehicle. This paper presents the design of an active suspension controller which maximizes the ride comfort of a vehicle and improves the handling performance considering the physical limits on the suspension travel. For this purpose the Model Predictive Control (MPC) is applied and it is assumed that the preview information of the oncoming road disturbance is available. Some of the prevalent techniques for the semi-active or active suspension control are sky-hook damping, optimal LQR and optimal LQR with preview (Park and Koo, 1994; Kim and Yoon, 1994; Hac, 1992; Bangsing *et al.*, 1997). None of these schemes take the previous information of the states into consideration as constraints. There were some researches about constrained semi-active suspension control (Cho and Yi, 1997; Aa *et al.*, 1997). These researches were very successful on smooth road but not over rough terrain. For vehicles over rough terrain active suspensions are more appropriate. The MPC framework (Clark, 1994; Medhra *et al.*, 1982) promises to be a suitable tool for this application since it allows the explicit considerations of the physical limits on

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suspension travel in the controller design. Furthermore, this framework offers the ability to switch suspension stiffness based on the predicted suspension travel. Previous research for a quarter car suspension model with this framework (Cho, 1999) shows that this controller can make great improvement on suspension performance. It is worthwhile to expand the model for half car suspension.

The predictive control approach uses the output prediction and receding-horizon approach. It uses a predictor to forecast the output over a time horizon and determines the future control over the horizon by minimizing the performance index. Of the future control determined, only the first control is used because of the receding horizon approach. The same steps are repeated for the next sampling instant. The constrained predictive control problem can be recast as constrained quadratic problem. It is important to consider the physical limits on the suspension deflection because a smaller suspension travel requires less packaging space of suspension system.

2. HALF CAR SUSPENSION MODEL

The four degrees-of-freedom half car model is considered to analyze the behavior of vehicle. Figure 1 shows a schematic diagram of a half car model. Vehicle is composed of three rigid bodies: one sprung mass and two unsprung masses. For the sprung mass, vertical and pitch movements are allowed and for the unsprung masses, only vertical movements of the front and rear wheels are considered. z_s is the vertical displacement of sprung mass, θ is the pitch angle and z_{rf}, z_{rr} are ground variation, and z_{uf}, z_{ur} are the vertical displacements of front and rear

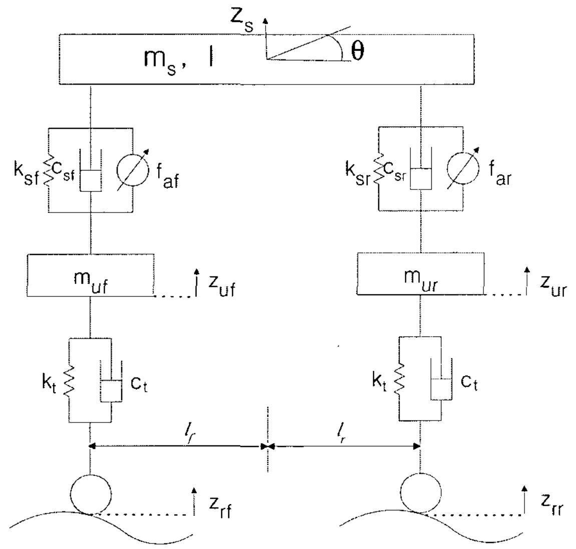


Figure 1. Half car suspension model.

unsprung masses respectively. m_S is the mass of sprung and I is the moment of inertia of the vehicle. m_{uf} and m_{ur} are the front and rear unsprung masses, k_{sf} and k_{sr} are the front and rear suspension stiffness and k_t is the tire stiffness. c_{sf} and c_{sr} are the front and rear suspension damping coefficients and c_t is the tire damping coefficient. f_{af} and f_{ar} are the control inputs of front and rear acuators, l_f and l_r are the distance from the front and rear wheel axes to the center of gravity, respectively. Then the equations of motion of the system can be expressed as

$$
\begin{aligned}
m_s\ddot{z}_s &= k_{sf}(z_{uf}-z_f)+c_{sf}(\dot{z}_{uf}-\dot{z}_f)+f_{af} \\
&\quad +k_{sr}(z_{ur}-z_r)+c_{sr}(\dot{z}_{ur}-\dot{z}_r)+f_{ar} \\
I\ddot{\theta} &= -l_f[k_{sf}(z_{uf}-z_f)+c_{sf}(\dot{z}_{uf}-\dot{z}_f)+f_{af}] \\
&\quad +l_r[k_{sr}(z_{ur}-z_r)+c_{sr}(\dot{z}_{ur}-\dot{z}_r)+f_{ar}] \\
m_{uf}\ddot{z}_{uf} &= -k_{sf}(z_{uf}-z_f)+c_{sf}(\dot{z}_{uf}-\dot{z}_f) \\
&\quad +k_t(z_{rf}-z_{uf})+c_t(\dot{z}_{rf}-\dot{z}_{uf})-f_{af} \\
m_{ur}\ddot{z}_{ur} &= -k_{sr}(z_{ur}-z_r)+c_{sr}(\dot{z}_{ur}-\dot{z}_r) \\
&\quad +k_t(z_{rr}-z_{ur})+c_t(\dot{z}_{rr}-\dot{z}_{ur})-f_{ar}
\end{aligned}
$$

These equations can be written in discretized matrix form as follows:

$$
\begin{aligned}
\dot{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}_u\boldsymbol{u}(k)+\boldsymbol{B}_v\boldsymbol{v}(k) \\
y(k) &= \boldsymbol{C}\boldsymbol{x}(k)+\boldsymbol{D}\boldsymbol{u}(k)
\end{aligned} \tag{1}
$$

with state vector,

$$x = [z_f-z_{uf}, z_r-z_{ur}, \dot{z}_s, \dot{\theta}, z_{uf}-z_{rf}, \dot{z}_{uf}, z_{ur}-z_{rr}, \dot{z}_{ur}]^T$$

where $z_f = z_s - l_f\theta,\ z_r = z_s + l_r\theta$.

$\boldsymbol{u} = [f_{af}, f_{ar}]^T$ and $\boldsymbol{v} = [\dot{z}_{rf}, \dot{z}_{rr}]^T$ are a control input vector and a road disturance vector, respectively. The output $\boldsymbol{y}$ is composed of suspension travels, tire deflections, vertical acceleration and pitch rate of sprung mass.

3. CONTROLLER DESIGN

In the previous study (Cho, 1999), an active suspension controller for a quarter car model was designed. The purpose of this study is to design an active suspension controller for a half car model. Basic concept of control strategy is the same as the previous study, but the controller should include the movements of front and rear suspension and pitch motion simultaneously. Therefore, the controller needs more sophisticated control strategy.

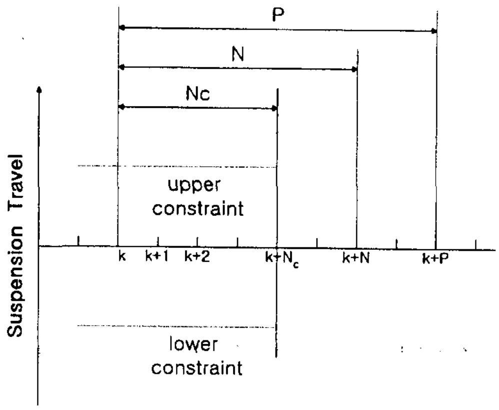


Figure 2. Predictive control horizon.

3.1. Control Law

For more efficient control three predictive control horizons are used. It is assumed that the road disturbance, v, is given accurately through the previous horizon, P, i.e. $v(k)$, $\cdots$, $v(k+p+1)$ are given at time step k with a road preview sensor - road previewing is beyond this paper. Control inputs are permitted to vary only within control horizon, N, and control inputs are constants between control horizon and preview horizon. Suspension limits are checked within constraint horizon, N_c. Three horizons, N, P and N_c involved in the predictive control formulation are shown in Figure 2.

Controller should minimize the quadratic performance index - output over preview horizon and control input over control horizon -

$$J = \sum_{i=1}^{P} \boldsymbol{y}^T(k+i)\bar{\boldsymbol{Q}}(k+i)\boldsymbol{y}(k+i) + \sum_{i=0}^{N} \boldsymbol{u}^T(k+i)\bar{\boldsymbol{R}}(k+i)\boldsymbol{u}(k+i) \tag{2}$$

where the weighting functions for output and input, $\bar{\boldsymbol{Q}}$ and $\bar{\boldsymbol{R}}$, are symmetric and positive definite matrices. This index can be written in the equivalent vector space form (Cho, 1999)

$$J = \hat{\boldsymbol{y}}^T\boldsymbol{Q}\hat{\boldsymbol{y}} + \hat{\boldsymbol{u}}^T\boldsymbol{R}\hat{\boldsymbol{u}} \tag{3}$$

where $\hat{\boldsymbol{y}} = [y(k+1)\ldots\ y(k+P)]^T$ and so on.

From Equation (1), output predictor over a preview horizon is calculated as,

$$\hat{\boldsymbol{y}} = \Lambda\boldsymbol{x}(k) + \Gamma_u\hat{\boldsymbol{u}} + \Gamma_v\hat{\boldsymbol{v}} \tag{4}$$

where $\hat{\boldsymbol{y}}$ is the output predictor over a preview horizon, $\hat{\boldsymbol{u}}$ is the calculated future control vector over a control horizon and $\hat{\boldsymbol{v}}$ is the road disturbance vector over a preview horizon.

Substituting the predictor equation given in Equation (4) into output in the performance index given in Equation (3) gives

$$J = (\Lambda\boldsymbol{x}(k) + \Gamma_u\hat{\boldsymbol{u}} + \Gamma_v\hat{\boldsymbol{v}})^T\boldsymbol{Q}(\Lambda\boldsymbol{x}(k) + \Gamma_u\hat{\boldsymbol{u}} + \Gamma_v\hat{\boldsymbol{v}}) + \hat{\boldsymbol{u}}^T\boldsymbol{R}\hat{\boldsymbol{u}} \tag{5}$$

Because only the terms are effective in $\hat{\boldsymbol{u}}$, the performance index may be rearranged as,

$$J = \frac{1}{2}\hat{\boldsymbol{u}}(\Gamma_u^T\boldsymbol{Q}\Gamma_u + \boldsymbol{R})\hat{\boldsymbol{u}} + \boldsymbol{x}^T\Lambda^T\boldsymbol{Q}\Gamma_u\hat{\boldsymbol{u}} + \boldsymbol{v}^T\Gamma_u^T\boldsymbol{Q}\Gamma_u\hat{\boldsymbol{u}} \tag{6}$$

There are some constraints over a constraint horizon,

$$low_c \le \boldsymbol{y}_{c(k+i)} \le up_c \quad i = 1, \cdots, N_c$$

This equation can be expressed in a vector form,

$$L_c \le \hat{\boldsymbol{y}}_c \le U_c \tag{7}$$

In a similar manner, the output constraint predictor is obtained as follows:

$$\hat{\boldsymbol{y}}_c = \Lambda_c\boldsymbol{x}(k) + \Gamma_{uc}\hat{\boldsymbol{u}} + \Gamma_{vc}\hat{\boldsymbol{v}} \tag{8}$$

Substituting Equation (8) into Equation (7), the constraints can be expressed in matrix form as follows:

$$\begin{bmatrix} \Gamma_{uc} \\ -\Gamma_{uc} \end{bmatrix}\hat{\boldsymbol{u}} \le \begin{bmatrix} U_c \\ -L_c \end{bmatrix} + \begin{bmatrix} -\Lambda_c & -\Gamma_{vc} \\ \Lambda_c & \Gamma_{vc} \end{bmatrix}\begin{bmatrix} \boldsymbol{x}(k) \\ \hat{\boldsymbol{v}} \end{bmatrix} \tag{9}$$

And actators have their saturated limits and this works as constraints,

$$u_{min} \le f_{af}, f_{ar} \le u_{max} \tag{10}$$

The controller should minimize performance index J in Equation (6) subject to constraints in Equation (9) and Equation (10).

3.2. Controller

Control inputs are affected by the front and rear suspension deflections which explain pitch motion as well as vertical motion. In this study suspension travel which is limited by the bump stopper is divided into three regions. Region I is a small suspension deflection region where reduction of the sprung mass acceleration and pitch rate are more important, and region II is a large suspension deflection region where reduction of the suspension deflection as well as that of the sprung mass acceleration is important. Region III is a bump stopper contact region where suspension deflection is constrained and reduction of the suspension deflection is urgent. Actuators are controlled depending on the region where the front and rear suspension travels belong.

The controller is composed of nine inner-loop

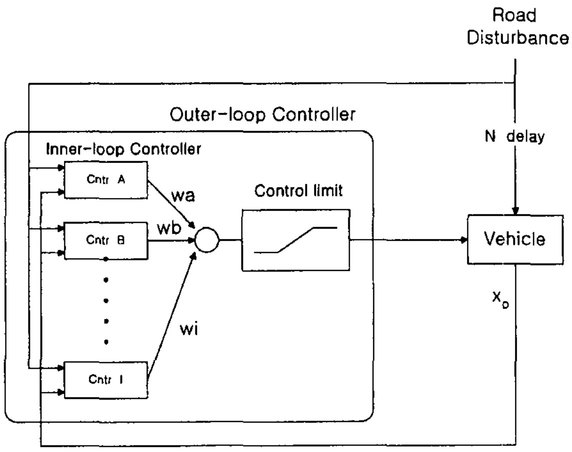


Figure 3. Controller design.

Table 1. Constitution of controller.

Rear \ Front	Region	Region	Region
Region I	Ctrl A	Ctrl B	Ctrl C
Region II	Ctrl D	Ctrl E	Ctrl F
Region III	Ctrl G	Ctrl H	Ctrl I

controllers and an outer-loop controller as shown in Figure 3. Inner-loop controller A is optimized for the case that both front and rear suspension travels are in Region I, controller B is optimized for the case that front wheel is on Region II and rear is on Region I, and so on as shown in Table 1. Each inner-loop controller has its own feedback gain predetermined by solving LQR problem for each situation.

The nine inner-loop controllers run in parallel and calculate their own control inputs. Outer-loop controller adjusts nine weighting factors based on the front and rear suspension deflections and computes the weighted sum for control input considering actuator limits as shown in Figure 3. The weighted sums for future control are calculated over the control horizon, N, according to the future front and rear suspension travels over the constraint horizon, N_c, considering future road input and predicted output over the preview horizon, P. Among the calculated future control inputs, only the first one is used at the instant and the same procedure is repeated at the next instant.

4. SIMULATION RESULTS

Simulations were performed on a rounded pulse bump and a random road as shown in Figure 4.

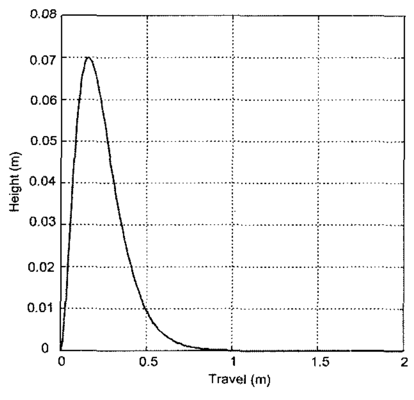
For comparative purpose, a sky-hook controller whose control gains were optimized by trial and error was simulated too. Table 2 shows the suspension parameters for simulation.

4.1. Rounded Pulse Road

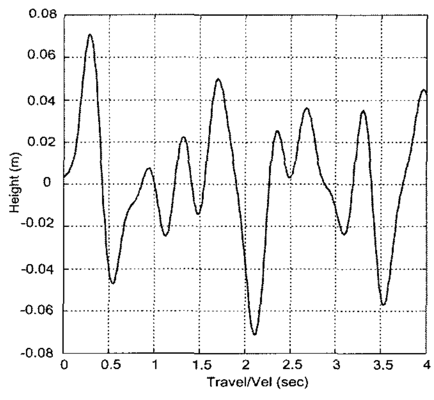
Rounded pulse is used to evaluate the performance of the suspension for deterministic road disturbance. A pulse shape is determined by height and characteristic length. In this study 0.07 m pulse height and 1 m characteristic length is used. The velocity of vehicle is 45 km/h, which means characteristic time for the pulse is about 0.08 second and it is appropriate to analyse the result since it is more than 8 times of sampling rate.

The sprung mass acceleration and pitch rate should be reduced to improve ride comfort (Arvidsson *et al.*, 2000) and tire deflection should be reduced to improve handling performance, and suspension travel should be reduced to minimize packaging space for suspension.

In Figure 5 the responses over the rounded pulse bump are shown. Solid line shows the MPC and dotted line



(a) Pulse



(b) Random

Figure 4. Road profiles.

Table 2. Suspension parameters.

Description		Symbol	Value
Mass	Sprung	m_s	570.6 *kg*
	Unsprung	m_{uf}, m_{ur}	56.5 *kg*
	Moment of inertia	I	768.9 kgm^2
Suspension stiffness	Front	k_f	16,812 *N/m*
	Rear	k_r	16,812 *N/m*
Suspension damping	Front	c_f	100 *N/m/sec*
	Rear	c_r	100 *N/m/sec*
Tire	Stiffness	k_t	190,000 *N/m*
	Damping	c_t	15 *N/m/sec*
Wheel base	Front	l_f	1.38 *m*
	Rear	l_r	1.36 *m*
Suspension limit		low_c, up_c	0.06 *m*

shows the sky-hook control.

The MPC compared to the sky-hook control reduces not only sprung mass acceleration but also oscillating period. This shows the MPC can damp down vibration more rapidly than the sky-hook control. Consequently the absorbed power by driver of MPC is 0.3346 kw and that of sky-hook control is 0.8118 kw. The absorbed power is calculated from the acceleration change. The MPC improved 58% in absorbed power. This means the MPC improves the ride comfort.

Pitch rate does not show significant difference because of running over only one bump.

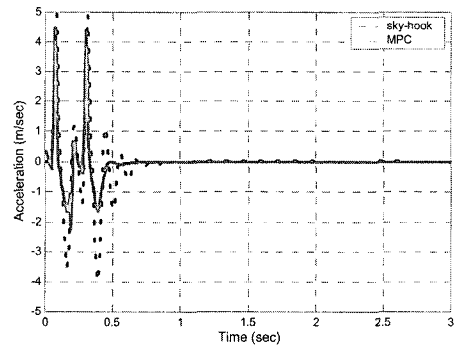
The suspension deflection and the tire deflection of the MPC are much less than those of the sky-hook controller. Small suspension deflection requires less packaging space for a suspension system and smaller tire deflection makes road holding and handling performance better. Consequently, it is apparent that the MPC can improve handling ability as well as reduce the suspension working space.

The MPC shows better performance compared to the sky-hook controller in ride and handling as well as packaging space.

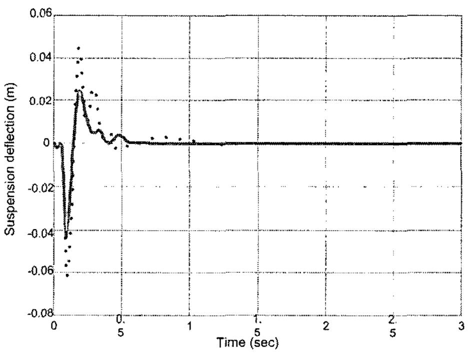
4.2. Random Road

The severe harsh random road was designed to test the robustness for the controller.

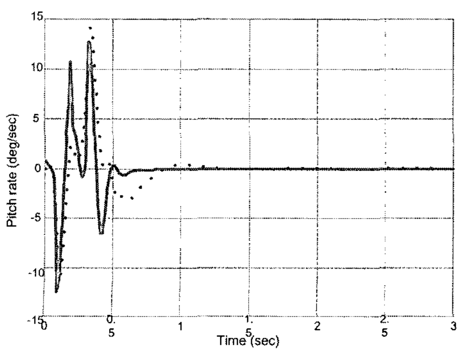
Figure 6 shows the responses over random road, as in Figure 5, solid line shows the MPC and dotted line shows



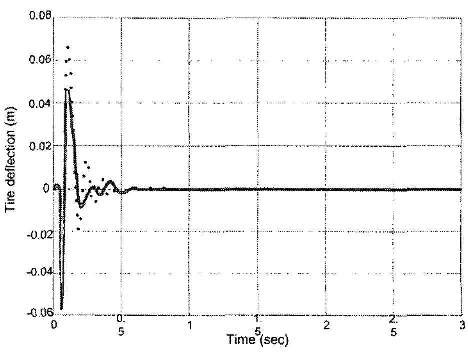
(a) Sprung mass acceleration



(c) Front suspension deflection



(b) Pitch rate



(d) Front tire deflection

Figure 5. Response over rounded pulse bump.

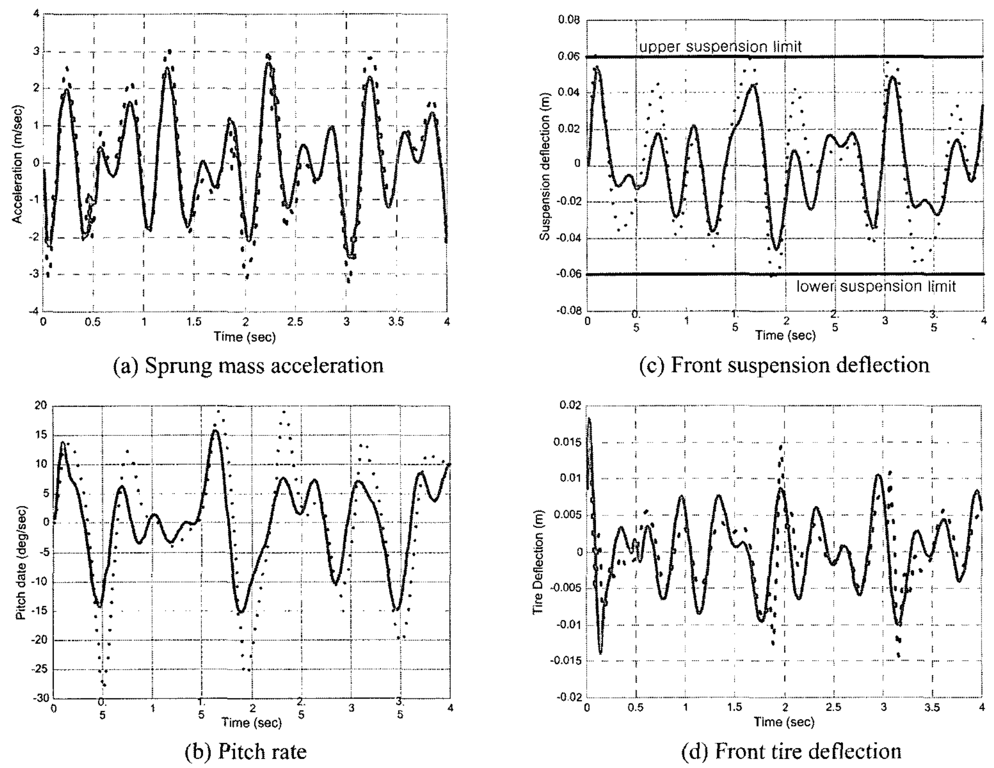


Figure 6. Response over random road.

the sky-hook control. As was expected, sprung mass acceleration, pitch rate, suspension deflection and tire deflection of the MPC are much smaller than those of the sky-hook control. The absorbed power by driver of the MPC is 1.1826 kw and that of the sky-hook controller is 1.5773 kw. The MPC improves 25% in absorbed power. This fact represents that the shock absorbing performance of the MPC is superior and the MPC can make passengers comfortable. Even on random road it is clear that the MPC improves ride and handling performance simutaneously.

According to Figure 6(c), in the sky-hook controller bump-stoppers hit their limits but in the MPC this situation does not happen. This means the MPC improves durability of the suspension as well as makes passenger comfortable.

Figure 7 shows the power spectral density of the sprung mass acceleration over the random road. The MPC shows better performance all over the frequencies than the sky-hook controller. In addition to the time domain response, frequency response shows the MPC improves ride quality greatly.

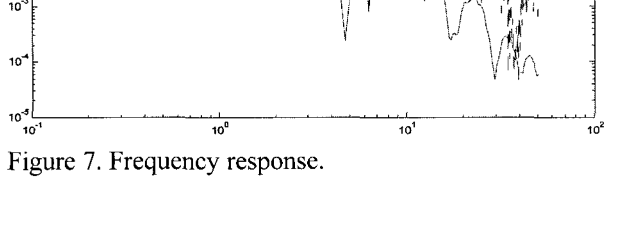


Figure 7. Frequency response.

5. CONCLUSION

In this paper, an active suspension controller for a half car model with MPC that incorporates preview information and constraints on the suspension travel was designed. The MPC controller greatly improves not only the ride comfort but also road holding compared with the sky-hook controller. Considering the preview information of road and the suspension travel constraints, the MPC manages suspension deflections within their limits and keeps the acceleration and the pitch rate of the sprung mass small. The MPC makes a passenger comfortable by reducing acceleration and pitch rate of the sprung mass, and packing space small by reducing the suspension deflections. It also improves handling performance and road holding by reducing the tire deflections.

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