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RELIABILITY-BASED DESIGN OPTIMIZATION OF AN AUTOMOTIVE SUSPENSION SYSTEM FOR ENHANCING KINEMATIC AND COMPLIANCE CHARACTERISTICS

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ABSTRACT−This study introduces the Reliability-Based Design Optimization (RBDO) to enhance the kinematic and compliance (K & C) characteristics of automotive suspension system. In previous studies, the deterministic optimization has been performed to enhance the K & C characteristics. Unfortunately, uncertainties in the real world have not been considered in the deterministic optimization. In the design of suspension system, design variables with the uncertainties, such as the bushing stiffness, have a great influence on the variation of the suspension performances. There is a need to quantify these uncertainties and to apply the RBDO to obtain the design, satisfying the target reliability level. In this research, design variables including uncertainties are dealt as random variables and reliability of the suspension performances, which are related the K & C characteristics, are quantified and the RBDO is performed. The RBD-optimum is compared with the deterministic optimum to verify the enhancement in reliability. Thus, the reliability of the suspension performances is estimated and the RBD-optimum, satisfying the target reliability level, is determined.

KEY WORDS : Suspension system, Reliability-based design optimization (RBDO), Kinematic and compliance characteristics, Uncertainty, Single-loop-single-vector (SLSV)

1. INTRODUCTION

As the automotive technologies have developed, there has been an increase in the need to improve the kinematic and compliance (K & C) characteristics. The design, improving the K & C characteristics, is a tradeoff and both vehicle dynamic and kinematic characteristics should be considered in order to definitely identify suspension performances. In general, the design which considers the vehicle dynamics characteristics is to determine the mass of the tire/wheel and dynamic characteristics of the bushing and so on. On the other hand, the design which considers the kinematic characteristics is to determine the position of the hardpoints of the suspension system. As the average product-development time has been reduced, the need of a design optimization capable of satisfying the design requirements during a short period of time has increased. The conventional deterministic optimization may present a low confidence level and violate design requirements. Uncertainties, such as material properties, geometries, and so on, bring about the critical variation of performances. Thus the RBDO, statistically estimating the reliability of suspension characteristics, is needed. Lately, the RBDO is applied to improve the crashworthiness of the side impact (Gu *et al.*, 2001) and to design the thin-walled beam of the vehicle structures (Lee *et al.*, 2003) and etc.

In this study, the RBDO is introduced to design a reliable Macpherson strut type front suspension system, improving the K & C characteristics. Probabilistic constraints are constructed by the K & C characteristics. The design variables, which affect the K & C characteristics, are dealt as random variables. The Single-Loop-Single-Vector method (SLSV), which is one of the RBDO methods, is used.

2. SUSPENSION ANALYSIS

The automotive suspension system satisfies the engineering requirements such as the K & C characteristics and so on. The suspension characteristics can be mathematically defined and divided by two groups. One is the kinematic characteristics, which influences handling. The other is

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the dynamic characteristics, such as forces and displacements which influence the ride comfort. In this research, the kinematic and compliance analysis, which combines kinematic analysis and quasi-static analysis, is performed using ADAMS/Car and an analysis model based on the data of the mass production model. The K & C characteristics are calculated through four analysis modes, which are the bump, roll, 30mm-trail lateral and fore & aft mode.

2.1. Bump Mode
As shown in Figure 1, the bump mode models bump and rebound motions. The change of toe angle, caster angle, camber angle and x-directional wheel center movement are calculated according to the variation (from −80 mm to 80 mm) of the wheel travel.

2.2. Roll Mode
As shown in Figure 2, the roll mode models rolling motions. The change of roll center height is calculated according to the variation (from −50 mm to 50 mm for left wheel, from 50 mm to −50 mm for right wheel) of the wheel travel.

2.3. 30 mm-Trail Lateral Mode
As shown in Figure 3, the lateral stiffness and lateral force steer are calculated according to the variation (from −2,000N to 2,000N) of the cornering force in the 30mm-trail lateral mode.

2.4. Fore and Aft Mode
As shown in Figure 4, the Fore & Aft mode models longitudinal motions. Longitudinal stiffness and longitu-

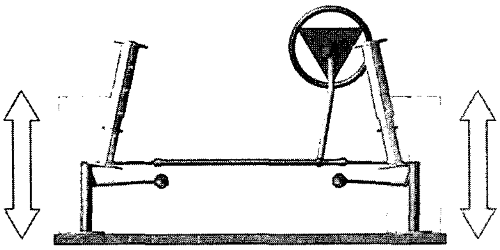


Figure 1. Bump mode.

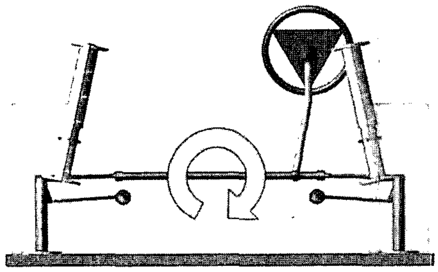


Figure 2. Roll mode.

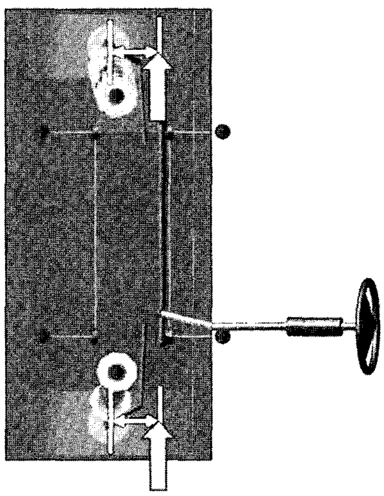


Figure 3. 30 mm-trail lateral mode.

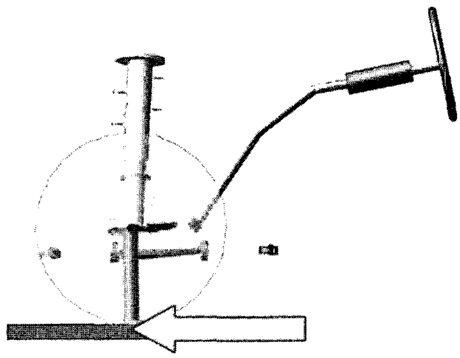


Figure 4. Fore and Aft mode.

Table 1. Various RBDO techniques.

	Basic concept	Efficiency	Accuracy	Robustness
Double loop (RIA, PMA)	Employs nested optimization formulation	Low	High	High
Single loop (SLSV)	Employs a single loop and uses update rule to search MPP	High	Medium	Medium
Serial single loop (SORA)	Employs a sequentially deterministic optimization and reliability analysis	High	High	Medium

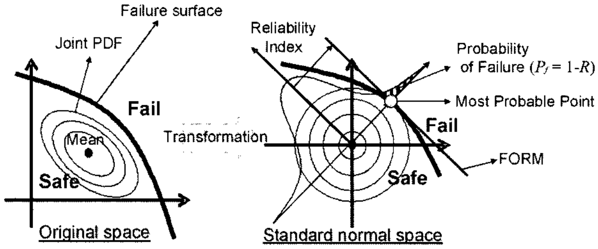


Figure 5. Reliability-analysis concept (FORM).

dinal force steer are calculated according to the variation (from −2,000N to 2,000N) of the longitudinal force.

3. RELIABILITY-BASED DESIGN OPTIMIZATION

3.1. Reliability Analysis

The goal in reliability analysis is to assess the reliability, given performance variation caused by the uncertainties. The reliability can be expressed as:

$$R = \int\int \dots \int_{g(X)\le 0} f_X(X_1, X_2, \dots, X_n)dX \tag{1}$$

where $X = [X_1, X_2, \dots X_n]^T$ is the vector of random variables and $g(X)$ is the limit state function and $f_X(X)$ is the corresponding joint probability function of the random variables X and reliability R.

In general, the explicit forms of the joint probability function are unknown and even if they are known, the multi-dimensional numerical integration of Equation (1) is extremely difficult and computationally expensive. To overcome these difficulties, an approximate reliability method, such as the first-order reliability method (FORM), has been proposed. The reliability index is defined as the shortest distance from the origin of the standard normal space to a point on the failure surface. At first, a transformation is introduced to map the original space to the uncorrelated standard normal space. Then mathematically, determining the reliability index is a minimization problem with one equality constraint. The solution of this problem is called the Most Probable Point (MPP). If the limit state function is linear in terms of the random variables that are normally distributed, the reliability is calculated as:

$$R = \Phi(\beta) \tag{2}$$

where Φ is the standard normal cumulative density function.

3.2. Reliability-Based Design Optimization Concept

If the probabilistic constraint is linear and normally distributed, the deterministic optimum has a reliability level of about 0.5. because of the uncertainty of design variables shown in Figure 6. It is RBDO that quantifies the uncertainty, and finds an optimum design to satisfy the target reliability of system performances.

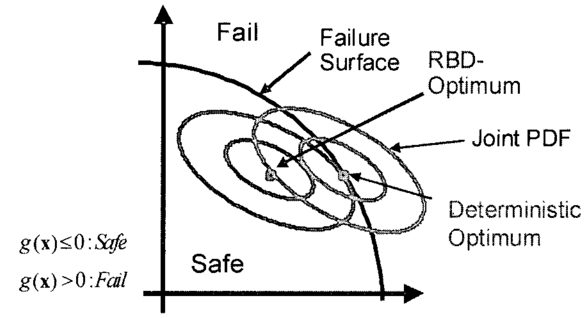


Figure 6. RBDO concept.

The RBDO can be divided into three methods, which are the Double loop method, Single loop method and Serial Single loop method depending on the numerical procedure. Various RBDO techniques are described in Table 1. The Double loop method (Frangopol, 1985; Belegundu, 1988; Thanedar and Kodiyalam, 1992; Tu et al., 1999) employs nested optimization loops that locate the most probable point (MPP) on each limit surface. The Single loop method employs a single loop to optimize the design and simultaneously update MPP values. It is efficient because there is no need to calculate a reliability index for each constraint. This eliminates the entire inner loop of the MPP search. The SLSV (Single-Loop-Single-Vector, Chen *et al.*, 1997) method belongs to the Single loop method. And The Serial Single loop method employs a sequentially equivalent deterministic optimization and reliability analysis. The SORA (Sequential Optimization and Reliability Analysis, Du and Chen, 2002) method is a Serial single loop method.

4. OPTIMIZATION FORMULATION

4.1. Deterministic Optimization Formulation

As shown in Figure 7, the design variables are six coordinate values of hardpoints. The tie rod outer point (x), front lower control arm (x, y, z), rear lower control arm (x, y), and five bushing stiffnesses, which are the front (x, z) and rear lower control arm bushings (x, y, z).

The requirements of the suspension design should be mathematically transformed to construct an optimization problem. The objective function shown in Figure 8, minimizes the deviation between the real toe angle curve and desired toe angle curve during the bump mode. Fifteen constraints are constructed by eight suspension characteristics from four analysis modes. As shown in Figure 9, suspension characteristics are linearized. The deterministic optimization (DO) formulation is formulated

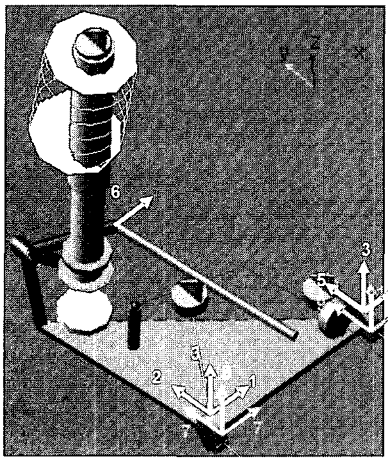


Figure 7. Design variables.

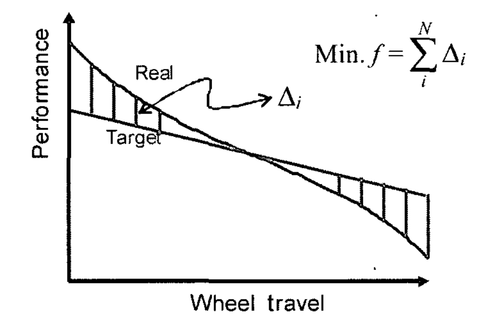


Figure 8. Definition of objective function.

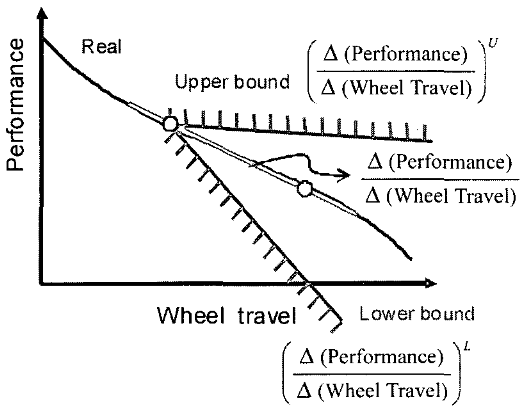


Figure 9. Definition of constraints.

as in Equation (3).

Find x_i, for $i = 1, 2, ..., \text{NDV}$,

$$\text{minimize } f(\mathbf{x}) = \sum_{k=1}^{N} \Delta_k, \tag{3a}$$

subject to

$$\text{TOE}^L \le \text{TOE}(\mathbf{x}) \le \text{TOE}^U, \tag{3b}$$
$$\text{CAS}^L \le \text{CAS}(\mathbf{x}) \le \text{CAS}^U, \tag{3c}$$
$$\text{CAM}^L \le \text{CAM}(\mathbf{x}) \le \text{CAM}^U, \tag{3d}$$
$$\text{LAS}^L \le \text{LAS}(\mathbf{x}), \tag{3e}$$
$$\text{LAFS}^L \le \text{LAFS}(\mathbf{x}) \le \text{LAFS}^U, \tag{3f}$$
$$\text{LOS}(\mathbf{x}) \le \text{LOS}^U, \tag{3g}$$
$$\text{LOFS}^L \le \text{LOFS}(\mathbf{x}) \le \text{LOFS}^U, \tag{3h}$$
$$\text{WCM}^L \le \text{WCM}(\mathbf{x}), \tag{3i}$$
$$\text{RCH}^L \le \text{RCH}(\mathbf{x}) \le \text{RCH}^U, \tag{3j}$$
$$x_i^L \le x_i \le x_i^U, \text{ for } i = 1, 2, ..., \text{NDV}, \tag{3k}$$

where

TOE: Toe angle
CAS: Caster angle
CAM: Camber angle
LAS: Lateral stiffness
LAFS: Lateral force steer
LOS: Longitudinal stiffness
LOFS: Longitudinal force steer
WCM: X-directional wheel center movement
RCH: Roll center height
NDV: No. of design variables

4.2. Reliability-Based Design Optimization Formulation

The bushing stiffnesses are dealt as the random design variables with normal distribution. The COVs (Coefficient of variation) of the bushing stiffnesses are 0.08. The design variables of the RBDO are the mean values of the random variables. This RBDO problem has 6 probabilistic constraints (limit state functions), which are the lower bound of the lateral stiffness, the upper and lower bound of the lateral force steer, the upper bound of the longitudinal stiffeness, and the upper and lower bound of the longitudinal force steer. In this problem, the joint probability density functions ($f_X(\mathbf{x})$) are unknown. The target reliability index is 1.0 (Probability level = 84.13%). The deterministic optimum is used as an initial design for the RBDO and the SLSV, which is one of the RBDO method, is applied. The RBDO formulation is formulated as in Equation (4) and Figure 10 shows the flowchart and basic concept of the SLSV.

Find μ_{X_i}, for $i = 1, 2, ..., \text{NDV}$,

$$\text{minimize } f(\mu_X) = \sum_{k=1}^{N} \Delta_k, \tag{4a}$$

$$\text{subject to } P[g_j(\mathbf{x}) \le 0] \ge R_j, \text{ for } j = 1, 2, ..., \text{NCON} \tag{4b}$$

$$\mu_{X_i}^L \le \mu_{X_i} \le \mu_{X_i}^U, \text{ for } i = 1, 2, ..., \text{NDV} \tag{4c}$$

where R: Target reliability
NDV: No. of design variables
NCON: No. of probabilistic constraints

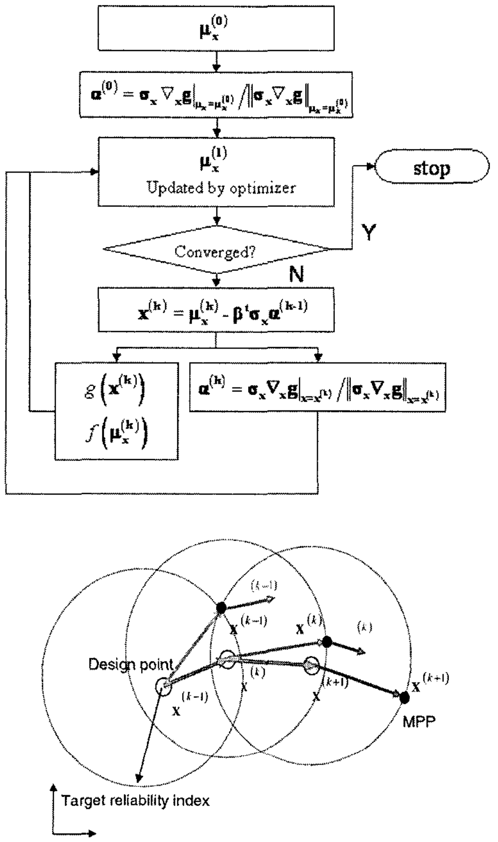


Figure 10. Flowchart and concept of RBDO (SLSV).

5. RESULTS

5.1. Objective Function

The DO and RBDO are sequentially performed by using an optimization program called DOT. The total number of function evaluations is 1068 The number of function evaluations for DO is 94 (3,022 sec.) and the that for RBDO is 974 (31,320 sec.). The objective function values are compared in Table 2. The objective function of the DO is reduced by 97.2%. However, the objective function of the RBDO is increased by 0.4% to improve the reliability of violated probabilistic constraints.

Figure 11 shows the toe angle curves, which is adopted as objective function. For the initial design, the maximum deviation is 0.45 deg at 80 mm wheel travel. However, for the DO and RBDO, the maximum deviation is 0.13 deg and 0.12 deg respectively. Consequently, an actual toe angle curve moves close to the target curve as shown in Figure 11.

Table 2. Comparison of objective function.

	Initial	DO	RBDO
Objective function	0.7093	0.0201	0.0202

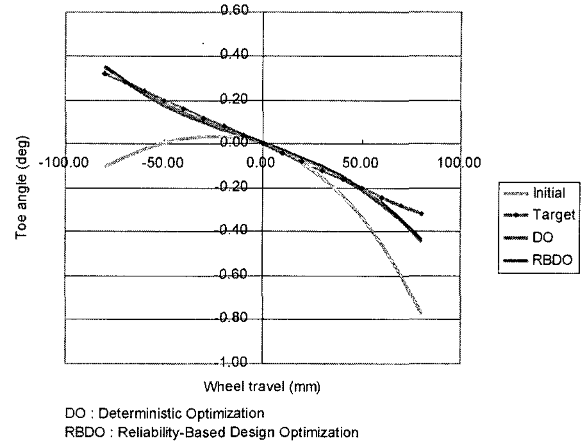


Figure 11. Comparison of toe angle curves.

5.2. Constraints

5.2.1. Deterministic optimization

In Figure 12(a) and 12(b), the change of caster angle and camber angle curves after the DO are shown but little change has appeared. This represents that the design variables are insensitive to the caster and camber angle. In order to improve these characteristics, new design variables are required from the sensitivity analysis or parametric study.

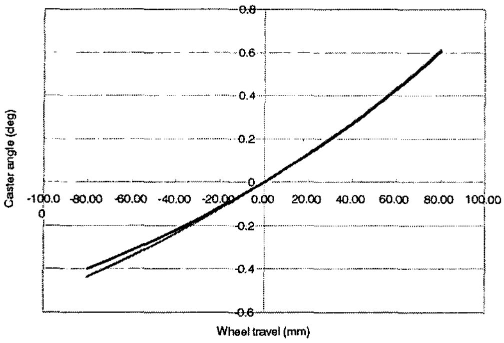
Figure 12(c) shows the change of the X-directional wheel center movement curve. The 7th constraint represents that the X-directional wheel center movement has a positive value only during bump motion.

5.2.2. Reliability-based design optimization

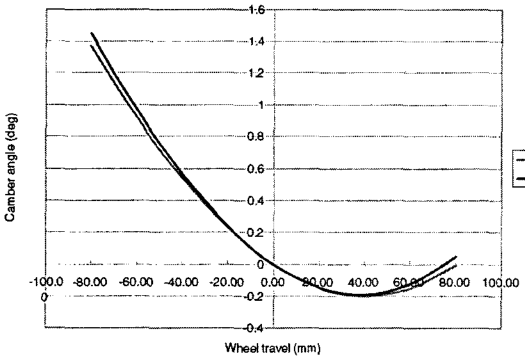
Figure 13 shows the histograms and empirical CDFs (cumulative distribution function) of probabilistic constraints. Before the RBDO, the 1st and 4th probabilistic constraints have a 46.6% and 51.7% reliability level. After the RBDO, reliability levels of violated probabilistic constraints satisfy the target reliability level. Table 3 shows the reliability level and the reliability index of active probabilistic constraints. This results represent that the RBD-optimum has 1-sigma level for the given design problem.

In Figure 14, the reliability levels of the active constraints for the DO and RBDO are compared. In order to analyze the reliability of probabilistic constraints, the descriptive sampling technique (Saliby, 1990; Ziha, 1995), which is one of the reliability analysis techniques, is used. The sample size is 1000. Prior to the RBDO, the 1st and 4th constraints are violated. After the RBDO, no constraint is violated, and the 1st constraint, which is related with lateral stiffness, is the most active.

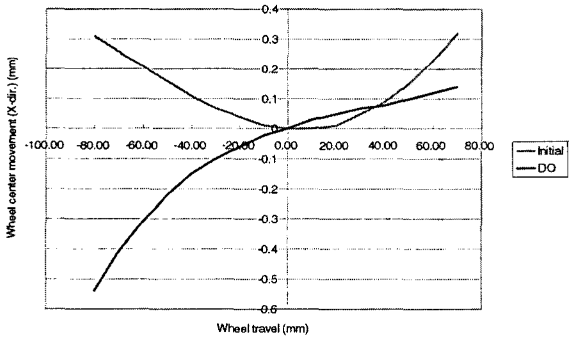
Figure 15(a) shows lateral stiffness curves. The design



(a) Caster angle



(b) Camber angle

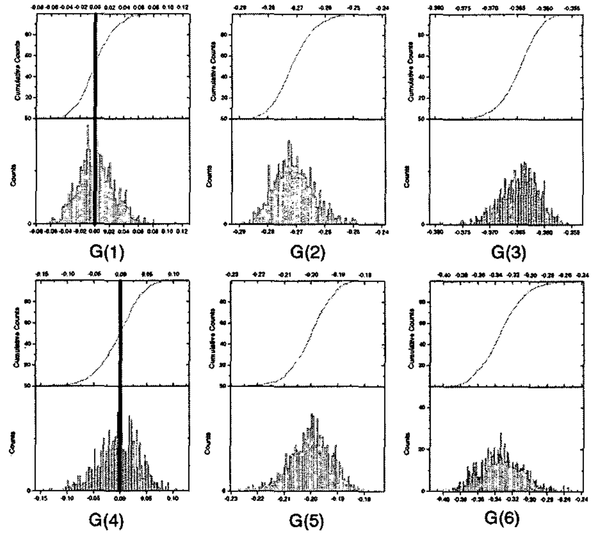


(c) X-directional wheel center movement

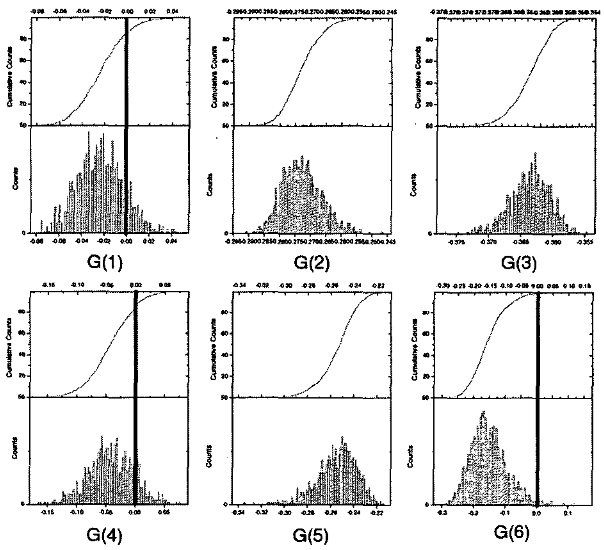
Figure 12. Comparison of suspension characteristics (bump mode).

Table 3. Reliability level and reliability index.

Active constraint no.	Reliability level		Reliability index	
	DO	RBDO	DO	RBDO
G(1)	0.466	0.849	−0.085	1.032
G(4)	0.517	0.851	0.043	1.041
G(6)	1.000	0.987	6.362	2.226



(a) Before RBDO



(b) After RBDO

Figure 13. Comparison of histograms and empirical CDF of probabilistic constraints.

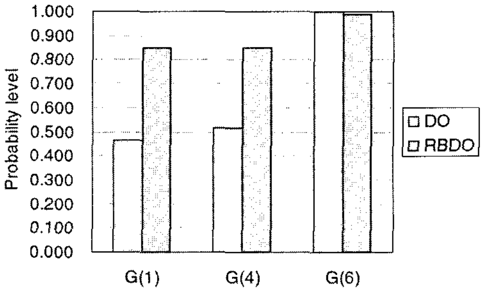
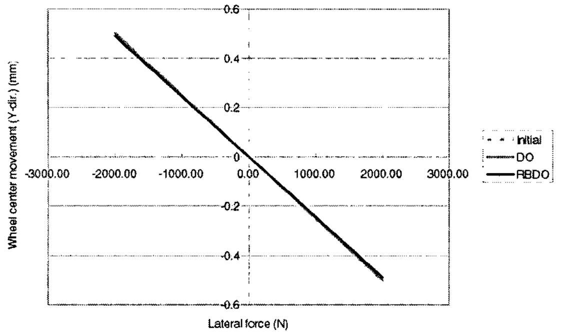
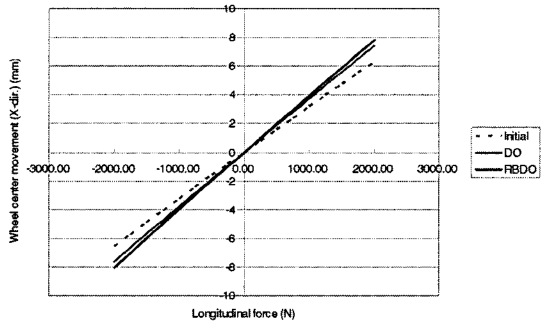


Figure 14. Compasrison of reliability levels for active probabilistic constraints.

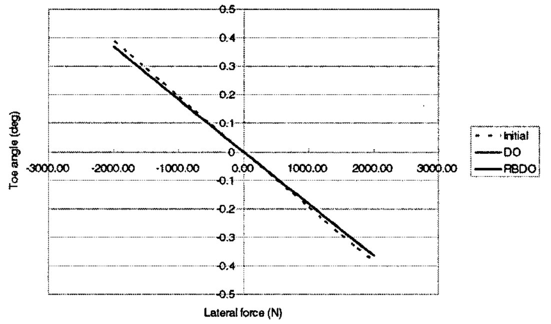
requirement is that the coefficient of lateral stiffness is set over 4000.0 N/mm. After DO, it is 4,000 N/mm and the reliability level is 46.6%. After the RBDO, it is 4098.4 N/mm and a 1-sigma level is satisfied. Figure 15(b) shows longitudinal stiffness curves and the design requirement is that the coefficient of longitudinal stiffness is set below



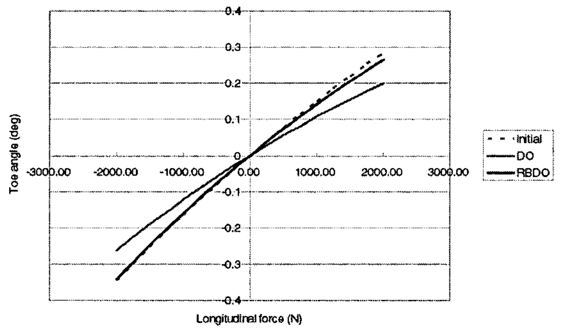
(a) Lateral stiffness



(b) Longitudinal stiffness



(c) Lateral force steer



(d) Longitudinal force steer

Figure 15. Comparison of suspension characteristics (30 mm-trail lateral and Fore & Aft mode).

270 N/mm. After the DO, it is 269.91 N/mm and the reliability level is 51.7%. After the RBDO, it is 255.75 N/mm and and a 1-sigma level is satisfied.

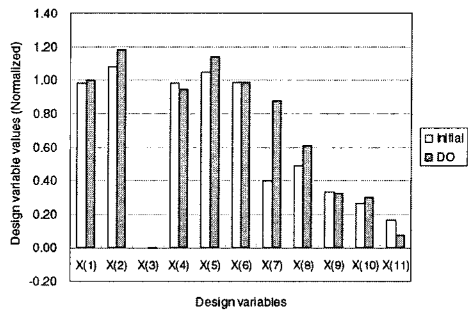


Figure 16. Comparison of design variables (normalized values, deterministic optimum).

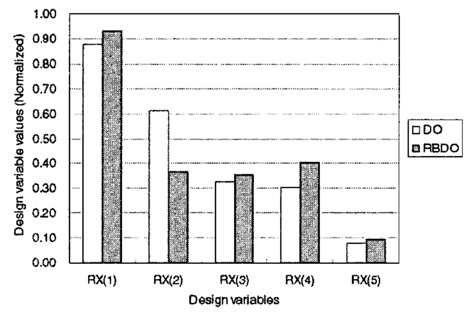


Figure 17. Comparison of design variables (normalized values, RBD-optimum).

Figure 15(c) and 15(d) show lateral force steer and longitudinal force steer curves. Both of them satisfy a 1-sigma level.

5.3. Design Variables

5.3.1. Deterministic optimization

Figure 16 shows the change of design variables. After the DO, the 2^{nd}, 5^{th}, 7^{th}, 8^{th}, 10^{th} and 11^{th} design variable have changed over 10%. In this research, the deterministic optimization is used to determine the hard points improving the K & C characteristics and to find an initial design of the RBDO.

5.3.2. Reliability-based design optimization

Figure 17 shows the change of random design variables after the RBDO. Bushing stiffnesses, which are from the 7^{th} to 11^{th} design variables for the DO, are dealt as random design variables with normal distribution. The front Z-directional bushing stiffness (2^{nd}) has changed by 39.8%. Also, the rear X-directional bushing stiffness (4^{th}) has changed by 33.5%.

6. CONCLUSIONS

In this paper, the DO and RBDO are sequentially performed for the Macpherson strut type front suspension and its results are as follows:

(1) As a result of the DO, the optimal position of hardpoints is determined to satisfy the requirement of kinematic characteristics. And the bushing stiffness is set in order to obtain an initial design for the RBDO.
(2) The toe angle curve, which is adopted as objective function, moves close to the target curve during the DO. however the toe angle curve makes little change during the RBDO. This result shows that the change of hardpoints make large influences on the toe angle characteristics.
(3) The reliability levels of active constraints has about a 50% reliability for the DO. However all constraints satisfy a 1-sigma level after the RBDO. This result represents that the reliable design for the given requirements of suspension characteristics is determined considering the uncertainty of bushing stiffness.
(4) As a result of the RBDO, the bushing stiffness largely affects the stiffness and compliance of the suspension system. And the distribution of probabilistic constraints for the DO and RBDO intuitively shows the change of the reliability level.

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