International Journal of Management Science
Vol 11, No 1, May 2005

Modeling Coordinated Contracts for a Supply Chain Consisting of Normal and Markdown Sale Markets

Chang Hwan Lee*

Associate Professor, Ajou University, School of Business Administration,
5 Wonchon-Dong, Paldal-Gu Suwon, 442-749, Korea

(Received Jul. 2004 ; Revised Mar. 2005/Apr. 2005 ; Accepted Apr. 2005)

ABSTRACT

The results of a study of the coordination effect in stocking and promotional markdown policies for a supply chain consisting of a retailer and a discount outlet (DCO) are reported here. We assume that the product is sold in two consecutive periods: the Normal Sales Period (NSP) and the subsequent Promotional Markdown Sales Period (PSP). We first study an integrated supply chain in which managers in the two periods design a common system so as to jointly decide the stocking quantities, markdown time schedule, and mark-down price to maximize mutual profit. Next, we consider a decentralized supply chain. An uncoordinated contract is designed in which decisions are decentralized to optimize the individual party's objective function. Here, three sources of system inefficiencies cause the decentralized system to earn a lower expected system profit than that in the integrated supply chain. The three sources are as follows: in the decentralized system the retailer tends to (1) stock less, and (2) keep a longer sales period, and the DCO tends to (3) stock fewer leftovers inventories and charge a higher markdown price. Finally, a numerical experiment is provided to compare the coordinated model with the uncoordinated model to explore factors that make coordination an effective approach.

Keywords: Supply Chain Management, Pricing, Inventory

* Email: chlee@magang.ajou.ac.kr

1. INTRODUCTION

The importance of coordination in the supply chain has recently been discussed in a considerable body of literature. The main argument states that while the importance of achieving integration in the supply chain is generally well recognized, for real-world applications designing a sophisticated integrated system is an arduous task. Few firms are so powerful that they can manage the entire provision of the supply chain so as to drive individual members to a superimposed integrated objective. Rather, a more realistic approach is to design a coordination contract with incentive to induce members of a supply chain to cooperate with others under a voluntary compliance base. This notion has drawn a great body of research focusing on designing supply chain coordinating contracts. Examples are coordination via buy back contracts (Pasternack [18]), markdown allowances contracts (Tsay [21]), price only contracts (Lariviere [8]), quantity flexibility contracts (Tsay [22]), price protection contracts (Taylor [21]), and Quantity Discount contracts (Jeuland and Shugan [3], Monahan [16], Lee and Rosenblatt [12], Kohli and Park [6, 7], and Weng [25]).

In this study, along a similar vein, we explore coordination effects in a supply chain consisting of a retailer and a discount outlet. Consider a supply chain consisting of a retailer and a discount outlet (DCO) selling a "short life cycle good" (e.g., personal computers, consumer electronics, fashion items) to possible consumers. If the product is not sold after the first Normal Sale Period (NSP), the supply chain has an opportunity to try a new price in a secondary Promotional Markdown Sale Period (PSP). When does the supply chain change price and how does the new price relate to the old? How many of these items should the supply chain produce in the first place, and how many should the second agent (discount outlet) stock in the secondary markdown market? Firms face very similar problems when they market a new product that has a limited product lifetime. Our study tries to provide answers to these questions. In particular, we study three issues of great importance in designing a coordinated supply chain consisting of normal and secondary markdown sale markets, namely markdown sale timing, markdown pricing and inventory stocking. A related model was studied in Lee [11], in which a two-periods newsvendor model (consisting of normal and markdown sale periods) was formulated to study supply chain coordination issues in inventory control and price setting. We expand Lee's model to include the issue of designing an integrated markdown pricing decision and markdown time schedule for launching a markdown sell event. The supply chain coordinated pricing decision studied in our model differs from the previous related studies in two aspects. First, our model attempts to explore the relationship between two pricing decisions, namely, normal and markdown prices; thus, the model is different from the single period models (see, for example, Emmons and Gilbert [2], Marvel and Peck [15], Kandel [4], and Lau, Lau, and Willett [10]). Second, we try to verify the relationship between the markdown price decision and markdown time schedule. These issues have not been discussed in the previous literature, which are aspects of coordination that are unique to the particular setting.

This paper is structured as follows. In section 2, a problem description, assumptions, and notations are presented. In section 3.1, an objective function of the retailer and the discount outlet is formulated. Then the optimal policies for the integrated supply chain are developed and analyzed. In section 3.2, we consider a decentralized supply chain. Here, decisions are made for optimizing objectives of the individual systems. The sources of supply chain distortions are verified, and coordinating strategies are suggested. In section 4, numerical examples are provided to explore the coordinated contract. A brief discussion in section 5 completes the paper.

2. THE MODELING ISSUES

Our problem is formulated as a two-period Newsboy model with the objective of maximizing the expected profit. (See Porteus [20] for a review of the Newsboy problem.) The chronology of events for the model is described as follows:

(1) In the beginning of NSP, the retailer observes the market for a short period of time labeled an *Observation Period* (OP) to evaluate market potential. After the observation period a signal about market potential is revealed. The retailer then decides an order size based on a forecasted expected demand conditional on the market signal. Since the forecasted demand is closely related to the time schedule, i.e., the lengths of NSP and PSP, to choose an optimal order size, the retailer needs to simultaneously estimate a time schedule. A similar approach can be seen in Bartmann and Beckmann [1] (a time-dependent demand Newsboy problem in which optimal order quantity and selling period are determined simultaneously). Notice that the estimated time schedule (the lengths of NSP and PSP) is the retailer's private information, and will only be used by the retailer as an internal aid for the order

quantity decision-making. It is not a firm commitment to the discount outlet, and can be changed in the later phase of the selling season. We assume inventory replenishment is allowed only once (at the beginning of the NSP). When the random demand in any point in time exceeds availability, selling opportunities are lost.

(2) At the end of NSP, if stocks are not completely depleted, the retailer initiates the second period (PSP) by offering a leftovers wholesale price, for which the retailer will sell the leftovers to the DCO.

(3) In PSP, the DCO determines a markdown sales quantity (≤ leftovers quantity) and a unit markdown sale price (≤ normal sale price) for which the DCO will sell the leftovers to the possible consumers.

Table 1 lists notations used in the paper.

Table 1. Notations

P	the NSP retail price
Q	the retailer's order quantity
C	the retailer's unit order cost
C_r	the wholesale price of leftovers at the end of the NSP
α	the length of the retailer's normal sales period ($\bar{\alpha}=1-\alpha$)
γ	the DCO's markdown price ratio ($\bar{\gamma}=1-\gamma$).For example, if a product with an initial list price of P =\$10 is later priced at 35% off the original face value, then $P\gamma$ =10(0.65)=\$6.50.
q	the DCO's markdown sales quantity
$D_N(\alpha)$	the expected random demand of the retailer in the NSP
$D_P(\alpha,\gamma)$	the expected random demand of the DCO in the PSP
$Y:=\tilde{Y}\mid Z \quad X:=\tilde{X}\mid K(Z)$	probabilistic scaling factor after information update (after OP)
$f(y)$ and $g(x)$	the probability densities of Y , X
$F(y)$ and $G(x)$	the cumulative distribution functions ($\bar{F}(y)$ and $\bar{G}(x)$ converse CDF)
$I(Q,\alpha)=\max\left[Q-yD_N(\alpha),0\right]$	the retailer's leftovers at the end of the NSP
$\xi_N(Q,\alpha):=Q/D_N(\alpha)$ and $\xi_P(q,\alpha,\gamma):=q/D_P(\alpha,\gamma)$	

The following assumptions and notations are used for modeling purposes:

Demands are probabilistic, depending on the lapse in the sales period and/or markdown price, and assumed to be comprised of two components. The first component, representing the expected demand or the location parameter of the random demand, is influenced by the lapse in the sales period and/or the markdown price. The second component, representing the probabilistic scaling parameter of the random demand, is independent of the lapse in the sales period and markdown price. This two-component approach was used in various literatures to formulate price-dependent random demands due to its simplicity and flexibility. Leland [13], for example, has considered two price-dependent random demand models-multiplicative and additive. The multiplicative model formulates random demand $d = h(P)\lambda$ as the product of an expected demand $h(P)$ (as a function of price P) and a probabilistic scaling component λ with $E(\lambda)$ =1 (see also Emmons and Gilbert [2]: linear expected demand $d = \lambda b(a - P)$ and Petruzzi and Dada [19]: iso-elastic expected demand $d = \lambda bP^{-a}$. Here, a and b are non-negative, and arbitrarily decided demand parameters). The additive model assumes random demand $d = h(P) + \lambda$ with $E(\lambda) = 0$ (see also Lau and Lau [9]: (model B) hyperbolic expected demand $d = a(bP^2 + P)^{-1} + \lambda$ and (model A) linear expected demand $d = a - bP + \lambda$). In this work, we will focus on studying the problem assuming that demand is multiplicative; however, we will also briefly discuss the results for the additive demand model in Section 3.1.

Define τ as the exogenously determined total life cycle. The lengths of NSP (PSP) are formulated as a fraction $0 \le \alpha \le 1$ of τ, i.e., $\alpha\tau$ $((1-\alpha)\tau)$. Let $D_P(\alpha,\gamma)$ $(D_N(\alpha))$ denote expected demand during the PSP (NSP). We assume a linear expected demand function, and two square root expected demand functions.

(i) Linear expected demand function assume $D_N(\alpha) = k_N\alpha$ and $D_P(\alpha,\gamma) = k_P(1-\gamma)(1-\alpha)$ (Hereafter, we will denote linear model as Model C), where k_N and k_P are positive constants of the expected demand functions.

(ii) Square root demand functions are respectively two factors (α,γ) multiplicative model $D_P(\alpha,\gamma) = \sqrt{k_P(1-\alpha)(1-\gamma)}$ (Model A) and two factors additive model $D_P(\alpha,\gamma) = \sqrt{k_{P1}(1-\alpha) - k_{P2}\gamma}$ with $0 \le \gamma \le \min\left[k_{P1}(1-\alpha)/k_{P2}, 1\right]$ (Model B) where k_{P1} and k_{P2} are positive constants of the expected demand function.. We assume $D_N(\alpha) = \sqrt{k_N\alpha}$ for both models.

Let $(\tilde{Y},\tilde{X})$ be the probabilistic scaling parameters of NSP and PSP without the prior knowledge of market signal. We assume that $(\tilde{Y},\tilde{X})$ can be expressed as a sum of two random components $\tilde{Y} = Z + \varepsilon_y$ and $\tilde{X} = K + \varepsilon_x$ with $\varepsilon_y(\varepsilon_x)$ and $Z(K)$ being independent, and $E(\tilde{Y}) = 1$ and $E(\tilde{X}) = 1$. Here, Z, denoting "the signal about market potential", can be observed after observation period (OP). This approach is similar to the random demand model formulated in Tsay [22]. However in his model the demand is not a function of the elapse of time. We assume that $\tilde{X}$ and $\tilde{Y}$ are correlated. For example, if $K(Z) = a + bZ$ then $Cov(\tilde{Y},\tilde{X}) = bVar(Z)$, and the random component of demand in the NSP ($\tilde{Y}$) is positively (negatively) correlated with that of the PSP ($\tilde{X}$) if $b > 0 (b < 0)$. Define $Y := \tilde{Y} \mid Z$ and $X := \tilde{X} \mid K(Z)$ as random variables given that the market signal Z has been observed. For example, if both ε_y and ε_x are normally distributed with variances $\sigma_Y{}^2$ and $\sigma_X{}^2$, then Y and X are normally distributed. Assuming now $\sigma_X = \sigma_Y = \sigma$, then the demand variance in the NSP and PSP can be expressed as

(i) Two factors multiplicative square root demand model (Model A): $\sigma_N^2\alpha$ and $\sigma_P^2(1-\alpha)$, where $\sigma_N^2 = k_N\sigma^2$ is a constant, and $\sigma_P^2 = k_P\sigma^2(1-\gamma)$ is a decreasing function of γ. We see that the variance in PSP, particularly σ_P^2, increases as the expected demand increases (due to price (γ) decrease). A justification for this result can be seen in Lau and Lau [9]. They state that for a high demand level (due to low price) beyond the normal operating range, the random demand may have a large variance due to a lack of past experience to draw on.

(ii) Two factors additive square root demand model (Model B): $\sigma_N^2\alpha$ and $\sigma_P^2(1-\alpha) - \tilde{\sigma}_P^2$, where $\sigma_N^2 = k_N\sigma^2$ and $\sigma_P^2 = k_{P1}\sigma^2$ are constants, and $\tilde{\sigma}_P^2 = k_{P2}\sigma^2\gamma$ is an increasing function of γ.

It is seen that in the square root demand functions when $\sigma_N = \sigma_P$, the total variance is not affected by where the period is divided, and the variance is allocated between the two parts of the period in proportion to their length.

(iii) Linear demand model (Model C): $\sigma_N^2\alpha^2$ and $\sigma_P^2(1-\alpha)^2$, where $\sigma_N^2 = (k_N\sigma)^2$ is a constant, and $\sigma_P^2 = \left[k_P\sigma(1-\gamma)\right]^2$ is a decreasing function of γ. We see that in the linear demand model total variance is closely related to the markdown sale time schedule. In particular, the total variance is proportional to the value $\alpha^2 + (1-\alpha)^2$ which is strictly convex between [0,1]; thus, the variance can be reduced to the minimum when the two periods are divided equally.

3. RETAILER-DCO DECISION-MAKING SYSTEMS

In section 3.1, to provide an efficient benchmark, we consider an integrated system in which the retailer and the DCO form a common system, share demand information, and jointly design an integrated ordering, time schedule, and markdown policy so as to deliver the greatest possible expected system profits. In section 3.2, a decentralized supply chain is considered. We will focus on verifying possible sources of inefficiencies that cause sub-optimality in the decentralized system.

3.1 Centralized Supply Chain Model

In the centralized system, the supply chain will maximize the joint objective function of the retailer and the DCO. Since the market signal Z is assumed to be known to both parties, decisions regarding $Q(\alpha)$ and α are made based on the forecast about (Y,X). Let $\Omega_N(\Omega_P)$ denote the retailer's (DCO's) expected profit excluding the wholesale revenue (cost) of leftovers. Denote $\xi_N(Q,\alpha) := Q / D_N(\alpha)$ and $\xi_P(q,\alpha,\gamma) := q / D_P(\alpha,\gamma)$ as in Table 1:

$$\begin{cases} \text{Retailer}: \ \Omega_N(Q,\alpha) := P\left\{ \int_0^{\xi_N} yD_N(\alpha)\, dF + \int_{\xi_N}^{\infty} QdF \right\} - CQ \\ \\ DCO: \ \Omega_P(q,\gamma \mid I(Q,\alpha),\alpha) := P\gamma\left\{ \int_0^{\xi_P} xD_P(\alpha,\gamma)\, dG + \int_{\xi_P}^{\infty} qdG \right\} \end{cases}.$$

The expected joint profit for the centralized model, which we denote as Π_{NP}^C is:

$$\max_{Q,\alpha} \Pi_{NP}^C(Q,\alpha) = \Omega_N(Q,\alpha) + E_y\left\{ \max_{q,0\le\gamma\le1} \Omega_P(q,\gamma \mid I(Q,\alpha),\alpha) \right\}. \qquad (1)$$

The objective function in (1) reveals that the goal of the integrated supply chain is to choose the centralized optimal policy (Q^C, α^C, q^C, γ^C). Solving $\partial\Omega_P / \partial q = 0$ leads to $q^C = I$. Clearly, making $q^C = I$ can maximize the supply of leftovers available for sale in the PSP. The only impact of making $q^C < I$ is to reduce the supply of goods available for sale in the PSP, tightening a constraint cannot produce a better solution. Substituting $q^C = I$ into the objective function in (1), and solving $\partial\Omega_P(q^C = I,\gamma)/\partial\gamma = 0$ give the optimal markdown price ratio

$\gamma^C(q^C = I)$, satisfying the following expression. Note that since $q^C = I$, $\xi_P := I(Q,\alpha)/D_P(\gamma,\alpha)$.

$$D_P(\gamma,\alpha) + \gamma \frac{dD_P(\gamma,\alpha)}{d\gamma} = \frac{-I(Q,\alpha)\bar{G}(\xi_P)}{\int_0^{\xi_P} xdG} . \tag{2}$$

The right-hand side in (2) is negative, and the absolute value decreases in γ and approaches 0 when γ approaches 1. Since $d^2(\gamma D_P)/d\gamma^2 < 0$, the left-hand side changes its sign from positive to negative and continuously decreases thereafter. The two properties give a unique optimal markdown price ratio γ^C that is greater than the risk-less price ratio satisfying $D_P + \gamma dD_P / d\gamma = 0$. Petruzzi and Dada [19] state that the optimal price depends on the nature of the uncertainty: additive uncertainty leads to an optimal price that is less than the risk-less price, and multiplicative uncertainty leads to an optimal price that is more than the risk-less price. This property also applies to our study. At the end of this section, we will show that the markdown price of the additive demand model is smaller than that in the deterministic case. The necessary conditions for (Q,α) that maximize (1) satisfy

$$\partial\Pi_{NP}^C / \partial Q = 0 \Rightarrow \bar{F}(\xi_N) - \frac{C}{P} + \int_0^{\xi_N} \gamma\bar{G}(\xi_P)dF = 0 \text{, and} \tag{3}$$

$\partial\Pi_{NP}^C / \partial\alpha = 0 \Rightarrow \partial\Omega_N / \partial\alpha + \partial E_y\{\Omega_P\} / \partial\alpha = 0$, where

$$\frac{\partial\Omega_N}{\partial\alpha} = \frac{dD_N}{d\alpha}\int_0^{\xi_N} ydF \geq 0 \text{ , and}$$

$$\frac{\partial E_y\{\Omega_P\}}{\partial\alpha} = \frac{\partial D_P}{\partial\alpha}\int_0^{\xi_N}\int_0^{\xi_P} \gamma xdGdF - \frac{dD_N}{d\alpha}\int_0^{\xi_N} \gamma\bar{G}(\xi_P)ydF \leq 0 \tag{4}$$

We see that increasing α by one unit will increase the retailer's objective function by $\partial\Omega_N / \partial\alpha$ but will reduce the DCO's profit by $-\partial E_y\{\Omega_P\}/\partial\alpha$. Clearly, the supply chain has a tradeoff problem at hand. Let $\Omega(1) := \partial\Omega_P / \partial Q = 0$ and $\Omega(2) := \partial\Omega_N / \partial Q = 0$. The optimal order quantity derived from (3) show that $\partial Q^*(\alpha)/\partial\alpha |_{\Omega(1)} < 0$ and $\partial Q^*(\alpha)/\partial\alpha |_{\Omega(2)} > 0$. This tells us that optimization from the view of the DCO requires the retailer to reduce order quantity so as to generate fewer leftovers when the normal sales period is extended ($\partial Q / \partial \alpha |_{\Omega(1)} < 0$). However, the optimization from the retailer's viewpoint reveals a totally different result ($\partial Q / \partial \alpha |_{\Omega(2)} > 0$). Therefore, a conflict of interest exists between the retailer and the DCO. Proposition 1 shows the properties of the optimal solution.

Proposition 1. (Proof. See Appendix 1.)

1.1 *For models A, B, and C the objective function* Π_{NP}^{C} *is jointly concave with respect to* (Q,γ,α) *if* $g(x)$ *has increasing or constant failure rates.*

1.2 *Comparative statics: Let* $\Pi(1) := \partial\Pi_{NP} / \partial\alpha = 0$ *and* $\Omega(3) := \partial\Omega_P / \partial\gamma = 0$. *The optimum* γ^* *and* α^* *satisfy (2) and (4) show (a)* $\partial\gamma^*(I,\alpha)/\partial I |_{\Omega(3)} < 0$, *(b)* $\partial\gamma(I,\alpha)/\partial\alpha |_{\Omega(3)} < 0$, *and (c)* $\partial\alpha(Q)/\partial Q |_{\Pi(1)} < 0$. □

Proposition 1.1 reveals that the objective function is concave if the density functions have an increasing or constant failure rate. While the proposition limits the distribution, increasing or constant failure rate class is broad enough to include most of the distribution one would choose to employ. For example, the normal and the exponential are both relatively widely used increasing or constant failure rate densities that are quite probable for formulating random demands (see, for example, Parlar and Weng [17] and Li, Lau, and Lau [14].) Proposition 1.2 shows that the system gives a greater discount as leftovers increase or the markdown period decreases. Proposition 1.2 also reveals that the supply chain reduces the length of NSP so that the leftovers can be moved to DCO in a more timely fashion as leftovers increase (I increases as Q increases).

Now, let us briefly discuss the result obtained from an additive random demand model. Recall that the necessary condition in (2) is obtained from solving for the optimal value of $q^C = I$ for a given γ first, and then substituting the result back into the objective function, and solving for the optimal γ. This approach was employed in Whitin [26]. Here, we will use a different approach in which we solve for the optimal γ for a given q first, and then solve for the optimal q (see Zabel [27]). The random demand in the additive model is formulated as $y + D_N(\alpha)$ with $E(Y) = 0$ (we assume $Y \in [B,A]$ is well defined so that $y + D_N(\alpha)$ does not become negative). Similarly, we assume random demand to be $x + D_P(\alpha,\gamma)$, $X \in [D,C]$ with $E(X) = 0$. Define $\xi_N := Q - D_N(\alpha)$, and $\xi_P := q - D_P(\alpha,\gamma)$. The DCO's expected profit is $\Omega_P = P\gamma\left\{\int_D^{\xi_P}(D_P + x)\, dG + \int_{\xi_P}^{C}(\xi_P + D_P)dG\right\}$. Solving

$\partial\Omega_P / \partial\gamma = 0$ reveals γ^C, satisfying $D_P + \gamma D_P' = \int_{\xi_P}^{C} (x - \xi_P) dG$. The right-hand side is positive; thus, γ^C is smaller than the risk-less price ratio satisfying $D_P + \gamma D_P' = 0$ (notice that, in the multiplicative model, γ^C is greater than the risk-less price ratio). Substituting the optimal price reveals $\partial\Omega_P / \partial q = \gamma\bar{G}(\xi_P) \geq 0$, and this leads to $q^* \to \infty \Rightarrow q^C = I$. The optimal solution for the order quantity Q^C is identical with that in (3). Solving $\partial\Pi^C / \partial\alpha = 0$ results in α^C, satisfying $D_N' \int_B^{\xi_N} dF + \int_B^{\xi_N} \gamma \left\{ D_P' \int_D^{\xi_P} dG - D_N' \int_{\xi_P}^{C} dG \right\} dF = 0$.

3.2 The Contract Models for the Decentralized Supply Chain

In this section we model a decentralized system as a Stackelberg Game in which the DCO, acting as a follower, chooses a markdown sale quantity $q \leq I$, pays C_r for each leftover item, and designs a markdown price γ. The retailer, on the other hand, acting as a channel leader, designs (Q, α, C_r) to maximize his/her individual objective function. In addition to the notations given in Table 1, Table 2 lists additional notations used in the paper.

Table 2. Notations

$\tilde{g}(x)$, $\tilde{G}(x)$	probability density and cumulative distribution of $\tilde{X}$
φ	denotes probability densities g or $\tilde{g}$
Γ	denotes cumulative distribution functions G or $\tilde{G}$
χ	denotes variables x or $\tilde{x}$
$\rho(\_) := \varphi(\_)/\bar{\Gamma}(\_)$	
$\xi_L(Q, \bar{q}, \alpha) = (Q - \bar{q}) / D_N$ (see equation (6) for the definition of $\bar{q}$)	
$\xi_{P1} = \bar{q} D_P(\gamma_1, \alpha)$, and $\xi_{P2} = I / D_P(\gamma_2, \alpha)$	

As in Section 3.1. let Ω_N^{UC} (Ω_P^{UC}) denote the retailer's (DCO's) expected profit excluding the wholesale revenue (cost) of leftovers in the decentralized supply chain. Also denote $\Pi_N^{UC}(Q, \alpha \mid q, \gamma))x := \Omega_N^{UC}(Q, \alpha) + E_Y\{\max_{C_r} C_r \times q\}$ as the retailer's expected profit (including the expected wholesale revenue of leftovers). The retailer's problem is $\max_{Q,\alpha} \Pi_N^{UC}(Q, \alpha \mid q, \gamma)$, and the DCO's optimization problem is $\max_{q,\gamma} \{\Omega_P^{UC}(\gamma,q \mid I,\alpha) - C_r \times q\}$. Two scenarios regarding information sharing are assumed. (1) Scenario 1: The retailer does not share the market signal Z with the DCO, and the DCO has no way to know market signal Z; thus, the DCO's decisions are made based on $\tilde{g}(\tilde{x}),\tilde{X}$, and (2) Scenario 2: the retailer truthfully shares the market signal Z with the DCO, or the DCO has a way to obtain information regarding Z; hence, the DCO's decisions are made based on $g(x), X$. The retailer's optimal wholesale price satisfies:

$$C_r^{UC} = \begin{cases} C_{r1} \quad satisfies \;\; \bar{q} + \dfrac{\partial \bar{q}}{\partial C_r} C_r = 0 & if \;\; \bar{q}(C_{r1}) \le I \\ C_{r2} = \;\; P\gamma \bar{\Gamma}(\xi_{P2}) & if \;\; \bar{q}(C_{r1}) > I \end{cases} . \tag{5}$$

The (q,γ) maximizing the DCO's objective function are given by the following expression:

$$q^{UC} = \begin{cases} \bar{q}(C_{r1}) & if \;\; \bar{q}(C_{r1}) = D_P(\gamma_1,\alpha)\,\Gamma^{-1}(1 - C_{r1} / P\gamma_1) \le I \\ I & if \;\; \bar{q}(C_{r1}) > I \end{cases} \text{, and}$$

$$\gamma^{UC} = \gamma_A \;\; \text{A=1, 2 satisfies} \;\; D_P(\gamma,\alpha) + \gamma \frac{dD_P}{d\gamma} = \frac{-q^{UC}\bar{\Gamma}(\xi_{PA})}{\int_0^{\xi_{PA}} \chi d\Gamma} . \tag{6}$$

Upon substituting $C_{r2} = P\gamma_2\bar{\Gamma}(\xi_{P2})$, the optimal order quantity Q^{UC} satisfies:

$$\bar{F}(\xi_N) - C/P + \int_{\xi_L}^{\xi_N} \gamma_2 \bar{\Gamma}(\xi_{P2})(1 - \rho(\xi_{P2})\xi_{P2}) \, dF = 0 , \tag{7}$$

and α^{UC} satisfies:

$$\int_0^{\xi_N} y \frac{dD_N}{d\alpha} dF + \int_{\xi_L}^{\xi_N} \gamma_2 \left\{ \varphi(\xi_{P2})(\xi_{P2})^2 \frac{\partial D_P}{\partial \alpha} - y\bar{\Gamma}(\xi_{P2})(1 - \rho(\xi_{P2})\xi_{P2}) \frac{dD_N}{d\alpha} \right\} dF$$
$$+ \int_0^{\xi_L} \xi_{P1}\gamma_1 \bar{\Gamma}(\xi_{P1}) \frac{\partial D_P}{\partial \alpha} dF . \tag{8}$$

Proposition 2. (See Appendix 2 for the Proof of Proposition 2.)

2.1 *For both models A and B,* $\Pi_N^{UC}(Q,\alpha,C_r^{UC}(Q,\alpha))$ *is concave in* (Q,α) *if* $\varphi(x)$ *has an IFR.* $\Pi_P^{UC}(q,\gamma)$ *is concave in* (q,γ).

2.2 *Comparative statics: Let* $\Pi(1) := \partial \Pi_N^{UC} / \partial \alpha = 0$ *and* $\Omega(1) := \partial \Omega^{UC} / \partial \gamma = 0$.

(a) $\partial\gamma / \partial I \,|_{\Omega(1)} < 0$,

(b) $\partial\gamma / \partial\alpha \,|_{\Omega(1)} < 0$, *and (c)* $\partial\alpha / \partial Q \,|_{\Pi(1)} < 0$.

***2.3** System Inefficiency (Double Marginalization): Under scenario 2* $(\varphi(x) = g(x))$, *if* $g(x)$ *has an increasing failure rate (IFR)*

(a) Given $(\alpha^C, q^C) = (\alpha^{UC}, q^{UC})$, $Q^{UC} \le Q^C$.

(b) Given $(Q^C, q^C) = (Q^{UC}, q^{UC})$, $\alpha^{UC} \ge \alpha^C$.

(c) Given $(\alpha^C, Q^C) = (\alpha^{UC}, Q^{UC})$, $q^C = I \ge q^{UC} \Rightarrow \gamma^{UC} \ge \gamma^C$. □

Proposition 2.3 reveals three sources of system distortions (Double Marginalization). In general, these distortions stem from making decisions based on "local costs or revenues" rather than on "system costs or revenues".

Factor 1: The DCO's local sale margin is less than the full system margin ($P\gamma - C_r \ge P\gamma$); thus, it drives the retailer to order less and price higher than the centralized policy $(q^C = I \ge q^{UC} \Rightarrow \gamma^{UC} \ge \gamma^C)$.

Factor 2: System's expected salvage revenues $\int_0^{\xi_N} P\gamma\Omega_P \, dF$ (salvage revenue in DCO) versus the retailer's local salvage revenue $\int_0^{\xi_N} C_r q \, dF$ (whole sale revenue of leftovers). This leads to the following two system distortions. (i) The retailer orders less than the centralized order quantity $Q^{UC} \le Q^C$. (ii) The time schedule in the decentralized system is made based on analyzing the trade-offs regarding the retailer's normal sale revenue and wholesale revenue of leftovers ($\int_0^{\xi_N} C_r q \, dF$) and thus, the result is confined to optimizing the retailer's individual benefit. Whereas in the centralized system, the trade-off analysis focuses on optimizing system-wide profits. It considers the retailer's normal sale revenue, and the true system salvage revenue from the DCO ($\int_0^{\xi_N} P\gamma\,\Omega_P \, dF$). Proposition 2.3 (b) reveals that in the decentralized system the retailer tends to keep a longer sales period. This result has not been discussed in the previous literature, which is an aspect of coordination that is unique to the particular setting. In the following Section 3.3 we will furnish a numerical experiment to provide a more detailed study of the systems distortions and their effects on the supply chain.

3.3 Analysis of Retailer-DCO Coordination: A Numerical Experiment

This section presents a numerical example designed to explore and compare centralized and decentralized decision-making processes. The following assumptions are used in the numerical computation: (a) ε_Y and ε_X are exponentially distributed with means $E(\varepsilon_Y) = E(\varepsilon_X) = 0.5$. $a = 0$, $b = 1$ $(K = Z)$, and K and Z are exponentially distributed with $E(K) = E(Z) = 0.5$. (b) Base parameters will take the following values: $P = 50$, $C = 30$, $z = 0.4$, $k_N = 700$, and $k_p = 1{,}500$. (c) We assume Scenario 1; thus, the retailer does not share the market signal Z with the DCO. (d) Expected demand in PSP is assumed to be a two factors multiplicative model $D_P(\alpha,\gamma) = \sqrt{k_P(1-\alpha)(1-\gamma)}$. Four parameters are varied to see the effect of coordination. z changes from 0.2 to 0.75; k_N changes from 650 to 1100; C changes from 0.1 to 45; and P changes from 50 to 95. Figures 1-1 to 4-1 show difference in supply chain expected profits between the centralized model and the decentralized model. Figures 1-2 to 4-2 show order quantities of the centralized model and the decentralized model. Figures 1-3 to 4-3 show normal and markdown schedules of the centralized model and the decentralized model. Figures 1-4 to 4-4 show markdown prices of the centralized model and the decentralized model.

Figures 1-1 to 4-1 indicate that the coordination approach outperforms the uncoordinated approach on every occasion, but the benefits are most significant when the market signal Z and price P are relatively high, or the production cost C is relatively small. For example, Figure 3-1 reveals that the supply chain profit generated from the centralized model is almost \$6,000 more than that generated from the decentralized model when $C = 0.1$. Thus, both parties can design a fair arrangement for sharing the additional benefits generated from adopting the coordinated policies. We also see from Figure 2-1 that the coordination model generates higher profits when normal sale demand rate k_N is relatively high or relatively low (demands are either extremely sensitive or extremely insensitive to the length of NSP).

Figure 1-2 illustrates optimal stocking policies for the centralized model and the decentralized model as a function of (Z). It shows that as a result of market signaling a high demand (Z), the retailer increases the stock quantity Q^{UC}, and unilaterally increases α^{UC} so as to generate more normal sales. Whereas, Figures 1-2 and 1-3 show that $\bar{q}^{UC}$ is closely related to the length of the markdown period $1-\alpha^{UC}$; thus, $\bar{q}^{UC}$ shows a decreasing trend (due to the α^{UC} increase). As a result, the supply chain shows a clear evidence of disconnected decision-making pattern in which leftovers (I increases as Q^{UC} increases) and markdown quantity reveals completely different behaviors, and generates a large quantity of wasted leftovers. We see that this phenomenon recurs in Figures 2 through 4. For example, Figure 2-2 illustrates optimal stocking policies as a function of k_N. Due to a high demand (k_N), the retailer increases the stock quantity Q^{UC}, and increases α^{UC} so as to generate more normal sales. Whereas, Figures 1-2 and 1-3 show that $\bar{q}^{UC}$ is closely related to the length of the markdown period $1-\alpha^{UC}$; thus, $\bar{q}^{UC}$ shows a decreasing trend (due to the α^{UC} increase). On the other hand, in the centralized model, Figure 1.3 shows that markdown period $1-\alpha^{C}$ increases as leftovers quantity increases (due to Q^{C} increases). Here, the retailer-DCO alliance knows that both parties' markets signal high demands; therefore, markdown period $1-\alpha^{C}$ increases to absorb excessive leftovers. We can also observe a similar pattern from Figure 2.4. Here, the retailer-DCO alliance knows that the retailer will face a high market demand; therefore, the markdown price γ^{C} decreases as leftovers quantity increases (due to Q^{C} increases).

Figures 1-2 through 4-2 reveal that, in general, the centralized model orders a larger quantity as described in Proposition 2.3 (a). We also see from Figures 1-3 through 4-3 that in the centralized model, the retailer tends to maintain a shorter sales period compared to those in the decentralized model so that the leftovers can be moved to the DCO in a more timely fashion in order to take advantage of a more time-elastic market. This unselfish spirit undoubtedly generates a higher profit for the coordination model.

Similar patterns can also be seen from Figures 1-4 through 4-4. They show that, in the centralized model, the retailer tends to maintain a lower markdown priced compared to those in the decentralized model so as to increase the markdown demand. Figures 1-4 through 4-4 reveal that in the centralized model, inasmuch as all of the leftovers are moved to the DCO for a markdown sale, the markdown price is designed as a decreasing function of order quantity Q^{C}. Thus, markdown price decreases (increases) as leftovers quantity increases (decreases). In addition, as a supporting mechanism the markdown sale period $1-\alpha^{C}$ also increases (decreases) as Q^{C} increases (decreases) so that the leftovers can be moved to the DCO in a more timely fashion. Except in Figure 2, here both α^{C} and Q^{C} increases as normal demand rate k_N increases (normal demand is more sensitive to the length of NSP).

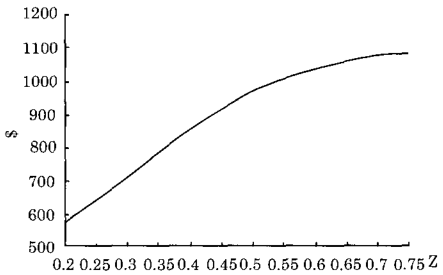

Figure 1-1. Difference in Expected Profits

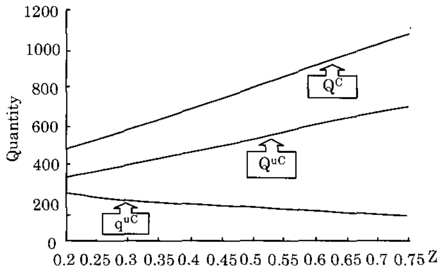

Figure 1-2. Oder Quantities as a function of Z

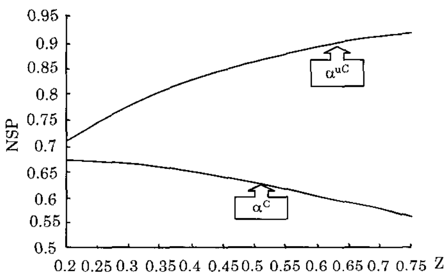

Figure 1-3. Lengths of NSP as a function of Z

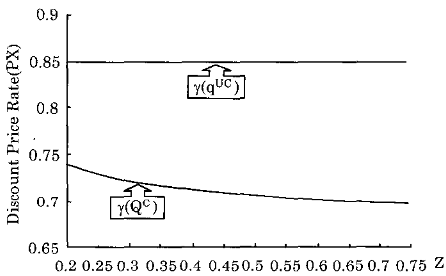

Figure 1-4. Markdown Prices as a function of Z

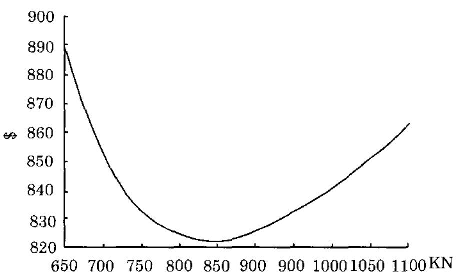

Figure 2-1. Difference in Expected Profits

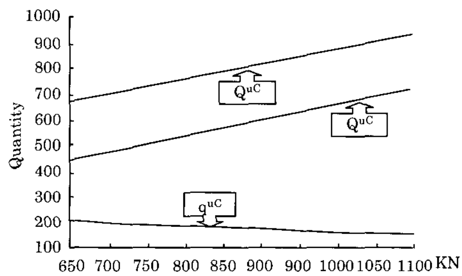

Figure 2-2. Oder Quantities as a function of KN

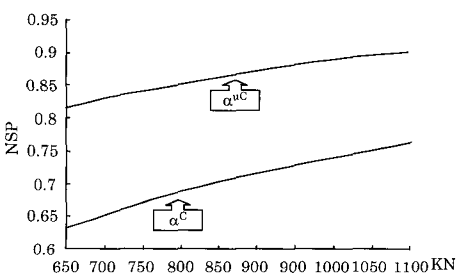

Figure 2-3. Lengths of NSP as a function of KN

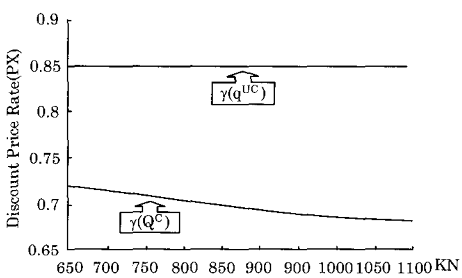

Figure 2-4. Markdown Prices as a function of KN

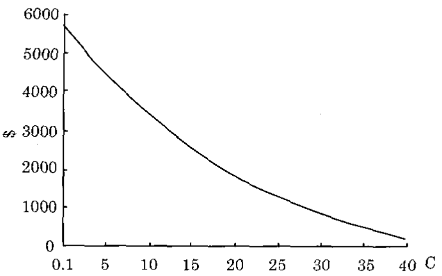

Figure 3-1. Difference in Expected Profits

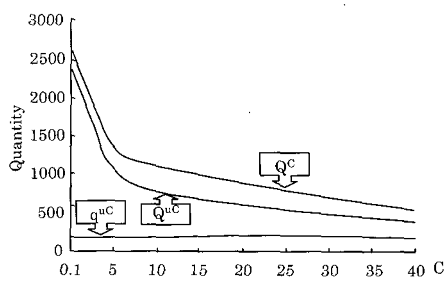

Figure 3-2. Oder Quantities as a function of C

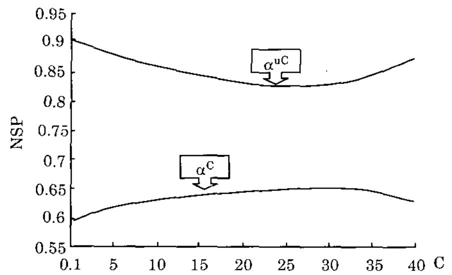

Figure 3-3. Lengths of NSP as a function of C

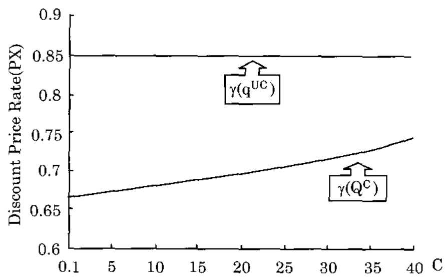

Figure 3-4. Markdown Prices as a function of C

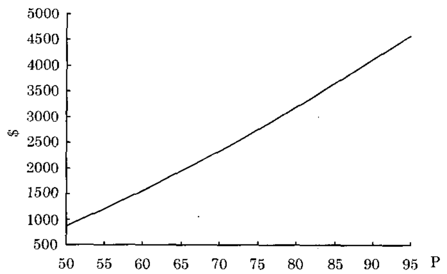

Figure 4-1. Difference in Expected Profits

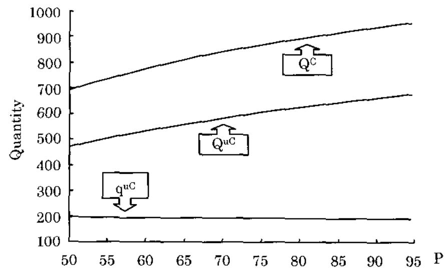

Figure 4-2. Oder Quantities as a function of P

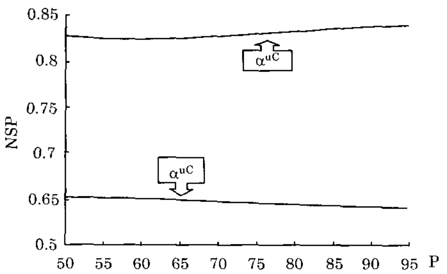

Figure 4-3. Lengths of NSP as a function of P

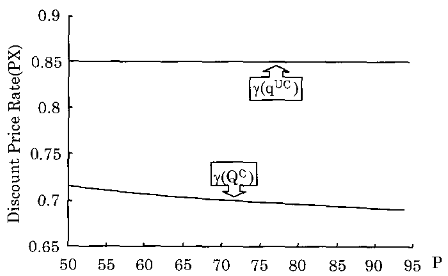

Figure 4-4. Markdown Prices as a function of P

4. DISCUSSION AND CONCLUSION

In this paper, we analyzed a model to study the effects of retailer-DCO coordination in supply chain stocking and promotional markdown operations. Our purpose was to develop an understanding of how, when, and why coordination helps to improve expected profits.

We first developed an integrated system as a benchmark case in which the retailer-DCO alliance jointly decides the stocking quantity, a plan for markdown time schedule, and markdown price to maximize mutual profit. Next, we considered a decentralized system in which the DCO, acting as a follower, individually optimizes his/her objective function by choosing a markdown sale quantity and a markdown price. The retailer, on the other hand, acting as a channel leader, designs order quantity, markdown time schedule, and wholesale price of leftovers to maximize his/her individual objective function. Three sources of system inefficiencies cause the decentralized system to generate a lower expected profit than that for the integrated system. These are as follows: in the decentralized system the retailer tends to (1) stock less, and (2) keep a longer sales period, and the DCO tends to (3) stock lesser inventories and charge a higher markdown price.

A numerical experiment is provided to compare centralized model and decentralized model. Our study indicates that the coordination approach outperforms the uncoordinated approach on every occasion, but the benefits are most significant when the market signal Z and price P are relatively high, or the production cost C is relatively small. We also see that the coordination model generates higher profits when normal sale demand rates k_N are relatively high or relatively low (demands are either extremely sensitive or extremely insensitive to the length of NSP).

We have observed several distinctive decision-making patterns that might contribute to the sub-optimality of the uncoordinated model. First, in the uncoordinated model, the two parties frequently show disconnected decision-making patterns. For example, when P increases, the retailer is overly aggressive while the DCO is overly pessimistic in their stocking policies. As a result the supply chain generates a large quantity of wasted leftovers. Second, we have observed that in the decentralized model, the retailer tends to hold the inventories for an excessive period of time. In the coordination model, the retailer reduces his/her sales period so that the leftovers can be moved to the DCO earlier to take advantage of a more time-elastic market. Finally, in the uncoordinated model the DCO stocks less inventory and charges higher prices for the leftover items, thereby resulting in a lower system profit.

Our focus thus far does not allow us to study the possibility of a situation involving multiple markdown periods. Generally, in a real-world application a markdown operation may consist of more than one discount period. The multiple markdown periods problem has been studied by Khouja [5] on a single company level, but in this model demand in the markdown period is treated independently of markdown rate and markdown timing. Future work on the two-party progressive promotional markdown model could certainly shed further light on the topic. Finally, the problem of jointly deciding normal sales price and markdown discount rates also should receive future research effort.

REFERENCES

[1] Bartmann, D. and M. Beckmann, Lecture Notes in Economics and Mathematical Systems, Inventory Control: Models and Method, Berlin Heidelberg Germany, Springer-Verlag 1992.

[2] Emmons, H. and S. M. Gilbert, "The Role of Return Policies in Pricing and Inventory Decision," *Management Science* 44 (1998), 276-283.

[3] Jeuland, A. P. and S. M. Shugan, "Managing Channel Profit," *Marketing Science* 3 (1983), 239-271.

[4] Kandel, E., "The Right to Return," *The Journal of Law and Economics* 39 (1996), 329-356.

[5] Khouja, M., "The Newsboy Problem Under Progressive Multiple Discount," *European Journal of Operational Research* 84 (1995), 458-466.

[6] R. Kohli and H. Park, "A Cooperative Game Theory Model of Quantity Discount," *Management Science* 35 (1989), 693-700.

[7] Kohli, R. and H. Park, "Coordinating Buyer-seller Transactions Across Multiple Products," *Management Science* 40 (1994), 1145-1150.

[8] Lariviere, M. A., "Supply Chain Contracting and Coordination with Stochastic Demand", in S. Tayur, R. Ganeshan, and M. Magazine (Eds.), *Quantitative Models for Supply Chain Management*, Kluwer's Academic Publisher, Norwell, MA, 1999.

[9] Lau, H. and H. Lau, "The Newsboy Problem with Price-Dependent Demand Distribution," *IIE Transactions*, (1988), 168-175.

[10] Lau, H. L., H. S. Lau and K. D. Willett, "Demand Uncertainty and Returns Policies For a Seasonal Product: An Alternative Model," *International*

Journal of Production Economics 66 (2000), 1-12.

[11] Lee, C. H., "Coordinated Stocking, Clearance Sale, and Return Policies for a Supply Chain," *European Journal of Operational Research* 131 (2001), 33-55.

[12] Lee, H. L. and M. J. Rosenblatt, "A Generalized Quantity Discount Pricing Model To Increase Supplier's Profit," *Management Science* 32 (1986), 1177-1184.

[13] Leland, H. E., "Theory of the Firm Facing Uncertain Demand," *The American Economic Review* 62 (1972), 278-291.

[14] Li, J., H. S. Lau and A. H. Lau, "A Two-Product Newsboy Problem with Satisficing Objective and Independent Exponential Demands," *IIE Transactions* 23 (1991), 29-39.

[15] Marvel, H. P. and J. Peck, "Demand Uncertainty and Return Policies," *International Economic Review* 36 (1995), 691-714.

[16] Monahan, J. P., "A Quantity Discount Pricing Model to Increase Vendor Profit," *Management Science* 30 (1984), 720-726.

[17] Parlar, M. and Z. K. Weng, "Designing a Firm's Coordinated Manufacturing and Supply Decisions with Short Product Life Cycles," *Management Science* 43 (1997), 1329-1344.

[18] Pasternack, B. A., "Optimal Pricing and Return Policies for Perishable Commodities," *Marketing Science* 4 (1985), 166-176.

[19] Petruzzi, N. C., and M. Dada, "Pricing and The Newsvendor Problem: A Review with Extension," *Operations Research* 47 (1999), 183-194.

[20] Porteus, E. L., "Stochastic Inventory Theory," in D. Heyman, and M. Sobel (Eds.), *Handbooks in Operations Research and Management Science VOL. 2: Stochastic Models*, North-Holland, Amsterdam, Netherlands, 1990.

[21] Taylor, T. A., "Channel Coordination Under Price Protection, Midlife Returns, and End of Life Returns in Dynamic Markets," *Management Science* 47 (2001), 1220-1234.

[22] Tsay, A. A., "The Quantity Flexibility Contract and Supplier Customer Incentives," *Management Science* 45 (1999), 1339-1358.

[23] Tsay, A. A., "Managing Retail Channel Overstock: Markdown Money and Return Policy," *Journal of Retailing* 77 (2001), 457-492.

[24] Webster, S. and K. Weng, "A Perishable Item Return Policy," *Manufacturing and Service Operations Management* 2 (2000), 100-106.

[25] Weng, Z. K., "Channel Coordination and Quantity Discount," *Management Science* 41 (1995), 1509-1522.

[26] Whitin, T. M., "Inventory Control and Price Theory," *Management Science* 2

(1955), 61-68.
[27] Zabel, E., "Monopoly and Uncertainty," *Review of Economic Studies* 37 (1970), 205-219.

Appendix 1.

Let $f'_{\mathbf{x}} := df(x)/dx$ (e.g., $D'_{P_\alpha} = \partial D_P/\partial\alpha$). The following notations will be used in the Appendix.

$\lambda_N := -(1-\gamma)f(\xi_N)/D_N < 0$, $\lambda_P := -\gamma g(\xi_P)/D_P < 0$, $\phi_N := \xi_N D'_{N\alpha} > 0$,

$\phi_P := \xi_P D'_{P_\gamma} < 0$, $\phi := \xi_P D'_{P\alpha} < 0$, $\phi_S := \xi_P D'_{P\alpha} + yD'_{N\alpha}$, and

$\theta := D''_{P\alpha\alpha} D'' \int_0^{\xi_P} x dG - yD''_{N\alpha\alpha}\bar{G}(\xi_P)$. Define $\Pi := \Pi^C_{NP}$ and $\Omega := \Omega_P$. The second derivatives are given as follows:

$\Pi''_{QQ} = \lambda_N + \int_0^{\xi_N} \Omega''_{QQ} dF \leq 0$, where $\Omega''_{QQ} = \lambda_P \leq 0$

$\Pi''_{\alpha\alpha} = D''_{N\alpha\alpha}\int_0^{\xi_N} ydF + \lambda_N {\phi_N}^2 + \int_0^{\xi_N} \Omega''_{\alpha\alpha} dF \leq 0$,

where $\Omega''_{\alpha\alpha} = \lambda_P {\phi_S}^2 + \theta \leq 0$

$\Pi''_{Q\alpha} = -\lambda_N \phi_N + \int_0^{\xi_N} \Omega''_{Q\alpha} dF$, where $\Omega''_{Q\alpha} = -\lambda_P \phi_S \leq 0$

$\Pi''_{\gamma\gamma} = \int_0^{\xi_N} \Omega''_{\gamma\gamma} dF \leq 0$, where $\Omega''_{\gamma\gamma} = 2D'_{P_\gamma}\int_0^{\xi_P} x dG + {\phi_P}^2 \lambda_P$

$+\gamma D''_{P\gamma\gamma}\int_0^{\xi_P} x dG \leq 0$,

$\Pi''_{\alpha\gamma} = \int_0^{\xi_N} \Omega''_{\alpha\gamma} dF$, where $\Omega''_{\alpha\gamma} = \int_0^{\xi_P} x D'_{P\alpha} dG + \int_0^{\xi_P} \gamma x D''_{P\alpha\gamma} dG$

$-yD'_{N\alpha}\bar{G}(\xi_P) + \phi_S \phi_P \lambda_P$,

$\Pi''_{Q\gamma} = \int_0^{\xi_N} \Omega''_{Q\gamma} dF \leq 0$, where $\Omega''_{Q\gamma} = \bar{G}(\xi_P) - \phi_P \lambda_P \leq 0$.

The following three lemmas will be used to prove Proposition 1:

Lemma 1. $\Omega''_{Q\gamma} < 0$ *if* $g(x)$ *has an Increasing Failure Rate (IFR): Let* $\bar{\eta} := \gamma D'_{P_\gamma}/D_p$ *and failure rate* $\rho := g/\bar{G}$, *then* $\Omega''_{Q\gamma} = \bar{G}(\xi_P)(1+\bar{\eta}\rho\xi_P)$. *Since* $1+\bar{\eta} = -\xi_P\bar{G}(\xi_P) < 0$ *and* $\xi_P G(\xi_P) > \int_0^{\xi_P} x dG$, *via necessary condition (2)* $(1+\bar{\eta})\xi_P G(\xi_P) + \xi_p \bar{G}(\xi_P)$

$< (1+\bar{\eta})\int_0^{\xi_P} xdG + \xi_P \bar{G}(\xi_P) = 0$. *Here,* $(1+\bar{\eta})\xi_P G(\xi_P) + \xi_p \bar{G}(\xi_P) < 0 \Rightarrow \bar{\eta} < -1/G(\xi_P)$. *Upon substitution reveals* $\Omega''_{Q\gamma} < \bar{G}(\xi_P)(1-\rho\xi_P/G(\xi_P))$ *; hence,* $\Omega''_{Q\gamma} < 0$ *if* $G(\xi_P) < \rho\xi_P$. *Denote* $\omega(\xi_P) := \left[\rho(\xi_P)\xi_P + \exp\left\{-\int_0^{\xi_P}\rho(s)ds\right\}\right]$, *it is well known that* $G(\xi_P) = 1-\exp\left\{-\int_0^{\xi_P}\rho(s)ds\right\}$*; thus,* $1-\exp\left\{-\int_0^{\xi_P}\rho(s)ds\right\} < \rho(\xi_P)\xi_P \Rightarrow 1 < \omega(\xi_P)$. *We see that* $\omega(\xi_P = 0) = 1$ *and* $\omega'_\xi > 0$ *if* $\rho'_\xi > 0$ *(IFR). Thus,* $\rho'_\xi > 0 \Rightarrow \omega > 1 \Rightarrow G < \rho\xi_P \Rightarrow \Omega''_{Q\gamma} < 0$. □

Lemma 2. $\phi_S := \xi_P D'_{P\alpha} + yD'_{N\alpha} \le 0$: Let $\bar{\xi}_P := -yD'_{N\alpha}/D'_{P\alpha} \ge 0$.

Proof: We see that $\phi_S \le 0$ *if* $\xi_P \ge \bar{\xi}_P$, *and* $\xi'_{P\alpha} = -D_P^{-1}\phi_S$*; hence,* $\xi'_{P\alpha}\left(\xi_P > \bar{\xi}_P\right) \ge 0$, $\xi'_{P\alpha}\left(\xi_P < \bar{\xi}_P\right) \le 0$ *, and* $\xi'_{P\alpha}\left(\bar{\xi}_P\right) = 0$. *It is seen that* $\xi_P(\alpha > 0) \ge \xi_P(\alpha = 0)$ *if* $\xi_P(\alpha = 0) \ge \bar{\xi}_P$ *since* $\xi'_{P\alpha}\left(\xi_P > \bar{\xi}_P\right) \ge 0$, *and* $\xi_P(\alpha > 0) \le \xi_P(\alpha = 0)$ *if* $\xi_P(\alpha = 0) \le \bar{\xi}_P$ *since* $\xi'_{P\alpha}\left(\xi_P < \bar{\xi}_P\right) \le 0$ *; thus, only one of* $\xi_P \ge \bar{\xi}_P(\xi'_{P\alpha} \ge 0)$ *or* $\xi_P \le \bar{\xi}_P(\xi'_{P\alpha} \le 0)$ *can apply. However,* $\xi_P \to \infty$ *as* $\alpha \to 1$*; hence, it must be that* $\xi_P \ge \bar{\xi}_P$ *and* $\phi_S \le 0 \quad \forall \alpha \in [0,1]$. □

Lemma 3.1 For models A, B, and C, $1 + \gamma D''_{P\alpha\gamma}/D'_{P\alpha} = 1 + \gamma D'_{P\gamma}/D_P$:

Proof: Model A: $D''_{P\alpha\gamma} = k_P/4D_P$, $D'_{P\alpha} = -k_P(1-\gamma)/2D_P$ *and* $D'_{P\gamma} = -k_P(1-\alpha)/2D_P$*; thus,* $D''_{P\alpha\gamma}/D'_{P\alpha} = -1/2(1-\gamma) = D'_{P\gamma}/D_P$. *Model B:* $D''_{P\alpha\gamma} = -k_{P1}k_{P2}/4D_P^3$, $D'_{P\alpha} = -k_{P1}/2D_P$ *and* $D'_{P\gamma} = -k_{P2}/2D_P$*; thus,* $D''_{P\alpha\gamma}/D'_{P\alpha} = k_{P2}/2D_P^2 = D'_{P\gamma}/D_P$. *Model C,* $D''_{P\alpha\gamma} = k_P$, $D'_{P\alpha} = -k_P(1-\gamma)$ *and* $D'_{P\gamma} = -k_P(1-\alpha)$*; thus,* $D''_{P\alpha\gamma}/D'_{P\alpha} = -1/(1-\gamma) = D'_{P\gamma}/D_P$. □

Lemma 3.2 $\Omega''_{\alpha\gamma} = -\Omega''_{Q\gamma}\phi_S < 0$ *: Denote* $\eta := 1 + \gamma D'_{P\gamma}/D_P$. $\Omega''_{\alpha\gamma} = D'_{P\alpha}\eta\int_0^{\xi_P} xdG + \phi\phi_P\lambda_P - yD'_{N\alpha}\Omega''_{Q\gamma}$ *(Lemma 3.1). Necessary condition (2) implies* $\eta\int_0^{\xi_P} xdG = -\xi_P\bar{G}(\xi_P) \Rightarrow D'_{P\alpha}\eta\int_0^{\xi_P} xdG + \phi\bar{G}(\xi_P) = 0 \Rightarrow D'_{P\alpha}\eta\int_0^{\xi_P} xdG + \phi\phi_P\lambda_P = -\Omega''_{Q\gamma}\phi$. *Therefore* $\Omega''_{\alpha\gamma} = -\Omega''_{Q\gamma}(\xi_P D'_{P\alpha} + yD'_{N\alpha}) = -\Omega''_{Q\gamma}\phi_S < 0$. □

Proof of Proposition 1.1 We will show that $\Omega_P(q^C = I,\gamma \mid I,\alpha)$ is concave in (Q,α,γ), and then show that $\Pi^C_{NP}(Q,\alpha,q^C = I,\gamma^C)$ is concave in (Q,α).

(a) $\Omega_P(q^C = I,\gamma \mid I,\alpha)$ is concave in (Q,α,γ) since

(a.1) $|H_1| = \Omega''_{QQ} \le 0$ $\Omega''_{QQ} \le 0$ (or $\Omega''_{\alpha\alpha} \le 0$, $\Omega''_{\gamma\gamma} \le 0$).

(a.2) $|H_2| > \Omega''_{\alpha\alpha}\Omega''_{QQ} - (\Omega''_{\alpha Q})^2 \ge 0$, $|H_2| = \Omega''_{\gamma\gamma}\Omega''_{QQ} - (\Omega''_{Q\gamma})^2 \ge 0$, and

$|H_2| = \Omega''_{\alpha\alpha}\Omega''_{\gamma\gamma} - (\Omega''_{\alpha\gamma})^2 \ge 0$.

(a.3) $|H_3| = \Omega''_{QQ}\left\{\Omega''_{\gamma\gamma}\Omega''_{\alpha\alpha} - (\Omega''_{\alpha\gamma})^2\right\} - \Omega''_{Q\gamma}\left\{\Omega''_{Q\gamma}\Omega''_{\alpha\alpha} - \Omega''_{\alpha Q}\Omega''_{\gamma\alpha}\right\}$

$+\Omega''_{\alpha Q}\left\{\Omega''_{Q\gamma}\Omega''_{\alpha\gamma} - \Omega''_{\alpha Q}\Omega''_{\gamma\gamma}\right\} < 0$

Proof:

(a.2) (i) $\Omega''_{\alpha\alpha}\Omega''_{QQ} - (\Omega''_{\alpha Q})^2 = \lambda_P{\phi_S}^2 \times \lambda_P - (\lambda_P\phi_S)^2 = 0$.

(ii) $|H_2| = \Omega''_{\gamma\gamma}\Omega''_{QQ} - (\Omega''_{Q\gamma})^2 \ge 0$: Let $a := 2\lambda_P D'_{P\gamma}\int_0^{\xi_P} xdG$, $b = \bar{G}(\xi_P)$, $c := \lambda_P\phi_P$, and $d = \gamma D''_{P\gamma\gamma}\int_0^{\xi_P} xdG$. It is seen that $|H_2| = \Omega''_{\gamma\gamma}\Omega''_{QQ} - (\Omega''_{Q\gamma})^2 = d\lambda_P + a + c^2 - b^2 - c^2$ $+2bc$. Since $d\lambda_P > 0$, $|H_2| > 0 \Rightarrow= a - b^2 + 2bc > a - b^2 + 2b^2 > 0$ (By Lemma 1, $b < c$).

(iii) $|H_2| = \Omega''_{\alpha\alpha}\Omega''_{\gamma\gamma} - (\Omega''_{\alpha\gamma})^2 \ge 0$: Define again $\eta := 1 + \gamma D'_{P\gamma}/D_P \le 0$ (via necessary condition (2)).

$$\Omega''_{\alpha\alpha}\Omega''_{\gamma\gamma} - (\Omega''_{\alpha\gamma})^2 \ge 2\lambda_P{\phi_S}^2 D'_{P\gamma}\int_0^{\xi_P} xdG + (\phi_P\lambda_P\phi_S)^2 - \left\{\bar{G}(\xi_P) - \phi_P\lambda_P\right\}^2 {\phi_S}^2$$

$$= {\phi_S}^2\left\{2\phi_P\lambda_P\bar{G}(\xi_P)\frac{\eta-1}{\eta} - \bar{G}(\xi_P)^2\right\} \qquad \text{via necessary condition (2) and Lemma 3.2}$$

$$\ge 2\left\{\phi_S\bar{G}(\xi_P)\right\}^2\left\{\frac{\eta-1}{\eta} - \frac{1}{2}\right\} = 2\left\{\phi_S\bar{G}(\xi_P)\right\}^2\left(\frac{\eta-2}{2\eta}\right) \ge 0 \qquad \text{via Lemma 1 } \bar{G}(\xi_P) \le \phi_P\lambda_P$$

(a.3) $|H_3| = \Omega''_{QQ}\left\{\Omega''_{\gamma\gamma}\Omega''_{\alpha\alpha} - (\Omega''_{\alpha\gamma})^2\right\} - \Omega''_{Q\gamma}\left\{\Omega''_{Q\gamma}\Omega''_{\alpha\alpha} - \Omega''_{\alpha Q}\Omega''_{\gamma\alpha}\right\}$

$+\Omega''_{\alpha Q}\left\{\Omega''_{Q\gamma}\Omega''_{\alpha\gamma} - \Omega''_{\alpha Q}\Omega''_{\gamma\gamma}\right\} < 0$

Proof: Define $a := 2D'_{P\gamma}\int_0^{\xi_P} xdG$ and $b := \int_0^{\xi_P} xD'_{P\alpha}dG + \int_0^{\xi_P}\gamma xD''_{P\alpha\gamma}dG - yD'_{N\alpha}\bar{G}(\xi_P)$.

It is seen that

$$\Omega''_{QQ}\left\{\Omega''_{\gamma\gamma}\Omega''_{\alpha\alpha}-(\Omega''_{\alpha\gamma})^2\right\}=\lambda_P{}^2\phi_S(a\phi_S-b^2/\lambda_P\phi_S-2b\phi_P)+\theta\Omega''_{QQ}\Omega''_{\gamma\gamma}$$

$$-\Omega''_{Q\gamma}\left\{\Omega''_{Q\gamma}\Omega''_{\alpha\alpha}-\Omega''_{\alpha Q}\Omega''_{\gamma\alpha}\right\}=-\lambda_P\phi_S\left(\bar{G}(\xi_P)-\phi_P\lambda_P\right)\left(\bar{G}(\xi_P)\phi_S+b\right)-\theta(\Omega''_{Q\gamma})^2$$

$$\Omega''_{\alpha Q}\left\{\Omega''_{Q\gamma}\Omega''_{\alpha\gamma}-\Omega''_{\alpha Q}\Omega''_{\gamma\gamma}\right\}=-\bar{G}(\xi_P)\lambda_P\phi_S(b+\phi_S\phi_P\lambda_P)+\lambda_P{}^2\phi_S\left\{b\phi_P-a\phi_S\right\}.$$

$|H_3|=\theta\left\{\Omega''_{\gamma\gamma}\Omega''_{QQ}-(\Omega''_{Q\gamma})^2\right\}-\lambda_P\left\{\phi_S\bar{G}(\xi_P)+b\right\}^2<0$, since

(i) $\theta\left\{\Omega''_{\gamma\gamma}\Omega''_{QQ}-(\Omega''_{Q\gamma})^2\right\}<0$, and

(ii) $\left\{\phi_S\bar{G}(\xi_P)+b\right\}=D'_{P\alpha}\left\{\left(1+\dfrac{\gamma D''_{P\alpha\gamma}}{D'_{P\alpha}}\right)\int_0^{\xi_P}x\,dG+\xi_P\bar{G}(\xi_P)\right\}$

$=D'_{P\alpha}\left\{\left(1+\dfrac{\gamma D'_{P\gamma}}{D_P}\right)\int_0^{\xi_P}x\,dG+\xi_P\bar{G}(\xi_P)\right\}=0$ via Lemma 3.1

By (a.1), (a.2), and (a.3), $\Omega_P(q^C=I,\gamma/I,\alpha)$ is concave in (Q,α,γ).

(b) $\Pi^C_{NP}(Q,\alpha,q^C=I,\gamma^C)$ is concave in (Q,α).

Proof: Denote $\tilde{\Pi}''_{\alpha\alpha}=\lambda_N\phi_N{}^2+\int_0^{\xi_N}\Omega''_{\alpha\alpha}dF\le 0$. Since $D''_{N\alpha\alpha}\le 0$, $\Pi''_{\alpha\alpha}\Pi''_{QQ}-(\Pi''_{\alpha Q})^2$ $>\tilde{\Pi}''_{\alpha\alpha}\Pi''_{QQ}-(\Pi''_{\alpha Q})^2=a+b>0$, where $b=\int_0^{\xi_N}\lambda_N\lambda_P(\phi_N-\phi_S)^2dF>0$, and $a=$ $a=\int_0^{\xi_N}\lambda_P dF\times\int_0^{\xi_N}\lambda_P\phi_S{}^2dF-\left(\int_0^{\xi_N}\lambda_P\phi_S dF\right)^2\ge 0$ (Cauchy-Swarz's inequality). Thus $\Pi^C_{NP}(Q,\alpha,q^C=I,\gamma^C)$ is concave in (Q,α). □

Proof of Proposition 1.2 (a) $\partial\gamma/\partial Q\,|_{\Omega(3)}=-\Omega''_{Q\gamma}/\Omega''_{\gamma\gamma}\le 0$ by Lemma 1. (b) $\partial\gamma/\partial\alpha\,|_{\Omega(3)}=-\Omega''_{\alpha\gamma}/\Omega''_{\gamma\gamma}<0$ by Lemma 3, and (c) $\partial\alpha/\partial Q\,|_{\Pi(1)}=-\Pi''_{Q\alpha}/\Pi''_{\alpha\alpha}<0$. □

APPENDIX 2. PROOF OF PROPOSITION 2

Proof of Propositions 2.1 and 2.2: The proofs are similar to those in Propositions 1.1 and 1.2: □

Lemma 4. $0 = 1 - \rho(\xi_{P1})\xi_{P1} \le 1 - \rho(\xi_{P2})\xi_{P2}$: Define $\omega := 1 - C_{r1} / P\gamma_1$. $\bar{q} + C_r \partial\bar{q} / \partial C_r = 0 \Rightarrow \varphi\Gamma^{-1}(\omega)/(1-\omega) = 1$. Since $\xi_{P1} = \Gamma^{-1}(\omega)$ and $\rho = \varphi(\xi_{P1})/\bar{\Gamma}(\xi_{P1}) = \varphi/(1-\omega)$, $\rho\xi_{P1} = \varphi\Gamma^{-1}(\omega)/(1-\omega) = 1$. Finally, $0 = 1 - \rho(\xi_{P1})\xi_{P1} \le 1 - \rho(\xi_{P2})\xi_{P2}$, since $\bar{q} > I$ and $\varphi(x)$ has an IFR or $\rho'_{\xi} > 0 \quad \forall y \in [\xi_L, \xi_N]$. □

Propositions 2.3: (a) Both $\Pi^{C}_{NP\mathbf{QQ}}{}'' \le 0$ and $\Pi^{UC}_{N\,QQ}{}'' \le 0$ are decreasing functions. Since $\Pi^{C}_{NPQQ}{}'' \ge \Pi^{UC}_{N\,QQ}{}''$, $Q^{UC} \le Q^{C}$.

(b) Since $\Pi^{C}_{NP\alpha\alpha}{}'' \le 0$ and $\Pi^{UC}_{N\alpha\alpha}{}'' \le 0$, both $\Pi^{C}_{NP\alpha}{}'$ and $\Pi^{UC}_{N\alpha}{}'$ are decreasing functions. Thus, $\alpha^{UC} \ge \alpha^{C}$ if $\Pi^{C}_{NP\alpha}{}' \le \Pi^{UC}_{N\alpha}{}'$. Denote $\upsilon(\xi_L) := \int_{\xi_L}^{\xi_N} \gamma_2\{\varphi(\xi_{P2})^2 D'_{P\alpha} - y\bar{\Gamma}(1-\rho\xi_{P2})D'_{N\alpha}\}dF + \int_0^{\xi_L} \xi_{P1}\gamma_1\bar{G}D'_{P\alpha}dF$, then $\Pi^{UC}_{N\alpha}{}' \Rightarrow \int_0^{\xi_N} yD'_{N\alpha}dF + \upsilon(\xi_L)$. It is seen that $\partial\upsilon(\xi_L)/\partial\xi_L = 0$ since $1 - \rho(\xi_{P1})\xi_{P1} = 0$ (see Lemma 4); thus, $\Pi^{UC}_{N\alpha}{}'$ is a constant (flat) function with respect to ξ_L. Therefore, it is appropriate to compare $\Pi^{C}_{NP\alpha}{}'$ and $\Pi^{UC}_{N\alpha}{}'$ at a specific point where $\xi_L = \xi_N$ ($\Pi^{UC}_{N\alpha}{}'(\xi_L = \xi_N) = \int_0^{\xi_N} yD'_{N\alpha}dF + \int_0^{\xi_N} \xi_P\gamma\bar{G}(\xi_P)D'_{P\alpha}dF$). Denote $\theta(\xi_P) := \int_0^{\xi_P} xdG - \xi_P\bar{G}(\xi_P)$. It is seen that $\Pi^{C}_{NP\alpha}{}' - \Pi^{UC}_{N\alpha}{}' \le \int_0^{\xi_N} \gamma D'_{P\alpha}\theta(\xi_P)dF \le 0$. $\theta' = 2\xi_P g - \bar{G} \ge 0$ (since $1 - \rho(\xi_{P1})\xi_{P1} = 0$) and $\theta(0) = 0$; thus, $\theta(\xi_P) \ge 0$ and $\alpha^{UC} \ge \alpha^{C}$.

(c) In both UC and centralized model $\partial\gamma/\partial q = -\Pi''_{P\mathbf{q}\gamma}/\Pi''_{P\gamma\gamma} < 0$ and $\partial\gamma/\partial I = -\Pi''_{NP\mathbf{Q}\gamma}/\Pi''_{NP\gamma\gamma} < 0$. Thus, $q^{C} = I \ge q^{UC} \Rightarrow \gamma^{UC} \ge \gamma^{C}$. □