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An analysis about the behavior of rubber component with large deformation

Moon-Sik Han*, Jae-Ung Cho+

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대변형을 하는 고무 부품의 거동에 관한 해석

한문식*, 조재웅+

Abstract

The non-linear finite element program of the large deformation analysis by computer simulation has been used in the prediction and evaluation of the behaviors of the non-linear rubber components. The analysis of rubber components requires the tools modelling the special materials that are quite different from those used for the metallic parts. The nonlinear simulation analysis used in this study is expected to be widely applied in the design analysis and the development of several rubber components which are used in the manufacturing process of many industries. By utilizing this method, the time and cost can also be saved in developing the new rubber product. The objective of this study is to analyze the rubber component with the large deformation and non-linear properties.

Key Words : Large deformation, Non-linear property, Hyperelasticity, Coefficients of Mooney-Rivlin, Self contact, Maximum equivalent stress and strain

1. Introduction

There is the elastic property returning to the original state in the linear range when the general elastic material is applied by load and released. Though the rubber shows the nonlinear relation, this material has the hyperelastic property as the large deformation. Rubberis the material having the large change of property according to the manufactural process. The database about the property of rubber must be collected and so, it is necessary to manage the effective use of these data.

In this study, the material is simulated by modelling

* 계명대학교 기계 · 자동차공학부 (sheffhan@kmu.ac.kr)
주소: 704-701 대구광역시 달서구 신당동 1000
+ 공주대학교 기계 · 자동차공학부

the same or the similar material to the practical material.

The properties of rubber are also investigated by many experiments and the material properties and database about components can be obtained. And so, these results can be utilized in the design of rubber component in case of no experimental data. Until now, the development of technology and the activity of study about rubber has been proceeded only by the area of chemical material[1~3]. Therefore the skill of CAE is not applied at composite material and other plastic product as well as metal component. The mechanics of rubber has been established theoretically nowadays. And the finite element analysis program of nonlinear and large deformation has been developed through the accumulation of rubber data. As the study on high level of design skill proceeds, the skill of CAE has been applied at the automobile and railroad coach[4~5].

The purpose of the study is to get the skill designing the rubber machinery component. And the performance and the reliability of product can be improved by understanding the particular property of rubber that is hyperelastic and nonlinear.

2. Material with hyperelasticity

The material with hyperelasticity has the possibility of great deformation and a few hundreds of strain rate(%)[6]. And It has the energy conservation because of pure elasticity in property and its behaviour is independent on the path applied by load. The model of hyper elasticity is used in modelling rubber or polymer generally.

Mooney-Rivlin model is suitable to be uncompressive material and Blatz-Ko model is suitable to be compressive material like form material. Rubber has uncompressive character nearly and uncompressive material occurs without the deformation of volume. Its poisson's ratio is about 0.48 to 0.5. Form material has the character of compressive hyperelasticity. The typical load-displacement curve of rubber is shown at Fig. 1[7].

The softened response is shown in tension and hardened response is shown again. And the extremely hardened response is shown in compression. The parameters of

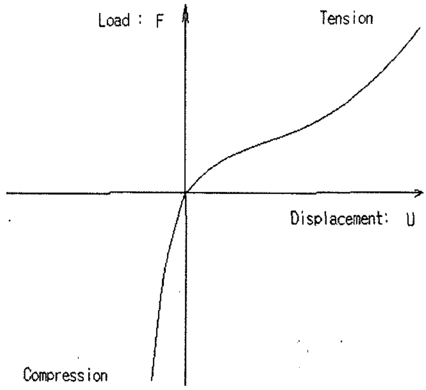

Fig. 1 Typical load-displacement curve of rubber

Mooney-Rivlin model satisfy the restricted conditions to represent the behavior of material adequately. The density function of strain energy must have the positive value. That is, the strain energy must be increased as material deforms. The both extreme stresses of uniaxial deformation must have the positive or minus value. The stress must be the continuous function about deformation. The hyperelastic problem occurs the great strain and the geometrical non-linearity must be activated. The hyperelastic problem deals with the nonlinearity of high difficulty. Rubber has the property that it can be elongated easily and return to the original length. It also has the great deformation from 500 to 900% and represent the behavior of tensile recover. Rubber as the compressive material is also called by Form[8].

The fundamental law of Hook between deformation and stress is established and stress(σ) is proportional to strain(ε) as indicated by

$$\sigma = E \cdot \varepsilon \qquad (1)$$

In case of the low modulus of elasticity, this material is flexible and easily elongated. Rubber has the modulus of elasticity of 0.2 kg/mm^2 nearly. As compared with the solid having strain below a few percents, rubber has the large deformation occurred by the contraction of area. When the elastic material is applied by load and then the load is released, this material shows the elastic behavior that will return to the initial state in the range of linear relation. The rubber has the hyperelastic property showing the behavior of elasticity even at a great deformation of the nonlinear relation between load and displacement.

3. Strain energy density function

The behavior of elastic material can be shown by the strain energy function according to the elastic theory(6,9~10). The stress at the hyperelastic material about the given strain can be obtained from the derived function of strain energy density function about the component of each strain as follows.

$$[S]=\frac{\partial W}{\partial [E]} \tag{2}$$

[E] is the known Green-Lagrange strain, and [S] is the calculated 2'nd Piola Kirchhoff stress. W is the term of the energy of the initial volume unit.

It is supposed that the hyperelastic material has the property of equivalent direction with constant material property in all axes. Therefore, the density function of strain energy can be shown by invariant quantities I_1, I_2, I3. In showing the order of strain, the strain invariants are used irrespectively of the axis system. the density functions of strain energy are represented by strain invariants in both Mooney-Rivlin and Blatz-Ko models(7). Most of hyperelastic materials have nearly uncompressible properties. And so, the uncompressible conditions must be considered in the procedure of analysis about the elements. The uncompressible hyperelastic elements are added with freedom of pressure and displacement. It is possible for the freedom of pressure not to decrease the overall accuracy of solution but to enable uncompressible constraint. The freedom of pressure is the internal freedom and it is concentrated in the internal element. Mooney-Rivlin model is useful in modelling the rubber or the uncompressible model similar to rubber.

The density function of strain energy is expressed by the formulas of multiple terms as follows.

$$W=W(I_1,I_2,I_3) \tag{3}$$

$$W=W(\lambda_1,\lambda_2,\lambda_3) \tag{4}$$

In this place,

λ_1, λ_2, λ_3 are the principal bulk moduli in case of the materials with equivalent properties and I_1, I_2, I_3 are the invriants about principal bulk moduli defined as follows.

$$\begin{aligned} I_1 &= \lambda_1^2+\lambda_2^2+\lambda_3^2 \\ I_2 &= \lambda_1^2\lambda_1^2+\lambda_2^2\lambda_3^2+\lambda_3^2\lambda_1^2 \\ I_3 &= \lambda_1^2\lambda_2^2\lambda_3^2 \end{aligned} \tag{5}$$

The density function of strain energy is represented by the function as follows.

$$W=W(I_1,I_2)=\sum_{i+j=1}^{N} C_{ij}(I_1-3)^i(I_2-3)^j \tag{6}$$

C_{ij} at the model of Mooney-Rivlin are the material constants obtained by experiment.

Two, five or nine material constants are used to represent the behavior of material. If the constants of Mooney-Rivlin can be known about the material that will be analyzed, these constants can be directly inputted in program.

4. Results of analysis

The values of stresses output the values of true stresses about the coordinate system of each element. This study is analyzed by ANSYS program to obtain the maximum equivalent stress and strain(7).

The configurations and dimensions of rigid body, rubber, wall and floor are shown in Fig. 2 The rigid body has the parabolic shape. The mesh of rubber is also shown in Fig. 3 and the numbers of its elements and nodes are 700 and 540 respectively.

The rubber has the element of HYPER56(7) that is hyperelastic and this analysis has been done by 2 dimension. The time that the rigid body is acting on the rubber is 1sec. As shown in Fig. 3, the rigid body acts on the rubber in the direction that are the axes of positive X and negative Y. This rigid body has been applying to the direction of positive X axis until the displacement of 33 inch for 1 second. It has also been applying to the

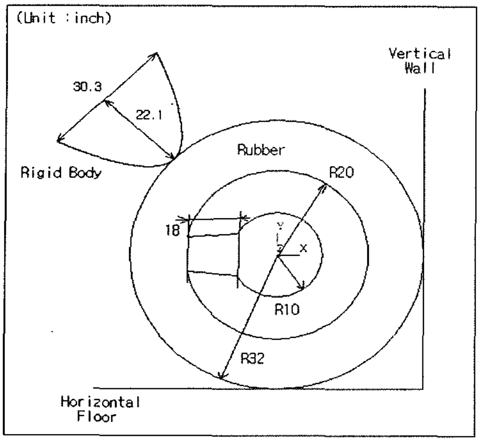

Fig. 2 The configurations and dimensions of rigid body, rubber, wall and floor

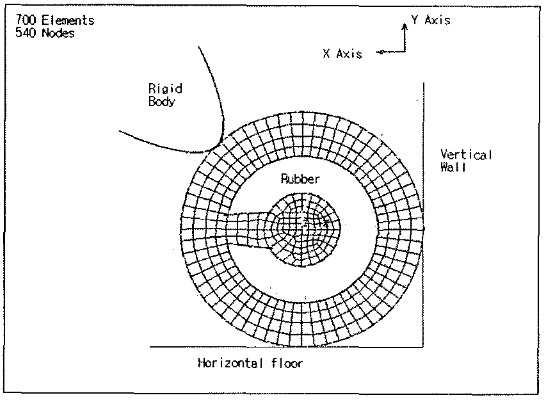

Fig. 3 The mesh of rubber

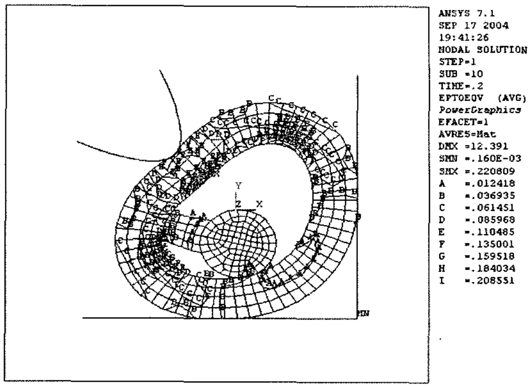

Fig. 4 The contours of maximum equivalent stress at the elapsed times of 0.2 second(Unit : lb/in^2)

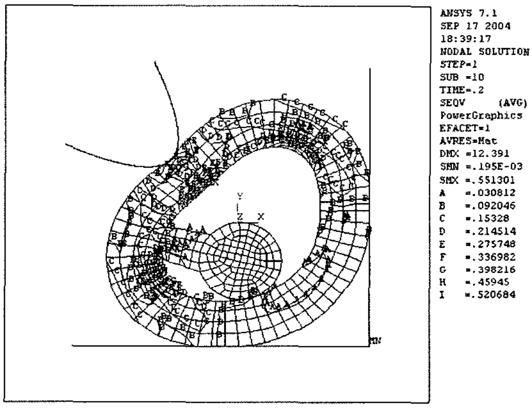

Fig. 5 The contours of maximum equivalent strain at the elapsed times of 0.2 second

direction of negative Y axis until the displacement of 33 inch downward for 1 second. The rubber is slided in x axis on the horizontal floor. The rubber is also slided in y axis on the vertical wall. The pushing body is rigid and the friction coefficient is 0.25 between rigid body, rubber, wall and floor. The self contact between inner rubber and outer rubber is established. Rubber component also has the property of linear isotropic material.

The Poisson's ratio is 0.49 and the coefficients of Mooney-Rivlin (C_{ij}) are 0.418 and 0.006[11]. The contours of maximum equivalent stress and strain at the elapsed times of 0.2, 0.4, 0.6, 0.8, 1.0 second are plotted by Fig. 4 to Fig. 13 respectively.

Until the elapsed time of 0.4 second since rigid body has been the contact with rubber, this stress and strain increases more and more at the inner left and center part of rubber folded by rigid body as shown in Fig. 6 and 7. The maximum equivalent stress and strain at this part are 1.169 lb/in2 and 0.439 respectively.

At the elapsed time of 0.6 second, the inner center part of rubber is contacted by rubber by itself as shown in Fig. 8 and 9. The maximum equivalent stress and strain are also shown at this part. This stress and strain at this part are 2.359lb/in^2 and 0.699 respectively.

At the elapsed time of 1 second, the inner upper part of rubber has been pressed flat finally after rigid body presses continuously as shown in Fig. 12 and 13. The

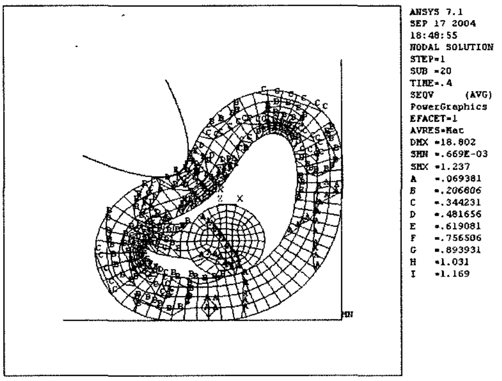

Fig. 6 The contours of maximum equivalent stress at the elapsed times of 0.4 second(Unit : lb/in^2)

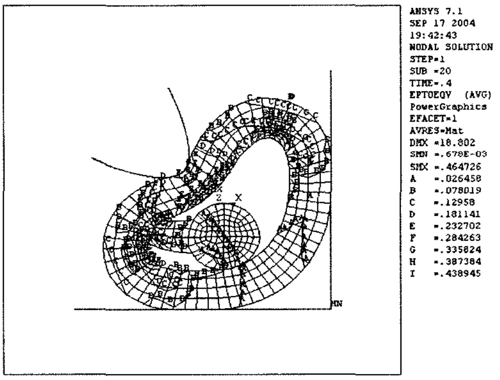

Fig. 7 The contours of maximum equivalent strain at the elapsed times of 0.4 second

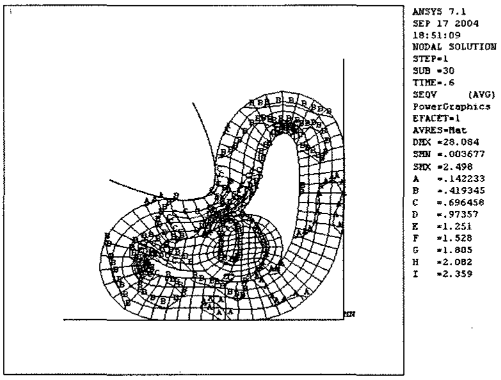

Fig. 8 The contours of maximum equivalent stress at the elapsed times of 0.6 second(Unit : lb/in^2)

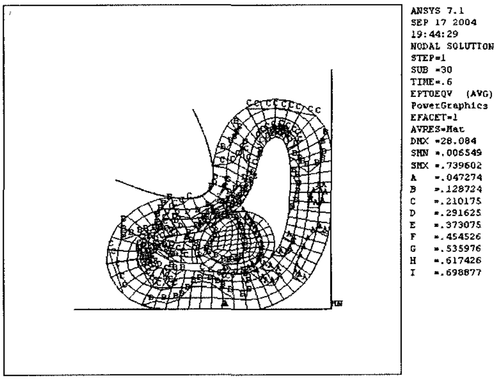

Fig. 9 The contours of maximum equivalent strain at the elapsed times of 0.6 second

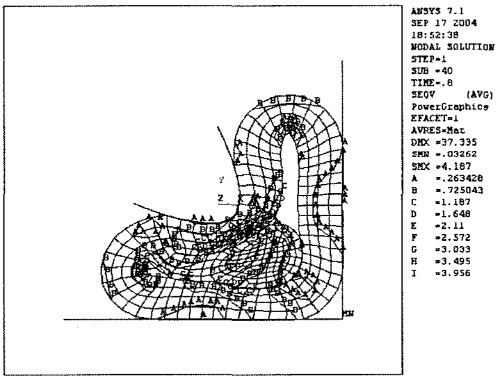

Fig. 10 The contours of maximum equivalent stress at the elapsed times of 0.8 second(Unit: lb/in^2)

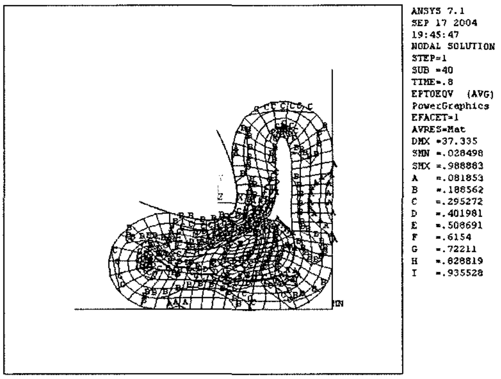

Fig. 11 The contours of maximum equivalent strain at the elapsed times of 0.8 second

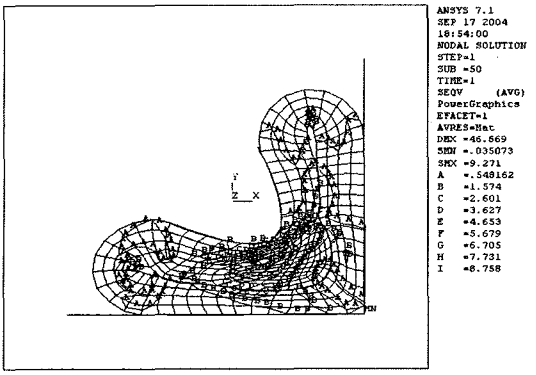

Fig. 12 The contours of maximum equivalent stress at the elapsed times of 1 second(Unit: lb/in^2)

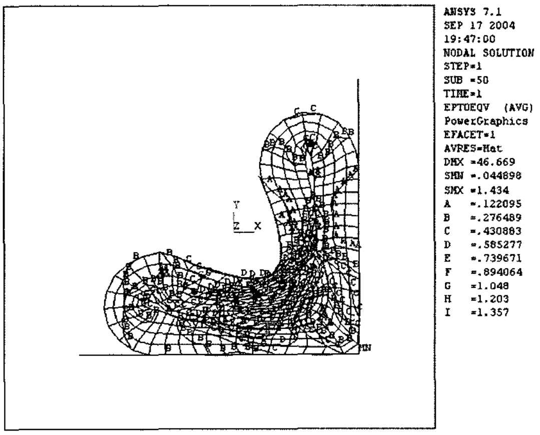

Fig. 13 The contours of maximum equivalent strain at the elapsed times of 1 second

maximum equivalent stress and strain are also shown at this part. This stress and strain at this part are 8.578lb/in^2 and 1.357 respectively. But the inner left part of rubber folded by rigid body as shown by Fig. 12 and 13 has been released and decreased in maximum equivalent stress and strain contrary to Fig. 6 and 7. The maximum equivalent stress and strain are also shown at this part. This stress and strain at this part are 1.574lb/in^2 and 0.277 respectively.

5. Conclusions

This result of study are shown by the contour lines of maximum equivalent stress and strain. This study analyzes the rubber component with large deformation and nonlinear properties, These conclusions are summarized as follows.

(1) Large displacement and rigidity about rubber component are expected by nonlinear and large deformation analysis in this study.
(2) The rigid body has the parabolic shape. Rubber is used by the model of Mooney-Rivlin and the self contact between inner rubber and outer rubber is established. There is also the friction between rigid body, rubber, wall and floor.
(3) The nonlinear simulation analysis used in this study is expected to be widely applied in design, analysis and development of several rubber components which are used in automotive, railroad, and mechanical elements etc. By utilizing this method, time and cost can also be saved in developing new rubber product.

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