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Optimum Shape Design of Magnetizing Yoke of 2 Pole PM Motor for Cogging Torque Reduction

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Abstract – A novel cogging torque reduction algorithm is presented for 2-pole permanent magnet DC motor. While the shape of the permanent magnet is changed in the conventional method, the pole shape of the magnetizing yoke is optimized in the presented algorithm. In order to parameterize the shape of the yoke, and the distribution of the residual magnetization of the permanent magnet, the Bezier spline is used. The shape of the magnetizing yoke is optimized using the design sensitivity analysis incorporated with the finite element method and Bezier spline.

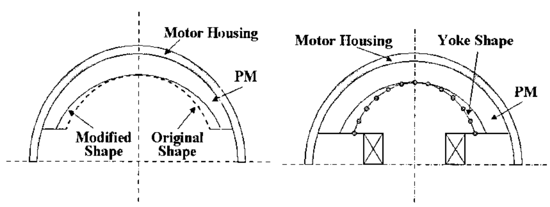
Keywords: Cogging torque, Magnetizing yoke, 2 Pole PM Motor, Design sensitivity analysis

1. Introduction

The cogging torque of the permanent magnet(PM) motor makes the torque ripples and undesirable mechanical vibrations. Especially, in case of 2-pole PM DC motor, where the number of armature winding slots should be an even number, the cogging torque is inherently quite big, and the vibration is also horrible. It is because the frequency of the cogging torque is very low due to the small LCM (least common multiple) of the number of the PM poles and that of armature winding slots [1].

Among the conventional cogging torque reduction methods, the most popular method is to change the number of armature winding slots, and modify the pole shape of the armature core[1]. In the former, the number of armature winding slots is chosen so that the LCM of the number of the PM poles and that of armature winding slots become large. It is because the cogging torque decreases as the LCM increases. This method, however, is not effective for the 2-pole PM DC motor because the number of armature winding slots should be a multiple of the number of the PM poles. In the latter, on the other hand, the auxiliary slots, usually, are put on the pole face of the armature core. However this method is also difficult to implement for 2 pole PM DC motor because the armature core is designed to have quite big number of winding slots and the teeth width is too small to put the auxiliary slots on its pole face. For this reason, in the design of 2 pole PM DC motor, an additional processing is carried out to modify the shape of the PM, as shown in Fig. 1, after the PM is adhered to the motor housing. In the viewpoint of mass production, this method is not preferred because the additional processing increases the production cost.

In this paper, in order to reduce the cogging torque, the pole shape of the magnetizing yoke for the PM is optimally designed, as shown in Fig. 1, while the shape of the PM is not modified. In order to do this, an optimal distribution of the residual magnetization of the PM, first, is found, and the pole shape of the magnetizing yoke, then, is optimized to get the desired distribution of the residual magnetization using the design sensitivity analysis.



(a) (b)

Fig. 1 Comparison of the cogging torque reduction methods. (a) Design of PM shape, (b) Design of magnetizing yoke shape.

2. Cogging Torque Reduction

The cogging torque of the PM motor comes from the uneven distribution of the magnetic energy stored in the motor as the armature rotates with respect to the PM without driving current [2]. Once the magnetic field distribution is found using finite element method, the cogging torque is computed by using the nodal force method [3]. The magnetic energy, W_m, stored in the motor is computed using the following equation:

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$$2W_m = \int_{Iron} \nu B^2 ds + \int_{Air} \nu_0 B^2 ds + \int_{PM} \nu (B - M_r)^2 ds + C \quad (1)$$

where C is the inherent magnetic energy stored in the PM when it is magnetized [2].

The azimuthal distribution of the residual magnetization of the PM is parameterized using Bezier spline [4]. When N_c control points $Z_i(\phi_i, M_{ri})$, $i = 1,2\cdots,N_c$ are given, the M_r at any azimuthal point is given as:

$$M_r(\phi(t)) = \sum_{i=1}^{N_c} J_{N_c,i}(t) \cdot Z_i(\phi_i, M_{ri}), \quad 0 \le t \le 1 \quad (2)$$

where t is the curvilinear coordinate from the first control point, and the N_c order basis function $J_{N_c,i}(t)$ is defined as follows:

$$J_{N_c,i}(t) = \binom{N_c}{i} \cdot t^i \cdot (1-t)^{N_c - i} \quad (3)$$

where $\binom{\cdot}{\cdot}$ presents the combination.

The magnetic energy stored in the motor is a function of only the distribution of the residual magnetization of the PM if the armature core design is fixed. The objective function for cogging torque reduction is defined as follows [2]:

$$F_{cog} = \sum_{i=1}^{N_r} \left(W_m(\theta_i) - \bar{W}_m \right)^2 \quad (4)$$

where θ is the relative rotation angle of the armature with respect to the PM, $\bar{W}_m$ is mean energy during the rotation, and N_r is the number of rotation for energy calculation.

The sensitivity of the objective function with respect to the control point of Bezier spline is computed as follows:

$$\frac{dF_c}{d[Z]} = 2\sum_{i=1}^{N_r} \left(W_m(\theta_i) - \bar{W}_m \right) \cdot \left\{ \left. \frac{\partial W_m(\theta_i)}{\partial [Z]} \right|_{[A]=c} + \frac{\partial W_m(\theta_i)}{\partial [A]^T} \cdot \frac{\partial [A]}{\partial [Z]} \right\} \quad (5)$$

where the second term can be neglected because the magnetic vector potential satisfies the stationary condition. The derivatives are computed as follows:

$$\left. \frac{\partial W_m(\theta_i)}{\partial [Z]} \right|_{A=c} = \sum_{e=1}^{N_m} \frac{\partial W_m^e(\theta_i)}{\partial M_r^e} \cdot \frac{\partial M_r^e}{\partial [Z]} \quad (6)$$

$$\frac{\partial W_m^e(\theta_i)}{\partial M_r^e} = \nu^e \Delta^e \left(\vec{M}_r^e - \vec{B}^e \right) \cdot (\cos\varphi_e, \sin\varphi_e), \quad \frac{\partial M_r^e}{\partial Z_k} = \binom{N_c}{k} t_e^k \cdot (1-t_e)^{N_c - k} \quad (7)$$

where the sub/superscript e denotes the element number.

The control points are updated using the steepest decent algorithm as follows:

$$[Z]_{new} = [Z]_{old} + \alpha F_c \cdot \frac{dF_c}{d[Z]} \Big/ \left| \frac{dF_c}{d[Z]} \right|^2 \quad (8)$$

where α is the relaxation factor. Finally, the optimum distribution of the residual magnetization is found using (2) with the optimized control points.

3. Shape Design of Magnetizing Yoke

A capacitor discharge pulse magnetizer is composed of RLC serial circuit with initially charged capacitor. The field governing equation taking into account the non-linearity and eddy current of the materials can be written as [5].

$$\nabla \times \nu \nabla \times \vec{A} = \vec{J} + \nu \nabla \times \vec{M}_r - \sigma \frac{\partial \vec{A}}{\partial t} \quad (9)$$

where the symbols have their usual meanings. Since the system is excited by the initially charged capacitor, the following circuit equation is coupled to (9):

$$\frac{d\Phi}{dt} + R\, i(t) + \frac{1}{C}\left(\int i(t)dt + Q_0 \right) = 0 \quad (10)$$

where R and C are the resistance and capacitance of the circuit, respectively, and Q_0 is the charge initially stored in the capacitor. Applying the time-stepping finite element method with Newton-Raphson algorithm to (9) and (10), the time variation of the magnetic flux density in each element can be found.

The residual magnetization of the PM at an element (e) is calculated from the maximum magnetic flux density, B_{max}^e using the $B_{\max} - M_r$ relation, as shown in Fig. 2, which is obtained using the magnetization and demagnetization curves.

In order to parameterize the shape of the magnetizing yoke the Bezier spline is also used. With the N_y control points, $Z(\phi, R)$, the shape of the magnetizing yoke is presented as

$$p\left(\varphi(t), r(t)\right) = \sum_{i=1}^{N_y} J_{N_y,i}(t) \cdot Z_i\left(\varphi_i, r_i\right) \quad 0 \le t \le 1 \quad (11)$$

The design target is finding the yoke shape that gives the optimum distribution of the residual magnetization.

According to the model for magnetization process, the distribution of the residual magnetization can be converted into that of maximum magnetic flux density using $B_{\max}-M_r$ relation.

The objective function to be minimized by optimizing the yoke shape is defined as follows:

$$F_y=\sum_{e=1}^{N_e}\left(B_{r,\max}^e-B_{r,opt}^e\right)^2 \tag{12}$$

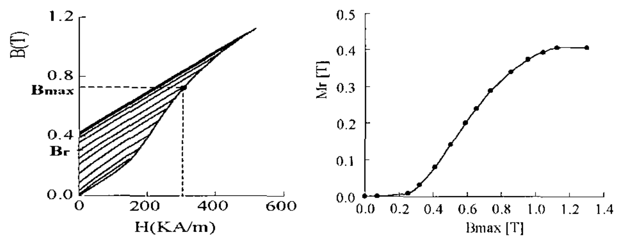
where N_e is the number of elements inside the PM, and $B_{r,\max}^e$, $B_{r,opt}^e$ are the maximum and desired values of the radial component of the magnetic flux density at element (e).

The design sensitivity can be derived as follows, considering the magnetizing yoke is not directly connected to the elements inside the PM:

$$\frac{dF_y}{d[Z]}=2\sum_{e=1}^{N_e}\left(B_{r,\max}^e-B_{r,opt}^e\right)\cdot\sum_{k=1}^{N_p}\frac{\partial P_k}{\partial[Z]}\cdot\frac{\partial B_{r,\max}^e}{\partial[A]^T}\cdot\frac{d[A]}{dP_k} \tag{13}$$

where P and N_p are the nodal points on the magnetizing yoke surface and the number of them, respectively.

The variation of the nodal point with respect to that of the control point, $\partial P_k/\partial[Z]$, can be computed using (3), (7), and (11). The second derivative in (13), $\partial B_{r,\max}^e/\partial[A]^T$ is computed as:



(a) (b)

Fig. 2 Model for magnetization process for the anisotropic sintered ferrite PM. (a) Magnetization and demagnetization curves, (b) $B_{\max}-M_r$ relation

$$\frac{\partial B_{r,\max}^e}{\partial[A]^T}=[S^e]^T\cdot[\cos\theta^e,\sin\theta^e]^T \tag{14}$$

where $[S^e]$ is defined as $[B^e]=[S^e][A^e]$.

The last derivative in (13) is computed using the discretized equation of (9) and (10). If B_r^e reaches its maximum value at time t, the following equation can be obtained by discretizing (9) and (10) [5]:

$$[K'][A']=[F'] \tag{15}$$

where $[K'],[A']$ and $[F']$ are the system matrix, state variable vector and forcing vector at time t, respectively. Taking into account the non-linearity of the material, differentiating (15) with respect to the nodal point P_i results in

$$\frac{\partial}{\partial P_i}([K'][A']-[F'])\bigg|_{[A]=c}+\frac{\partial}{\partial[A']^T}([K'][A']-[F'])\bigg|_{P_i=c}\cdot\frac{d[A']}{dP_i}=0 \tag{16}$$

After some mathematical manipulations, (16) can be written as

$$\left([K']+[\bar{K}']\right)\frac{d[A']}{dP_i}=[M']+[\bar{M}'] \tag{17}$$

where the symbols are defined as follows:

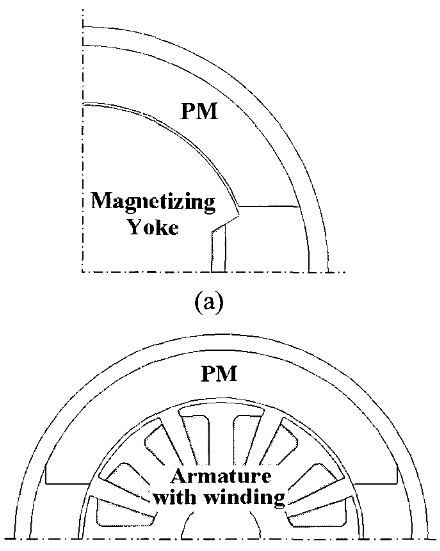
$$[\bar{K}]=\frac{\partial\nu}{\partial B^2}\frac{\partial B^2}{\partial[A']^T}[K'][\tilde{A}'],\ [M]=\frac{\partial[K']}{\partial P_i}[\tilde{A}'],\ [\bar{M}]=\frac{\partial\nu}{\partial B^2}\frac{\partial B^2}{\partial P_i}[K'][\tilde{A}'] \tag{18}$$

where $[\tilde{A}']$ is the solution of (15). The design sensitivity (13) is expressed as follows using the adjoint variable:

$$\frac{dF_y}{d[Z]}=2\sum_{e=1}^{N_e}\left(B_{r,\max}^e-B_{r,opt}^e\right)\cdot\sum_{k=1}^{N_p}\frac{\partial P_k}{\partial[Z]}\cdot[\lambda]^T\left([M']+[\bar{M}']\right) \tag{19}$$

where the adjoint variable vector $[\lambda]$ is defined as follows:

$$\left([K']+[\bar{K}']\right)^T[\lambda]=\frac{\partial B_{r,\max}^e}{\partial[A']} \tag{20}$$



(b)

Fig. 3 A 2-pole PM DC motor and its magnetizing fixture: (a) Magnetizing fixtures, (b) Motor with winding.

Table 1 Motor and Magnetizing Fixture Specifications

Motor		Magnetizing Fixture	
Number of slots	8	Number of pole	2
Winding pitch	3	Winding	1 turn
	Double layer Lap		
	38 turn/slot	Resistance	0.11 Ω
Rating Current	5.5 A	Capacitor	800 μF
Rating Voltage	13.2 V	Voltage	680 V
Axial length	25.1 mm	Axial length	32 mm

4. Numerical Design Results

Fig. 3 shows the 2-pole PM DC motor, of which the cogging torque is to be minimized, and magnetizing fixture. Table 1 shows the specifications of the motor and magnetizing fixture.

An optimum distribution of the residual magnetization, at first, which gives minimum cogging torque, is obtained. At the optimization iterations, the mean magnetic flux at the air gap is maintained not to be smaller than 95% of initial shape. This is to keep the rating torque not too small.

Fig. 4 compares the optimized distributions of the residual magnetization and control points, which are obtained after 17 iterations, with the initial ones. At the figure the first 6 control points are allowed to move while the last 2 are fixed. Fig. 5 and Fig. 6 compare the cogging and rating torques for the initial and optimized distributions, where it is found that 73% of the cogging torque is reduced while only 3% of rating torque is sacrificed.

The optimum shape of the magnetizing yoke, second, is found. In order to describe the magnetizing yoke shape, the Bezier spline is used with the 7 control points, as shown in Fig. 7, and the 6 control points are defined as the design variables and allowed to move in radial direction.

The initial charging voltage is reduced to 600[V] during the optimization process. After 21 iterations, the optimized shape of the magnetizing yoke is obtained, and compared to the initial one at the Fig. 7, where it is found that, owing to the Bezier spline, the final optimized shape is smooth enough to be constructed. Fig. 8 shows the maximum values of the radial component of the magnetic flux density inside the PM with the optimized yoke shape.

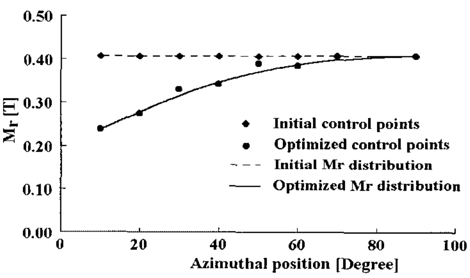


Fig. 4 Comparison of the control points and distribution of residual magnetizations

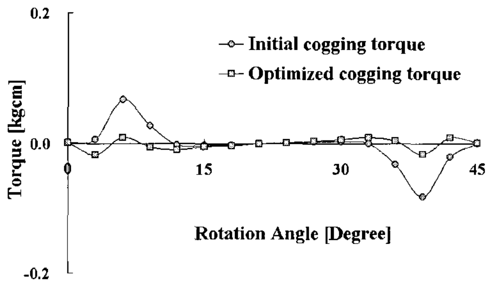


Fig. 5 Comparison of the cogging torque for the initial and optimized shapes

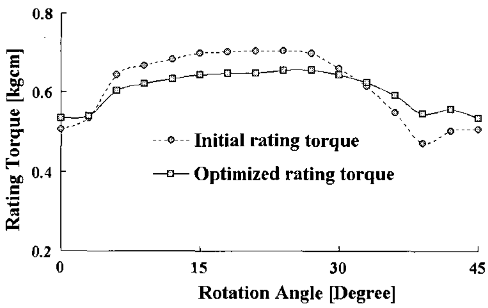


Fig. 6 Comparison of the rating torque for the initial and optimized shapes

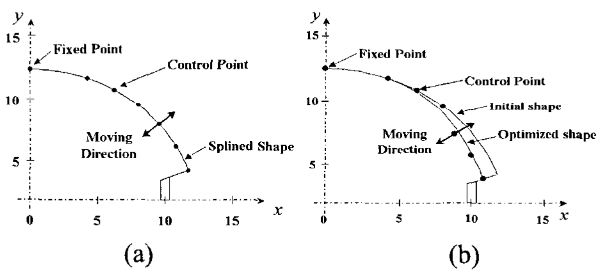


Fig. 7 Comparison of the initial and optimized yoke shapes: (a) Initial shape and design variables, (b) Optimized shape.

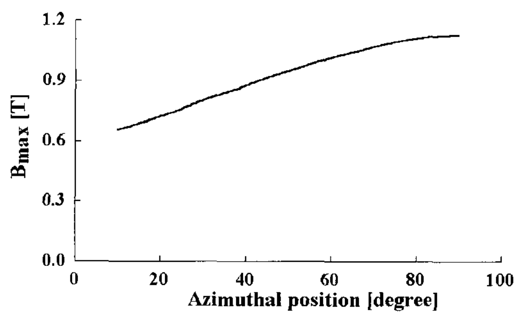


Fig. 8 Distribution of the maximum values of radial component of the magnetic flux density at the optimized shape.

5. Conclusion

A novel cogging torque reduction algorithm for the 2 pole PM DC motor is developed by combining the finite element method, the design sensitivity and the Bezier spline. In order to reduce the cogging torque, the distribution of the residual magnetization of the PM is, first, optimized. The magnetizing yoke shape is, then, optimized to get the desired distribution of the residual magnetization. The Bezier spline, through a numerical design example, is proved to be very effective for the shape optimization because it reduces the number of design variables and guarantees the smooth shape.

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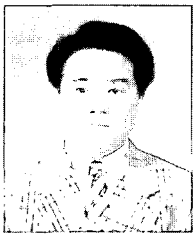

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