A New Method to Handle Transmission Losses using LDFs in Electricity Market Operation

Kyoung-Soo Ro† and Se-Young Han*

Abstract - This paper proposes a new method to handle transmission line losses using loss distribution factors (LDF) rather than marginal loss factors (MLF) in electricity market operation. Under a competitive electricity market, the bidding data are adjusted to reflect transmission line losses. To date the most proposed approach is using MLFs. The MLFs are reflected to bidding prices and market clearing price during the trading and settlement of the electricity market. In the proposed algorithm, the LDFs are reflected to bidding quantities and actual generations/ loads. Computer simulations on a 9-bus sample system will verify the effectiveness of the algorithm proposed. Moreover, the proposed approach using LDFs does not make any payments residual while the approach using MLFs induces payments residual.

Keywords: electricity market, loss distribution factor, marginal loss factor, transmission loss

1. Introduction

The onset of deregulation for electric power industries has promoted competitive electricity markets by enabling access to transmission services for all wholesale buyers and sellers of electricity alike. Transmission utilities must provide non-discriminatory transmission service to third parties at cost-based rates. Under a competitive electricity market, the pricing mechanism is based on the bidding data of generators and energy purchasers. Since a power transmission network intrinsically involves transmission losses, the bidding data should be adjusted to reflect transmission line losses during the electricity market operation. Transmission line losses can generally not be measured directly but are calculated with power-flow computer programs. These programs help to calculate actual power losses in transmission lines in near real time.

To date, the most proposed approach used to reflect transmission line losses is the transmission marginal loss factor(MLF)[1, 2]. MLF is the change in total transmission line loss with respect to the change in real power at a bus. During the trading and settlement of the electricity market, MLFs are reflected to the bidding prices and the market clearing price (MCP).

This paper proposes a new method to handle transmission losses using loss distribution factors(LDF) rather than MLFs in electricity market operation. LDFs are not marginal values but distribution factors of the existing transmission losses. In the proposed algorithm, the LDFs are reflected to the bidding quantities and actual generations/loads rather than to the bidding prices and MCP during the trading and settlement of the electricity market. A LDF represents the responsible share of generators or loads for the loss of a transmission line. The LDFs are calculated using the power tracing algorithm. A few approaches for power tracing have been developed to determine the shares of generators or loads for a particular transmission line flow[3, 4].

Computer simulations on a 9-bus sample power network will show a comparison of the results for the two cases, and will verify the effectiveness of the algorithm proposed.

2. Handling of Transmission Losses using MLFs

MLFs are defined by the following equation, which calculates the change in total real power loss with respect to a change in real power at each bus i. [5]

$$MLF_i = \frac{\partial P_{loss}}{\partial P_i} = \eta_i \tag{1}$$

Since losses are deemed to be supplied from the slack bus in power flow calculations, total power losses are insensitive to a change in real power at the slack bus, which means that

$$\eta_s = \frac{\partial P_{loss}}{\partial P_s} = 0 \tag{2}$$

where the subscript s denotes the slack bus. Thus, the

† Corresponding Author: Dept. of Electrical Engineering, Dongguk University, Korea. (ksro@dgu.ac.kr)
* Dept. of Electrical Engineering, Dongguk University, Korea. (iamhanse@nate.com)
Received November 3, 2004 ; Accepted March 24, 2005

location of the slack bus has a considerable impact on the values of MLFs.

Computation of MLFs starts with the results of power flow calculations for a system operating point and then applies the chain rule at that point to derive the following equation when assuming that bus 1 is the slack bus.

$$\begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_2} & \cdots & \frac{\partial P_n}{\partial \theta_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial P_2}{\partial \theta_n} & \frac{\partial P_3}{\partial \theta_n} & \cdots & \frac{\partial P_n}{\partial \theta_n} \end{bmatrix} \begin{bmatrix} \frac{\partial P_{loss}}{\partial P_2} \\ \frac{\partial P_{loss}}{\partial P_3} \\ \vdots \\ \frac{\partial P_{loss}}{\partial P_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{loss}}{\partial \theta_2} \\ \frac{\partial P_{loss}}{\partial \theta_3} \\ \vdots \\ \frac{\partial P_{loss}}{\partial \theta_n} \end{bmatrix} \quad (3)$$

The matrix in equation (3) is the transposition of the jacobian matrix in the Newton-Raphson method for solving the power-flow problem. Therefore, the MLFs are computed by multiplying both sides of equation (3) by the inverse of the matrix.

The right-hand side represents sensitivities of total power loss with respect to voltage angles. Since the power loss in a transmission line *i-j* is represented in equation (4), total power loss can be calculated by equation (5).

$$P_{loss,ij} = G_{ij}|E_i|^2 + G_{ij}|E_j|^2 - 2G_{ij}|E_i||E_j|\cos(\theta_i - \theta_j) \quad (4)$$

$$P_{loss} = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij}[|E_i|^2 + |E_j|^2 - 2|E_i||E_j|\cos(\theta_i - \theta_j)] \quad (5)$$

where G_{ij} is the conductance of a line *i-j*, $|E_i|$ is the voltage magnitude of a bus *i*, θ_i is the voltage angle of a bus *i*, and $P_{loss,ij}$ is the real power loss of a line *i-j*.

Then, the elements of the right-hand side of equation (3) can be developed like

$$\frac{\partial P_{loss}}{\partial \theta_i} = 2\sum_{j=1}^{n} G_{ij}|E_i||E_j|\sin(\theta_i - \theta_j) \quad (6)$$

Next, the bidding and settlement processes of the electricity market using MLFs are briefly described as follows. [1, 2]

i) Generators and energy purchasers bid quantities and prices.
ii) Bidding prices are adjusted by dividing the bid prices by(1-MLF) for generators. For energy purchasers, the adjustment is achieved by reversing the (-) sign.
iii) MCP is determined from the market rule.
iv) Settlement is implemented by multiplying metered generations/loads and MCP by the previous MLF terms.

According to the market code [1], the transmission owner receives payments for the total payments residual obtained as a result of the market settlement. The total payments residual is calculated by subtracting the total purchase charge to all energy purchasers from the total energy payment to all generators.

3. Handling of Transmission Losses using LDFs

This chapter describes the proposed scheme to handle transmission losses using LDFs in electricity market operation. The total transmission loss is equally divided, and one half of the total transmission loss is imposed on generators while the other half is imposed on loads. Each process to allocate half of the total transmission loss to generators/loads is explained in detail in sections 3.1 and 3.2.

After imposing transmission losses on generators and loads, the bidding and settlement processes of the electricity market using LDFs are briefly described as follows.

i) Generators and energy purchasers bid quantities and prices.
ii) Bidding quantities of generators are adjusted by adding the shares of generators for transmission line losses to the actual generations. Bidding quantities of loads are adjusted by adding the shares of loads for transmission line losses to the actual loads.
iii) MCP is determined from the market rule.
iv) Settlement is implemented by multiplying the adjusted generations/loads by MCP.

The approach handling transmission losses using LDFs is tested in the case study of the next chapter, with comparison of the result using MLFs.

3.1 Imposing Transmission Losses on Generators

Running a power-flow program can result in computing voltage values at all buses and real-power losses in all transmission lines. Assuming that it is possible to break up the total transmission loss into components added to individual loads, the total actual generation is equal to the fictitious demand, i.e. the sum of both the actual demand of a load bus plus the allocated component of the total transmission loss at the bus. Then, the sharing of the *k*-th generator for the actual power flow in a transmission line *i-j*, $R^u_{ij,k}$, can be calculated as follows. [4]

$$R^u_{ij,k} = \frac{P_{ij}}{P_i}[A_u^{-1}]_{ik} \quad (7)$$

where P_i is a bus power at bus i, and P_{ij} is a line flow in line i-j. A_u is the matrix that satisfies the equation below,

$$A_u P^{u,f} = P_G \tag{8}$$

Equation (8) is the matrix notation of the following equation.

$$P_i^{u,f} - \sum_{j \in \gamma_i} \frac{|P_{ji}|}{P_j} P_j^{u,f} = P_{Gi} \tag{9}$$

where $P^{u,f}$ is an unknown vector of fictitious bus powers, P_G is a vector of generations, and γ_i is a set of buses whose power flows into bus i.(The detailed derivation for the above equations is shown in Reference [4].)

Then, the responsible shares of generators for the real power losses in transmission lines can be represented as

$$P^u_{loss,ij,k} = U^u_{ij,k} P_{loss,ij} \tag{10}$$

$$U^u_{ij,k} = \frac{R^u_{ij,k} P_{Gk}}{\sum_l R^u_{ij,l} P_{Gl}} \tag{11}$$

where, $P^u_{loss,ij,k}$ is the responsible share of the k-th generator for the loss in a transmission line i-j, and $P_{loss,ij}$ is the real power loss in a transmission line i-j. $U^u_{ij,k}$ is now referred to as a LDF for the k-th generator.

Next, the total power loss responsible for the k-th generator is computed in the following equation.

$$P^u_{loss,k} = \sum_{ij \in \alpha_l} U^u_{ij,k} P_{loss,ij} \tag{12}$$

where α_1 is a set of all transmission lines. Equation (12) says that summing up the shares of a generator for individual line losses becomes the generator's total responsible loss.

3.2 Imposing Transmission Losses on Loads

The sharing of the k-th load for the real power flow in a transmission line i-j, $R^d_{ij,k}$, can be calculated as follows. [4, 6]

$$R^d_{ij,k} = \frac{P_{ij}}{P_i} [A_d^{-1}]_{ik} \tag{13}$$

where A_d is the matrix that satisfies the equation below,

$$A_d P^{d,f} = P_R \tag{14}$$

Equation (14) is the matrix notation of the following equation.

$$P_i^{d,f} - \sum_{j \in \beta_i} \frac{|P_{ji}|}{P_j} P_j^{d,f} = P_{Ri} \tag{15}$$

where $P^{d,f}$ is an unknown vector of fictitious bus powers, P_R is a vector of load demands, and β_i is a set of buses whose power flows from bus i.

Then, the responsible shares of loads for the actual power losses in transmission lines can be represented as

$$P^d_{loss,ij,k} = U^d_{ij,k} P_{loss,ij} \tag{16}$$

$$U^d_{ij,k} = \frac{R^d_{ij,k} P_{Rk}}{\sum_l R^d_{ij,l} P_{Rl}} \tag{17}$$

where, $P^d_{loss,ij,k}$ is the responsible share of the k-th load for the loss in a transmission line i-j. $U^d_{ij,k}$ is referred to as a LDF for the k-th load.

Next, the total power loss responsible for the k-th load is computed in the following equation.

$$P^d_{loss,k} = \sum_{ij \in \alpha_l} U^d_{ij,k} P_{loss,ij} \tag{18}$$

4. Case Studies

A 9-bus sample system, shown in Fig. 1, is used to verify the two algorithms presented in the previous chapter. The sample system has 3 generators and 6 loads (energy purchasers).

4.1 Results using MLFs

This section starts with assuming the bidding data for generators and energy purchasers, as shown in Table 1. Without considering transmission losses, the MCP is determined as 53$/MW and the trading quantity is 400MW.

The approach (called case 1) using MLFs applies Equation (1) to get Table 2, which gives rise to MLFs for each bus, assuming bus 1 as the reference bus.

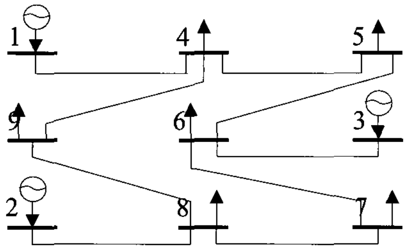


Fig. 1 A 9-bus sample system.

Table 1 Bidding data for generators and energy purchasers.

Generator			Energy purchaser		
Bus	Quantity (MW)	Price ($/MW)	Bus	Quantity (MW)	Price ($/MW)
1	50	32	4	160	53
	100	41	5	50	57
	150	50	6	50	55
2	100	38	7	100	58
	150	47	8	50	54
	200	60	9	100	56
3	50	35			
	100	44			
	150	56			

Table 2 MLFs for each bus

Bus	MLFs
1	0.0000
2	-0.0539
3	-0.0325
4	0.0007
5	0.0424
6	-0.0127
7	0.0163
8	-0.0288
9	0.0512

The bidding prices of Table 1 are adjusted by dividing the bidding prices by (1 +/- MLFs). Table 3 indicates the results of adjusting the bidding prices. Based on the adjusted bidding data, MCP is calculated as 52.963 $/MW. The trading result for this case will be illustrated in section 4.3 with comparison of the result using LDFs.

Table 3 Adjusted bidding data using MLFs

Generator			Energy purchaser		
Bus	Quantity (MW)	Price ($/MW)	Bus	Quantity (MW)	Price ($/MW)
1	50	32	4	160	52.963
	100	41	5	50	54.682
	150	50	6	50	55.708
2	100	36.057	7	100	57.070
	150	44.596	8	50	55.601
	200	56.931	9	100	53.273
3	50	33.898			
	100	41.615			
	150	54.237			

4.2 Results using LDFs

Table 4 illustrates the result of power flow calculations for the sample system of Fig. 1. Positive values imply that power flow direction is out of the bus and the opposite direction is for negative values. The sum of the two power flow values for each line represents the real power loss in the line and the total line loss is the sum of the individual line losses, which is 12.341 MW.

Table 4 Result of power flow calculations for real power(MW)

From	To	P(from)	P(to)	Loss
1	4	111.34	-110.26	1.082
4	5	46.88	-46.01	0.871
5	6	-43.99	45.26	1.264
3	6	85.00	-84.13	0.868
6	7	28.88	-28.38	0.499
7	8	-71.62	73.24	1.621
8	2	-160.99	163.00	2.009
8	9	73.75	-71.22	2.529
9	4	-53.78	55.38	1.598

The approach (called case 2) of imposing transmission losses equally on generators and loads using LDFs is applied to the same system to get Table 5, which represents the transmission line losses allocated to generators and loads. The sum of the line losses allocated to each generator and load is identical to the total line loss. Table 6 shows adjusted bidding quantities of generators (energy purchasers) by subtracting (adding) the shares of generators (loads) for transmission line losses from the actual generations (loads).

Table 5 Transmission line losses allocated to generators and loads (MW)

G-1	G-2	G-3	L-4	L-5	L-6	L-7	L-8	L-9	Total
1.776	3.080	1.316	0.040	1.530	0.053	1.668	0.090	2.790	12.341

Table 6 Adjusted bidding data using LDFs

Generator			Energy purchaser		
Bus	Quantity (MW)	Price ($/MW)	Bus	Quantity (MW)	Price ($/MW)
1	51.78	32	4	160.04	53
	101.78	41	5	51.53	57
	151.78	50	6	50.05	55
2	103.08	38	7	101.67	58
	153.08	47	8	50.09	54
	203.08	60	9	102.79	56
3	51.32	35			
	101.32	44			
	151.32	56			

4.3 Comparison and Discussion

Table 7 indicates the MCPs and trading quantities for the two cases. The trading quantity of case 2 is 406.17 MW, which reflects half of the total transmission loss of 12.341 MW. Fig. 2 shows the determination of MCPs of both case 1 and case 2.

Table 7 Comparison of MCPs and trading quantities

	Case 1	Case 2
MCP ($/MW)	52.963	53
Trading quantity (MW)	400	406.17

Table 8 illustrates settlement for generators and energy purchasers. Case 1 indicates constant trading quantities whereas case 2 exhibits adjusted trading quantities. Market prices are adjusted in case 1 whereas they are constant in case 2. It is more reasonable to think that the transmission losses should be reflected in the trading quantities instead of the prices. Energy purchasers' total payments for case 2 are less by $19.87 compared to case 1. Moreover, case 1 induces a payment residual as a result of market settlement. However, case 2 does not create any payment residual.

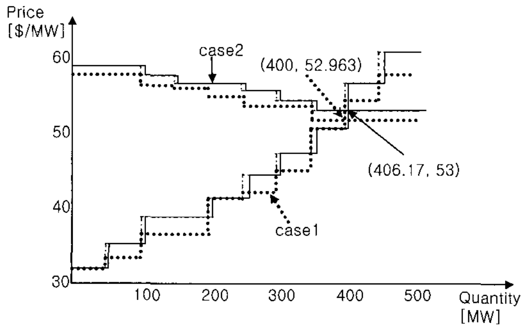


Fig. 2 Determination of MCPs of the two cases

Table 8 Settlement for generators and energy purchasers
(a) Generators' incomes

	Generation [MW]		Market price [$/MW]		Income [$]	
	case1	case2	case1	case2	case1	case2
G-1	150	151.78	52.963	53	7944.44	8044.10
G-2	150	153.08	55.818	53	8372.64	8113.21
G-3	100	101.32	54.684	53	5468.42	5369.72
sum of incomes [$]					21785.50	21527.03

(b) Energy purchasers' payments

	Load [MW]		Market price [$/MW]		Payment [$]	
	case1	case2	case1	case2	case1	case2
L-4	50	50.04	53	53	2650.00	2652.13
L-5	50	51.53	55.209	53	2760.43	2731.10
L-6	50	50.05	52.290	53	2614.51	2652.79
L-7	100	101.67	53.826	53	5382.62	5388.41
L-8	50	50.09	51.438	53	2571.88	2654.76
L-9	100	102.79	55.675	53	5567.46	5447.84
Sum of payments					21546.90	21527.03
Residual surplus of settlement					238.60	0

5. Conclusion

This paper proposed a new approach to handle transmission losses in electricity market operation using LDFs instead of MLFs. The main idea here is that it is more reasonable that the transmission losses should be reflected in the trading quantities rather than in the prices. Based on the result of the case study, the approach using MLFs (case 1) adjusts the bidding quantities whereas the approach using LDFs (case 2) adjusts the bidding prices. Since transmission losses are power quantities, it is natural to handle transmission losses in trading quantities rather than in trading prices. Settlement results indicate that the two cases show quite similar results for generators' incomes and energy purchasers' payments.

Moreover, the approach using MLFs induces a payment residual as a result of market settlement. That money has fallen into the transmission owner's hand. However, the approach using LDFs does not make any payment residual. Thus, it can be anticipated that the approach proposed in this paper promotes the operation of a competitive electricity market.

References

[1] Korea Power Exchange (KPX), Market Code, 2002.

[2] Jong-Bae Park, Ki-Song Lee, Joong-Rin Shin, Sung-Soo Kim "The Development of the Transmission Marginal Loss Factors with Consideration of the Reactive Power and its Application to Energy Spot Market" *The Transactions of the KIEE*, Vol. 52, No. 7, pp. 429-436, 2003

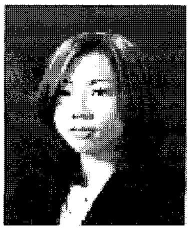
[3] W.Y. Ng, "Generalized Generation Distribution Factors for Power System Security Evaluations", *IEEE Trans. on Power Apparatus and Systems*, Vol. PAS-100, No. 3, pp. 1001-1005, March 1981

[4] J. Bialek, "Topological Generation and Load Distribution Factors for Supplement Charge Allocation in Transmission Open Access", *IEEE Trans. on Power Systems*, Vol. 12, No. 3, pp. 1185-1193, August 1997

[5] J. Mutale, G. Strbac, S. Curcic and N. Jenkins. "Allocation of losses in distribution systems with embedded generation", *IEE Proc.-Gener. Transm. Distrib.*, Vol. 147, No. 1, pp. 7-14, January 2000

[6] K. Ro, "An Approach to Allocate Real Power Losses of Transmission Lines to Individual Loads", *The Transactions of the KIEE*, Vol. 49A, No. 1, pp. 13-18, 2000

Kyoung-Soo Ro
He obtained his B.S. and M.S. degrees from Seoul National University, Seoul, Korea in 1985 and 1987, respectively. He graduated from the Virginia Polytechnic Institute and State University with a Ph.D degree in 1997. He is now an Associate Professor at Dongguk University, Seoul, Korea. His main areas of interest are power system operation and control, applications of renewable energy systems, and power system deregulation.



Se-Young Han
She received her B.S. degree in Electrical Engineering from Dongguk University, Seoul, Korea in 2003. She is currently a graduate student pursuing her Master's degree at Dongguk University, Seoul, Korea. Her research interests are power system deregulation and renewable energy systems.