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SELF-SIMILAR SOLUTIONS OF ADVECTION-DOMINATED ACCRETION FLOWS REVISITED

Heon-Young Chang
Department of Astronomy and Atmospheric Sciences, Kyungpook National University
1370 Sankyuk-dong, Buk-gu, Daegu 702-701, Korea
email: hyc@knu.ac.kr

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ABSTRACT

A model of advection-dominated accretion flows has been highlighted in the last decade. Most of calculations are based on self-similar solutions of equations governing the accreting flows. We revisit self-similar solutions of the simplest form of advection-dominated accretion flows. We explore the parameter space thoroughly and seek another category of self-similar solutions. In this study we allow the parameter f less than zero, which denotes the fraction of energy transported through advection. We have found followings: 1. For $f > 0$, in real ADAF solutions the ratio of specific heats γ satisfies $1 < \gamma < 5/3$ for $0 \leq f \leq 1$. On the other hands, in wind solutions a rotating disk does not exist. 2. For $f < 0$, in real ADAF solutions with ϵ greater than zero γ requires rather exotic range, that is, $\gamma < 1$ or $\gamma > 5/3$. When $-5/2 < \epsilon' < 0$, however, allowable γ can be found in $\gamma < 5/3$, in which case $\Omega_{0,-}$ is imaginary. 3. For a negative $u_{0,+}$ with $f > 0$, solutions are only allowed with exotic γ, that is, $1 < \gamma$ or $\gamma > (5f/2-5/3)/(5f/2-1)$ when $0 < f < 2/5$, $(5f/2-5/3)/(5f/2-1) < \gamma < 1$ when $f > 2/5$. Since ϵ' is negative, $\Omega_{0,+}$ is again an imaginary quantity. For a negative $u_{0,+}$ with $f < 0$, γ is allowed in $1 < \gamma < (5|f|/2 + 5/3)/(5|f|/2 + 1)$. We briefly discuss physical implications of what we have found.

Keywords: accretion, accretion disk – black hole physics – hydrodynamics

1. INTRODUCTION

Accretion is one of the most important physical processes in astrophysics. It is a widely accepted idea that the accreting matter toward the central compact objects is a source of power of active galactic nuclei (AGNs), and Galactic X-ray sources (see for a review, e.g., Frank, King, & Raine 2002). This idea is also well-applicable to interpret many observations of astrophysical phenomena, such as, proto-type stellar objects, (Suh 1996, Kenyon, Yi, & Hartmann 1996, Chang & Choi 2002), symbiotic stars (Lee & Park 1999), gamma-ray bursts (Brown et al. 2000). The most natural mode of accretion is through disks, geometrically thick or thin. There are three families of stable accretion disk models studied upto date : cool and thin disk (Shakura & Sunyaev 1973), slim disk (Abramowicz et al. 1988), and various versions of advection-dominated accretion flows (ADAF: Ichimaru 1977, Narayan & Yi 1994, ADIOS: Xu & Chen 1997, Blandford & Begelman 1999, Turolla & Dullemond 2000, Misra & Taam 2001, CDAF: Stone, Pringle, & Begelman 1999, Narayan, Igumenshchev, & Abramowicz 2000, Quataert & Gruzinov 2000, Abramowicz et al. 2002). Among these, a model of advection-dominated accretion flows (ADAFs) has been highlighted in recent years, particularly to account for dim galactic nuclei in X-rays including Low-Ionization Nuclear Emission Resions (LINERs), Low Luminosity AGNs (LLAGNs), Sgr A* in our own Galaxy center (Narayan, Yi, & Mahadevan 1995, Lasota et al. 1996, Ulvestad & Ho 2001, Chang & Choi 2003), and hard (low) state of Low Mass X-ray binary systems (Esin et al. 2001).

These studies on ADAF-variations have been carried out mostly based on self-similar solutions derived by Narayan & Yi (1994: hereafter NY94), in which all the unknown quantities scale as powers of normalized distance from the central object by the gravitational radius, r, in accretion flows. Though it may be too simple to describe complete characteristics as a whole, such a self-similar solution is easy both to handle and to understand. This is a reason why self-similar solutions of accretion flows have been investigated in the first place by picking up the most essential key feature of accretion flows to work on (e.g., NY94; Blandford & Begelman 1999, Wang & Zhou 1999, Narayan, Igumenshchev, & Abramowicz 2000, Medvedev & Narayan 2001). Though pursuing global solutions and/or sophisticated features can be timely and valuable (e.g., Park 1995, 2001, Honma 1996, Park & Ostriker 1999, 2001, Narayan, Kato, & Honma 1997, Mukhopadhyay & Ghosh 2003, Lu, Li, & Gu 2004), looking at self-similar solutions is still of interests.

Self-similar solutions for various conditions have been studied by many authors. Even in the case of ADAFs, equations of time-dependent quasi-spherical accretion are solved in a simplified one-dimensional model neglecting the latitudinal dependence of the flow (Ogilvie 1999). Considering ADAFs connected at a finite transition radius to an outer standard cool disk, a different power index for the angular velocity Ω has been discovered (e.g., Honma 1996). A self-similar solution for self-gravitating viscous disks is also studied (Mineshige & Umemura 1996, 1997, Mineshige, Nakayama, & Umemura 1997, Bertin & Lodato 1999, Boss & Hartmann 2001). Another interesting study has been done by Beloborodov & Illarionov (2001). They considered inviscid disk around black holes, that is, spherical infalling onto the accretion disk.

In this paper we revisit self-similar solutions of the simplest ADAFs which NY94 studied, and discuss other possible forms of accretion and their physical implications. We briefly review what the problem is, and discuss solutions of equations in §2. We then describe what are missing parts of self-similar solutions discussed earlier by NY94 in §3. And finally we conclude by summarizing and making some comments in §4.

2. QUICK SUMMARY OF SELF-SIMILAR SOLUTIONS OF SIMPLE ADAF

In the following we briefly summarize what NY94 did. Advection-dominated accretion flows are known to be quasi-spherical; therefore, they have implicitly assumed spherical symmetry in the derivation. In fact, they discuss (Newtonian) accretion flows as polar-averaged, one-dimension flows with only a radial coordinate r. They have considered a steady state, axisymmetric flow so that $\partial/\partial t = \partial/\partial \phi = 0$. Mass is accreted by the central object with a rate $\dot{M} = -4\pi r u H \rho$, where ρ is the volume density, u is the accretion velocity being negative for inflowing, H is the vertical scale height which is assumed proportional to r such that $H \sim c_s/\Omega_K$, sound speed c_s being the isothermal sound speed and Ω_K being the Keplerian angular velocity. Note that the H given above is always satisfied if one assumes a vertically stratified disk. For the viscosity ν, they adopt the α prescription introduced by Shakura & Sunyaev (1973), where ν is given as $\nu = \alpha c_s^2/\Omega_K$. They have defined a parameter $\epsilon = (5/3-\gamma)/(\gamma-1)$, where γ is the ratio of specific heats. Another predefined parameter f measures the degree to which the flow is advection-dominated. They defined $\epsilon' = \epsilon/f$, which plays a crucial role in determining the nature of the flows as shown below. Note also that the quantity f is designated as a positive quantity in their discussions for the inward advection flow.

Four differential equations of steady state and axisymmetric accretion disk are given as follows:

$$
\begin{aligned}
\frac{d}{dr}(\rho r H u) &= 0, \\
u\frac{du}{dr} - \Omega^2 r &= -\Omega_K^2 r - \frac{1}{\rho}\frac{d}{dr}(\rho c_s^2), \\
u\frac{d(\Omega r^2)}{dr} &= \frac{1}{\rho r H}\frac{d}{dr}\left(\frac{\alpha\rho c_s^2 r^3 H}{\Omega_K}\frac{d\Omega}{dr}\right), \\
\Sigma u T\frac{ds}{dr} = \frac{3+3\epsilon}{2}2\rho H u\frac{dc_s^2}{dr} - 2c_s^2 H u\frac{d\rho}{dr} &= Q^+ - Q^-, \\
Q^+ - Q^- &\equiv f\frac{2\alpha\rho c_s^2 r^2 H}{\Omega_K}\left(\frac{d\Omega}{dr}\right)^2
\end{aligned}
\tag{1}
$$

where notations follow conventional meanings.

Now self-similar solutions can be found for density ρ, radial accretion velocity u, angular velocity Ω, and isothermal sound speed c_s. By substituting

$$
\begin{aligned}
\rho(r) &= \rho_0 r^a, \\
u(r) &= u_0 r^b, \\
\Omega(r) &= \Omega_0 r^c, \\
c_s^2(r) &= c_{s0}^2 r^d
\end{aligned}
\tag{2}
$$

into four basic equations describing accreting flows and by equating exponents of r in the various terms, one obtains sets of algebraic equations of the exponents of r. By solving those algebraic equations one can straightforwardly show that self-similar solutions exist if

$$
\begin{aligned}
\rho(r) &= \rho_0 r^{-3/2}, \\
u(r) &= u_0 r^{-1/2}, \\
\Omega(r) &= \Omega_0 r^{-3/2}, \\
c_s^2(r) &= c_{s0}^2 r^{-1},
\end{aligned}
\tag{3}
$$

with the radial coordinate scaled to the gravitational radius of the central mass, where

$$
\begin{aligned}
\rho_{0,\pm} &= \frac{-\dot{M}\sqrt{GM}}{u_{0,\pm}c_{s0,\pm}}, \\
u_{0,\pm} &= \frac{\sqrt{GM}}{3\alpha}(2\epsilon'+5)\left[1 \pm \left(1 + \frac{18\alpha^2}{(2\epsilon'+5)^2}\right)^{1/2}\right], \\
\Omega_{0,\pm} &= \left(\frac{-2\sqrt{GM}}{3\alpha}\epsilon' u_{0,\pm}\right)^{1/2}, \\
c_{s0,\pm}^2 &= \frac{-2\sqrt{GM}}{3\alpha}u_{0,\pm},
\end{aligned}
\tag{4}
$$

where $g = [(1 + 18\alpha^2/(2\epsilon'+5)^2)^{1/2} - 1]$.

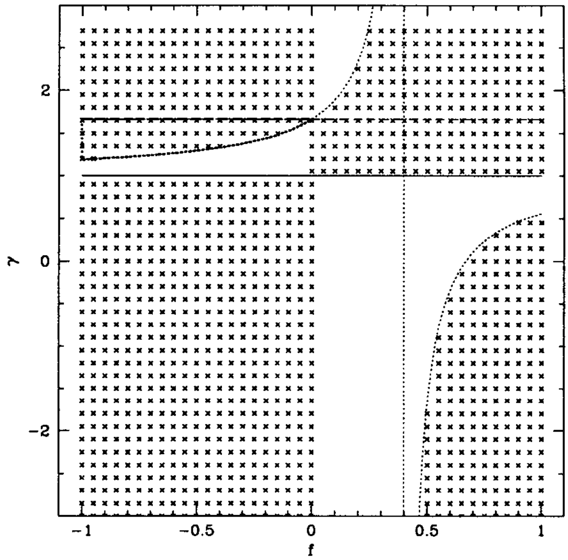

Figure 1. Allowed regions of γ in the $f - \gamma$ plane for the condition of $2\epsilon' + 5 > 0$ as $u_{0,-}$ has a negative sign. The dotted curves represent solution of equations for γ (see text), the solid line and the dashed indicate $\gamma = 1$ and $\gamma = 5/3$, respectively. Note that Narayan & Yi (1994) discuss only when f is greater than zero. The area enclosed by thick lines indicates the allowable γ when $-5/2 < \epsilon' < 0$ for a negative f value.

NY94 have taken $u_{0,-}$ as an inflow solution and discarded $u_{0,+}$ so that all other coefficients listed in eq. 4 are real quantities. In this section all discussions are restricted to the case of $u_{0,-}$. Since g is always positive for real ϵ', $2\epsilon' + 5$ should also be greater than zero for an inflow solution. Hence, γ is given such that, for a positive f as they assumed, $1 < \gamma < (5f/2 - 5/3)/(5f/2 - 1)$ when $f < 2/5$, and $\gamma < (5f/2 - 5/3)/(5f/2 - 1)$ or $\gamma > 1$ when $f > 2/5$. Allowed regions in the $f - \gamma$ plane are shown marked with crosses in Fig. 1. It should be noted that there is one more contingent condition, that is, $\Omega_{0,-}$ should be *real* for a rotating inflow solution. Therefore, ϵ' must be positive, rather than $2\epsilon' + 5 > 0$. This constrains the allowed regions for a positive f to $1 < \gamma < 5/3$ in the γ direction as shown with open circles in Fig. 2. It simply implies that self-similar solutions for inflowing ADAFs exist with $1 < \gamma < 5/3$ when $f > 0$. Main properties of self-similar solutions are as follows: Firstly, they seems to agree quite well with numerical solutions except regions close to boundaries. It makes self-similar solutions useful and widely used. Secondly, when the ratio of specific heats γ approaches $5/3$ only the non-rotating flow is allowed in these self-similar solutions. For when $\gamma = 5/3$, ϵ' is equal to zero meaning $\Omega_{0,-} = 0$ (see eq. 4). Thirdly, these flows, according to the self-similar solutions, are convectively unstable and subject to outflows. These facts actually lead to derivation of ADIOS and CDAF models.

It is worth making several remarks on the missing part of self-similar solutions of NY94 at this point, which they failed to discuss in their original paper. Firstly, they briefly mentioned the case of $\gamma > 5/3$, and they referred to this second class of solutions as a rotating wind solution where $u_{0,-} > 0$. However, when $f > 0$ and $\gamma > 5/3$, there can be two possibile regimes. That is, one is $-5/2 < \epsilon' < 0$ and the other is $\epsilon' < -5/2$. In the former case, one may still have an accreting solution, and $\Omega_{0,-}$ is an imaginary quantity. In the latter case, one has a *wind* solution with

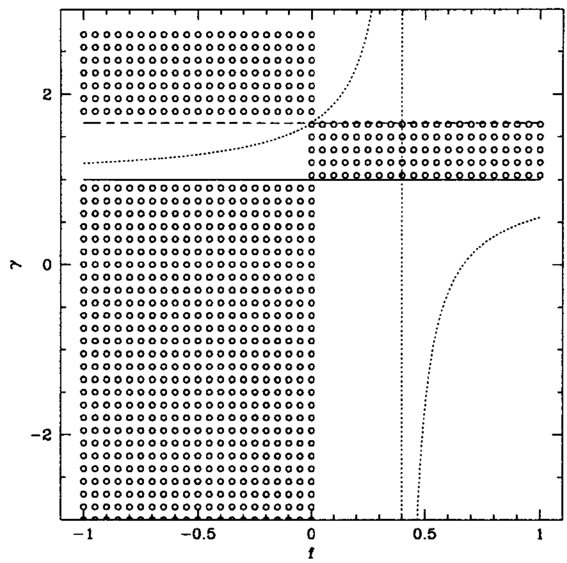

Figure 2. Allowed regions of γ in the $f-\gamma$ plane for the condition of $\epsilon' > 0$ as $\Omega_{0,-}$ is real, as well as $u_{0,-}$ has a negative sign. The dotted curves represent solution of equations for γ (see text), the solid line and the dashed indicate $\gamma = 1$ and $\gamma = 5/3$, respectively. Note that Narayan & Yi (1994) discuss only when f is greater than zero.

$u_{0,-} > 0$. This is what NY94 probably meant. But in this case, $c_{s0,-}$ and $\rho_{0,-}$ become imaginary quantities. Consequently, it turns out that figuring out $\gamma > 5/3$ case under the condition of positive f is not as simple as they mentioned. Secondly, if one allows a negative f in the ADAF self-similar solutions, while keeping positive $2\epsilon' + 5$ so that $u_{0,-}$ is still negative (accreting inward), allowed regions become $\gamma < 1$ and $\gamma > (5|f|/2 + 5/3)/(5|f|/2 + 1)$, as shown marked with crosses in Fig. 1. The negative f means cooling dominates over viscous heating for some reasons, which we discuss later on. It may imply that if $-5/2 < \epsilon' < 0$ then $\Omega_{0,-}$ is imaginary with γ smaller than $5/3$, and yet greater than $(5|f|/2 + 5/3)/(5|f|/2 + 1)$. Of course, even when γ is greater than $5/3$ an imaginary $\Omega_{0,-}$ is possible. We suspect these cases can be regarded as an over-damped oscillator. In this case, the infalling fluid is unable to make a complete rotation around the central object before getting to the final destination in the sense that $\Omega_{0,-}$ is not physically defined. For positive ϵ' and real $\Omega_{0,-}$, allowed region of γ satisfies $\gamma < 1$ or $\gamma > 5/3$, as shown in Fig. 2. In a sense, therefore, for a negative f only possible self-similar solutions of simple ADAF equations with real quantities are available only when γ is greater than $5/3$.

3. ANOTHER FAMILY OF SELF-SIMILAR SOLUTIONS OF ADAF

In this section we seek self-similar solutions in which the negative f can be accommodated. We explore the case of $u_{0,+}$ in this section, which NY94 discarded. Since $g + 2$ is always positive, there exists an inflow solution, that is, negative $u_{0,+}$, when $2\epsilon' + 5$ is less than zero (see eq. 4). In this case, however, $\Omega_{0,+}$ becomes again an imaginary quantity since ϵ' has a negative sign. We regard this case as an over-damped oscillator, as discussed in the case of $u_{0,-}$ with a negative f in the last section.

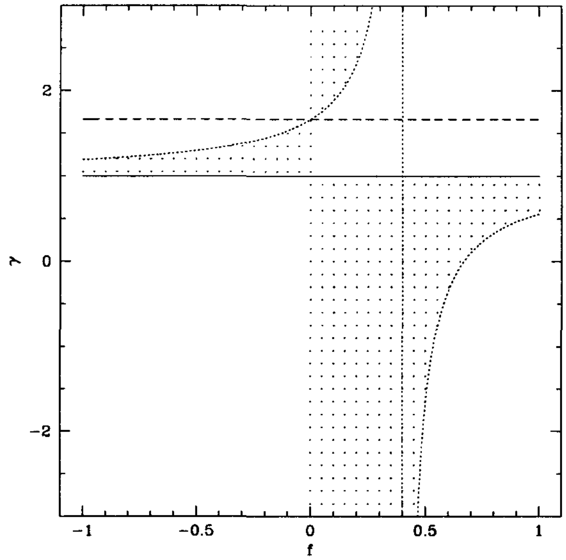

Figure 3. Allowed regions of γ in the $f - \gamma$ plane for negative $u_{0,+}$. The dotted curves represent solution of equations for γ (see text), the solid line and the dashed indicate $\gamma = 1$ and $\gamma = 5/3$, respectively.

To see possible range of γ for negative $u_{0,+}$, we solve the inequality equation, $2\epsilon' + 5 < 0$. Allowed regions of γ in the $f - \gamma$ plane are shown in Fig. 3. Solutions are obtained in three separate domains of f, in which when $f < 0$, γ should satisfy $1 < \gamma < (5|f|/2 + 5/3)/(5|f|/2 + 1)$, when $0 < f < 2/5$, $1 < \gamma$ or $\gamma > (5f/2 - 5/3)/(5f/2 - 1)$, and when $f > 2/5$, $(5f/2 - 5/3)/(5f/2 - 1) < \gamma < 1$, as shown in Fig. 3 marked with dots. We note that only when f is less than zero smaller γ than $5/3$ is available. It should be also noted that when f is greater than $2/5$ possible γ range is smaller than unity.

For a positive $u_{0,+}$, $2\epsilon' + 5$ should be greater than zero. In this case, however, sound speed $c_{s0,+}$ and density $\rho_{0,+}$ become imaginary quantities, which is hard to make a physical sense. Therefore, as of the case of $u_{0,-}$, self-similar solutions with positive radial velocity are unlikely to exist.

4. DISCUSSION AND CONCLUSION

We have revisited four basic equations for the simplest form of ADAFs, deriving self-similar solutions of them. We attempt to find out any other *missing* family of self-similar solutions in the setting similar to that of NY94. We take into account of possibility of negative f and consider another branch of solutions, which is ignored by NY94. The transition of the advection factor f from $f \approx 0$ to $f > 0$ corresponds to the transition from the standard thin disk to the geometrically thick disk. It happens when the accretion rate passes across $\sim \dot{m}_c$, where $\dot{m}_c$ is the critical mass accretion rate below which the ADAF exists. On the other hand, the transition from $f > 0$ to $f < 0$ occurs when the cooling dominates the viscous dissipation. Cooling may dominate under various circumstances. For instance, if there is a source of enough soft photons (either from the central star, or a flying-by star in the case of accretion disk around supermassive black holes) it can cool the disk by comptonization (Chang 2001). It is acting like a negative f. Along the line of a negative f one may be interested in pursuing the possibility of the existence of a self-similar solution corresponding to the Luminous Hot Accretion Flow (LHAF) solution for $f < 0$ (Yuan 2001). Or one may ask under what condition of γ the self-similar solution of LHAF exists.

We summarize our findings as follows:

1. For $f > 0$, ADAF solutions (any 1-dimensional accreting solution with advection) are required to satisfy $2\epsilon' + 5 > 0$ due to negative $u_{0,-}$ (inflow) and $\epsilon' > 0$ due to real $\Omega_{0,-}$ at the same time. This leads γ to satisfy $1 < \gamma < 5/3$ for $0 \leq f \leq 1$. In this case, all the quantities to describe inflowing accretion disks are real quantities. Outflowing solutions, or positive $u_{0,-}$ are not a form of rotating disk. When ϵ' is given so as $\epsilon' < -5/2$, $c_{s0,-}$ and $\rho_{0,-}$ turn out to be imaginary quantities.

2. For $f < 0$, self-similar ADAF solutions should satisfy $2\epsilon' + 5 > 0$ and $\epsilon' > 0$ like the case of positive f. When $\epsilon' > 0$, allowable γ is located at $\gamma < 1$ or $\gamma > 5/3$. Though all the quantities to describe inflowing accretion disks are real quantities, the physical condition to allow such an extreme γ is difficult to realize physically. When $-5/2 < \epsilon' < 0$, however, allowable γ can be found in $\gamma < 5/3$ as can be seen in Fig. 1. Yet, in this case, $\Omega_{0,-}$ is imaginary. We regard this is the case where infalling material cannot make a complete rotation before being swallowed by the central object.

3. For a negative $u_{0,+}$ with $f > 0$, ϵ' should satisfy $2\epsilon' + 5 < 0$. Solutions are allowed with exotic γ, that is, $1 < \gamma$ or $\gamma > (5f/2-5/3)/(5f/2-1)$ when $0 < f < 2/5$, $(5f/2-5/3)/(5f/2-1) < \gamma < 1$ when $f > 2/5$. Since ϵ' is negative, $\Omega_{0,+}$ is a imaginary quantity. For a negative $u_{0,+}$ with $f < 0$, γ is allowed in $1 < \gamma < (5|f|/2 + 5/3)/(5|f|/2 + 1)$, as shown marked with dots in Fig. 3. To make available smaller γ than 5/3, only negative f is allowable.

We have found no other *rotating* flows corresponding to negative f, under the condition of $\gamma < 5/3$, since the rotational velocity turns out to be an *imaginary* quantity. These are same for both $u_{0,-}$ and $u_{0,+}$. They end up before making a complete circle while accreted. Therefore, these self-similar solutions cannot represent the most common form of accretion, that is, a *rotating* disk. When the viscous heating is insufficient to compensate coolings, the hot accretion flows will collapse. These solutions might describe a homologous collapse of a disk with viscosity. These two solutions, however, may end up with the different surface density, etc. An interesting question to ask is what could separate these two kinds of solutions. From Figs. 1 and 3, for a given value of f, only difference is in the value of γ. So when fluid comes into experiencing different environment resulting in change in γ, as a result of that accreting fluid may suddenly show different appearance.

On the other hand, a self-similar solution of $u_{0,-}$ for rotating inflows may be allowed for a negative f. In this case, γ should be able to have *extraordinary* values such as, $\gamma < 1$ or $\gamma > 5/3$. There are no self-similar solutions of the NY94 form for $1 < \gamma < 5/3$. With the Baym-Bethe-Pethick equation of state, γ can exceed 5/3 (Shapiro & Teukolsky 1983). It could be achieved when the density is comparable to that of nucleus. We suspect that this might be a self-similar solution corresponding to a neutrino cooling disk whose density is extremely high (e.g., Kohri & Mineshige 2002).

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