

# Predicting the Failure of Slope by Mathematical Model

## 수학적 모델을 이용한 사면파괴예측

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### 요 지

사면 붕괴를 예측하기 위해서 적당한 수학적 모델을 선택하는 것은 매우 유용하다. 시간열로 실시간 계측된 자료를 통하여 합리적인 사면붕괴 예측용 수학적 모델을 선정할 수 있다. 3차 방정식을 이용한 2가지 형태의 이론적 모델이 이 연구에서 사용되었다(Polynomial 및 Growth 형). 사면의 변위각 및 침하를 계측할 수 있는 계측기가 느릅재 및 북실 현장에 적용되어 모델의 적용가능성을 점검하였다. 그 결과 계측 자료와 두 가지 수학적 모델과 아주 높은 일치성을 보였다.

### Abstract

It is useful to select an appropriate mathematical model to predict landslide. Through observation and analysis of real-time measured time series, a reasonable mathematic model is chosen to do prediction of landslide. Two theoretical models, such as polynomial function and growth model, are suggested for the description and analysis of measured deformation from an active landslides. These models are applied herein to describe the main characteristics of deformation process for two types of landslide, namely polynomial and growth models. The TRS (tension, rotation and settlement) sensors are applied to adopt two models, and the data analysis of two field (Nerupjae and Buksil) resulted in good coincidence between measured data and models.

**Keywords** : Growth model, Landslide, Polynomial model, Slope

## 1. Introduction

Throughout the years, field engineers have collected a large amount of data from monitoring engineering projects that are subjected to potential landslide hazards. In the civil engineering field, practitioners have accumulate rich monitoring data. Nevertheless, the question of how these data can be systematically used to provide information to facilitate the prediction of future landslide activities has not been given sufficiently addressed. The frequent occurrence of disastrous landslides in recent

years in Korea and Chain have reiterated the urgent need for geotechnical professionals to forecast more exactly the sliding event. A review of current works shows that existing prediction methods for landslide have seldom been based on basic physical principles or concepts. However, observations have confirmed repeatedly that the classical growth models about the phenomenon of growth could be applicable to provide predicting ability for landslide. For example, the deformation of rock avalanche grows exponentially or with power function, at least for certain period. According to a model devised

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by Verhulst, the deformation of a landslide with foot of the slope facing river bottom may follow the logistic distribution. The present paper shows the methodology involved in utilizing two kinds of theoretical models for the description and analysis of monitoring data and how the resulting information can be interpreted to facilitate prediction.

## 2. Prerequisites for Prediction Model

It is very important to choose an appropriate mathematical model to do the prediction of landslide. It will be totally wrong to think that only if fitting is very well by a suggested model or the residual sum of squares is minimum to the model, then saying the model is the best as well. This is because fitting and prediction are totally different concepts. Fitting just means past and now, and fitting well just means good interpolation estimation. But it would be groundless to extend such interpolation estimation into future practice. Very often, this kind of extension is sometimes good and is sometimes bad. The only way to achieve better prediction effect is to understand the mechanism and behavior of landslide moves, for example linearity tendency, periodical fluctuation, season transform, growing, and some efficiently proved differential equations. Generally speaking, through obser-

vation and analysis of real-time measured time series, we can choose a reasonable mathematic model to do prediction of landslide. By fitting the suggested model to the raw data, one can get corresponding parameters involved in the model. Once having the parameters, the prediction of failure time will be an easy and routine work [1,2].

Prerequisites for a reasonable short-term prediction model satisfy following items:

- (a) A good prediction model should be based on a large number of laboratory creep experiments and field investigation. sometimes, it needs to combine with field engineer's judgments. Certainly, the model should have physical meaning;
- (b) The model has to enclose the parameter  $t_f$  (failure time), or time left before failure ( $t_f - t$ );
- (c) When  $t \rightarrow t_f$  (a finite number), deformation  $D \rightarrow \infty$ , displacement velocity  $dD/dt \rightarrow \infty$ , and displacement acceleration  $d^2 D/dt^2 \rightarrow \infty$ ;
- (d) Some examples have proved the model valid and applicable.

## 3. Two Proposed Models

From experience, the deformation for a landslide with characteristics of avalanche is mainly presented as "straight-forward accelerated failure", with little or without inherent

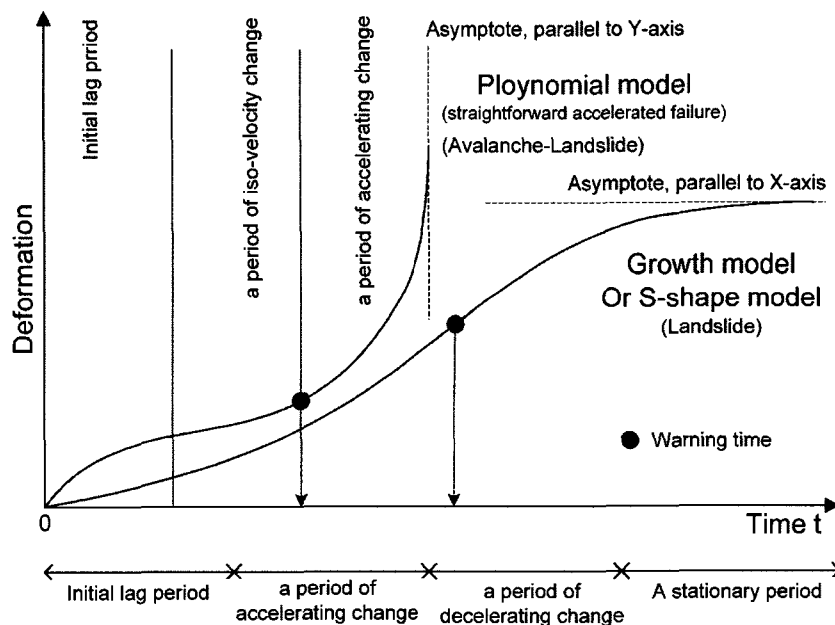


Fig. 1. Sketch map of slope movement process [3]

or natural constraint. The deformation appears to follow exponential, power or polynomial growth. On the other hand, the deformation of a landslide into a riverbed or subjected to unavoidable inherent or natural constraint shows approximately an S-shaped curve. Both failures have inflection points and maximum curvature points in their respective deformation curves as shown in Fig. 1.

For predicting landslide based on deformation observed in the early stage, the determination of an appropriate model and its “best-fit” parameters is still a frequently used method. The key is to select a best-fitted model considering both engineering geology survey and minimizing the squared errors between model and data. We propose the following two models, 3-degree polynomial models.

For “straightforward accelerated failure” case (polynomial model), we assume the time function of the deformation  $N$  as

$$N(t) = a_3t^3 + a_2t^2 + a_1t + a_0 \quad (1)$$

The coefficients ( $a_3, a_2, a_1, a_0$ ) could be determined by curve fitting technique by spread sheet. The next step in the analysis is to determine the asymptote and maximum points in this curve. In this case, the asymptote means the failure of slope. Because asymptote is the infinity of deformation curve of slope, therefore the maximum deformation of slope results in failure. If the curve approach to the point of curve slope change, then it is assumed that landslide will come soon. To find out the point of curve slope change, Eq. (2) is rearranged,

$$dN/dt = 3a_3t^2 + 2a_2t + a_1 \quad (2)$$

For “Failure with Inherent” case (growth model), we also assume the time function of the deformation  $N$  as

$$N(t) = a_7t^3 + a_6t^2 + a_5t + a_4 \quad (3)$$

In this case, the value of  $a_7$ , has minus value compared to  $a^3$  of the polynomial model. This model has also asymptote, therefore, the next step in the analysis is to determine the asymptote and maximum points in this curve. In this case, the asymptote means also the failure

of slope. Because asymptote is the maximum value of deformation curve of slope, it will result in failure. At the point of curve slope change, landslide should be on the alert.

## 4. Applied Model

Fig. 2 shows the TRS (Tension Rotation Settlement) sensor used for this paper to measure the displacement of the slope. This TRS sensor is applied to the Nerupjae, Jaechon and Buksil, Jungsun.

General failure models are divided as two main model, polynomial model and growth model. In case of Nerupjae, it follows the polynomial model, Buksil model is coincided to growth model.

### 4.1 Nerupjae, Jaechon

16 TRS Sensors and a rain gauge were installed to analyze the slope behavior in Nerupjae, Jaechon, which is the cut-slope near national road. The deformation shape versus time of Nerupjae has followed the typical 3-degree polynomial equation. Fig. 3 shows the slope view, sensor and the deformation graph. The data were analyzed, and that of sensor No. 12 showed the typical failure type. It is 3-degree polynomial having the deformation equation of trend line by time is  $y = 1E-08x^3 - 3E-05x^2 + 0.0316x - 0.3472$ , and  $R^2 = 0.929$ . As you can see in Fig. 4, the trend line of the slope deformation is very close to

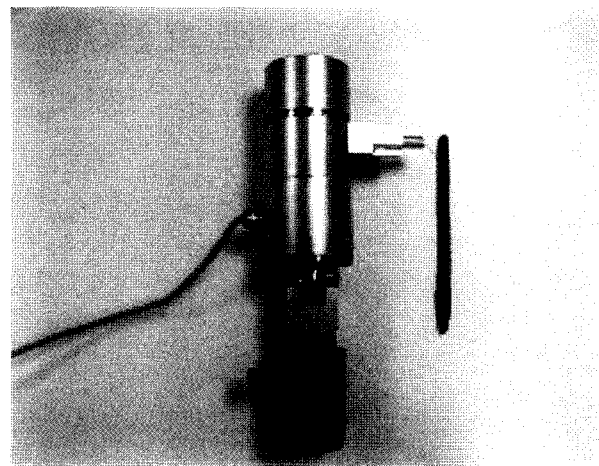


Fig. 2. TRS (Tension Rotation Settlement) sensor

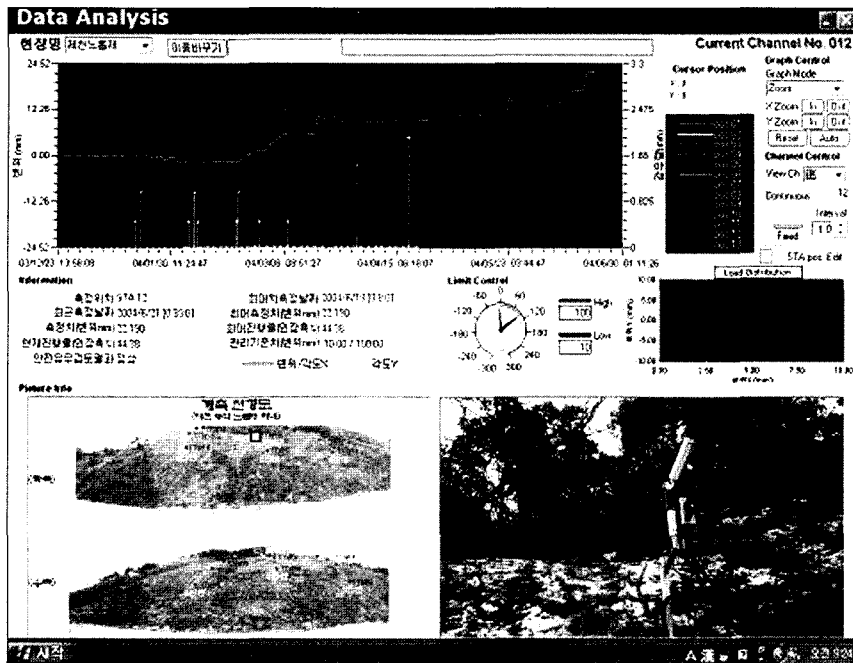


Fig. 3. Computer screen of TRS sensor at Nerupjae

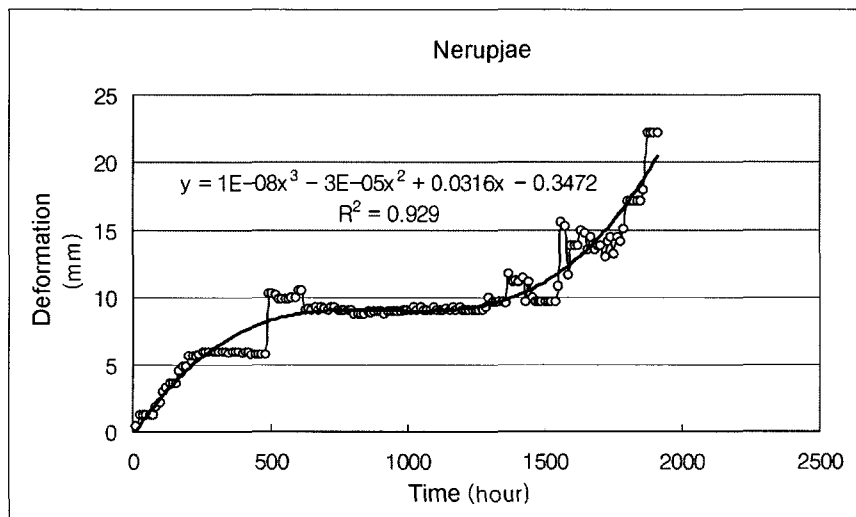


Fig. 4. Data analysis of Nerupjae

asymptote, therefore it is estimated that the failure time is very close.

#### 4.2 Buksil, Jungsun

13 TRS Sensors and a rain gauge were installed in Buksil, Jungsun. This slope is located by national road with 30 m in height and 200 m in length. The deformation graph of Buksil has followed the typical

growth model. Fig. 5 shows the slope view and data of Buksil. The data of sensor installed showed the typical growth type. The deformation equation of trend line by time is  $y = -8E-11x^3 + 6E-07x^2 + 0.0011x - 0.4764$ , and  $R^2 = 0.9694$ . As you can see in Fig. 6, the trend line of the slope deformation is very close to asymptote, therefore it is also estimated that the failure time is very close.

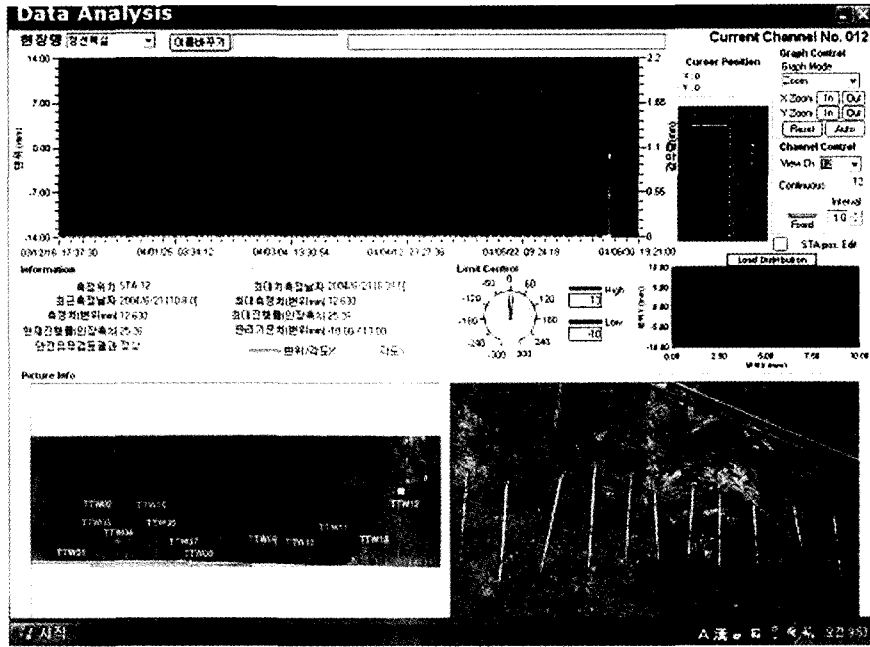


Fig. 5. Computer screen of TRS sensor at Buksil

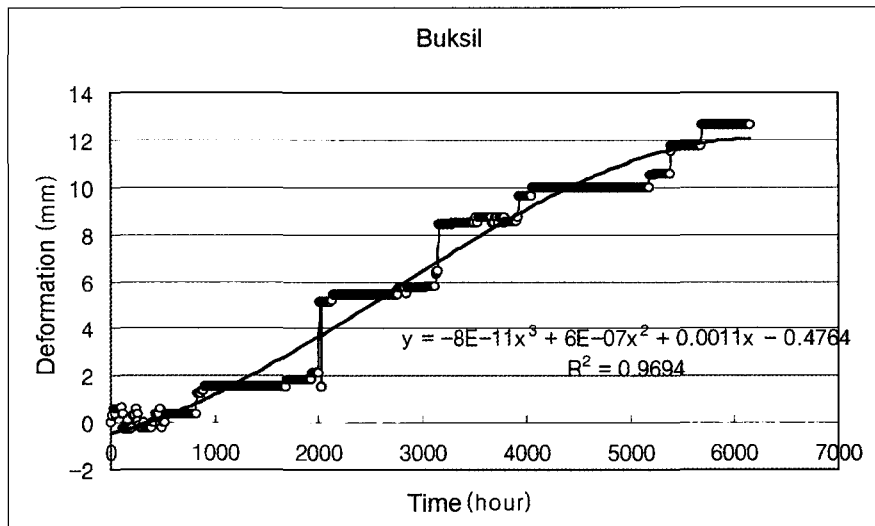


Fig. 6. Data analysis of Buksil

## 5. Conclusion

Compared with other pure mathematic models, the presented model has much more obvious physical meaning, especially, based on polynomial function form, and similarly, we found  $dD/dt$  and asymptote in the curve are doing key role to predict landslide. Again, compared with other existed models, the suggested models are much more generalized and simple because they are just 3-degree polynomial function both.

However, there are still influential factors not involved in the presented model, for example, geometry shape, amount of precipitation, etc. And, sometimes, the presented model shows still some unstabilities and some certainties. This presented model only considers the monitoring data from the tertiary creeping stage not including data from the first and second stages. In addition, the physical meaning of  $a$  value is not yet clear. Therefore, there are still problems and questions to be researched on this topic in the future.

Table 1. Summary of Nerupjae and Buksil

field	failure model	trend line	R <sup>2</sup>
Nerup	polynomial	$1E-08x^3-3E-05x^2+0.0316x-0.3472$	0.929
Buksil	S-type polynomial	$-8E-11x^3+6E-07x^2+0.0011x-0.4764$	0.9694

The summary of Nerupjae and Buksil is shown in Table 1, and the equations of the deformation trend line by time is represented to the 3-degree polynomials ( $y = ax^3 + bx^2 + cx + d$ ).

As you can see in Table 1, the trend line is believable because of high R<sup>2</sup> value.

The coefficient 'a' of Nerupjae equation is plus value, however, that of Buksil is minus value. Therefore, the

asymptote of Nerupjae is located at x-axis (time-axis), to the contrary, that of Buksil is at y-axis (deformation-axis). Anyway, the trend lines of two fields were going to the asymptotes. It is estimated that their failures are very close.

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(received on Mar. 9, 2005, accepted on Mar. 27, 2005)