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A Review on Ultimate Lateral Capacity Prediction of Rigid Drilled Shafts Installed in Sand

사질토에 설치된 강성현장타설말뚝의 극한수평지지력 예측에 관한 재고

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요 지

수평하중을 받는 현장타설말뚝을 합리적이고 경제적으로 설계하기 위해서 가장 중요한 것은 지반-구조 사이의 상호작용을 이해하는 것이다. 그러나 지난 수십년 동안 수평하중을 받는 깊은 기초의 거동에 대한 많은 연구가 있었음에도 불구하고, 문제의 성격상 삼차원적이며 비대칭성으로 인하여 더해지는 지반고유의 비선형성, 불균일성, 복잡성 때문에 극한수평지지력을 공식화하기란 매우 어렵다. 본 연구에서는 특정한 현장조건, 기초의 기하학적 특성(D/B비), 하중조건 등에 따른 많은 설계 방법들 중에서 가장 널리 알려진 네 가지의 방법(즉, Reese, Broms, Hansen, 그리고 Davidson)에 대해서 재검토하였다. 그리고 본 연구의 일환으로 행한 모형실험으로 얻어진 하중-변위곡선을 쌍곡선으로 변환하여 해석된 쌍곡선수평지지력(H_h)과 위의 네 가지 방법들에 의하여 예측되는 극한수평지지력(H_u)을 비교하였다. Reese와 Hansen의 방법에 의해 구한 H_u / H_h 비는 각각 0.966와 1.015로서 실험결과와 매우 근사한 극한수평지지력을 제시하고 있다. 반면에 Davidson의 방법에 의해 구한 H_u는 H_h에 비하여 30% 가량 큰 것으로 예측하고 있으나 네 가지 방법중에서 예측 수평지지력값에 대한 C.O.V.가 가장 작다. 네 가지 방법 중 가장 단순한 Broms의 방법은 H_u / H_h = 0.896으로서 네 방법 중에서 극한수평지지력을 가장 작게 평가하는 것으로 나타나지만 극한수평지지력값을 예측함에 있어서 가장 작은 S.D.를 보인다. 결론적으로, 네 가지의 방법 중 그 어느 것도 극한수평지지력을 정확하게 예측한다는 면에서 다른 방법보다 더 우수하다고 할 수는 없다. 또한, 계산과정이 얼마나 정교하거나 복잡한 것과는 상관 없이 극한수평지지력을 예측하는데 있어서 신뢰도는 또 다른 문제인 것 같다.

Abstract

An understanding of soil-structure interaction is the key to rational and economical design for laterally loaded drilled shafts. It is very difficult to formulate the ultimate lateral capacity into a general equation because of the inherent soil nonlinearity, nonhomogeneity, and complexity enhanced by the three dimensional and asymmetric nature of the problem though extensive research works on the behavior of deep foundations subjected to lateral loads have been conducted for several decades. This study reviews the four most well known methods (i.e., Reese, Broms, Hansen, and Davidson) among many design methods according to the specific site conditions, the drilled shaft geometric characteristics (D/B ratios), and the loading conditions. And the hyperbolic lateral capacities (H_h) interpreted by the hyperbolic transformation of the load-displacement curves obtained from model tests carried out as a part of this research have been compared with the ultimate lateral capacities (H_u) predicted by the four methods. The H_u / H_h ratios from Reese's and Hansen's methods are 0.966 and 1.015, respectively, which shows both the

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two methods yield results very close to the test results. Whereas the H_u predicted by Davidson's method is larger than H_h by about 30%, the C.O.V. of the predicted lateral capacities by Davidson is the smallest among the four. Broms' method, the simplest among the four methods, gives H_u / H_h = 0.896, which estimates the ultimate lateral capacity smaller than the others because some other resisting sources against lateral loading are neglected in this method. But it results in one of the most reliable methods with the smallest S.D. in predicting the ultimate lateral capacity. Conclusively, none of the four can be superior to the others in a sense of the accuracy of predicting the ultimate lateral capacity. Also, regardless of how sophisticated or complicated the calculating procedures are, the reliability in the lateral capacity predictions seems to be a different issue.

Keywords : Foundation geometries, Hyperbolic lateral capacity, Lateral loads, Soil-structure interaction, Ultimate lateral capacity

1. Introduction

1.1 General Background

It is often necessary to make simple assumptions that have only a negligible influence on the results to address complicated geotechnical problems. These simplifications lead to simpler methods of analysis and design for laterally loaded drilled shafts. Although the behavior of deep foundations is to be influenced by the methods of construction (driven, jetted, drilled, etc.), the analysis and design methods generally used in practice do not include a "method of construction" factor explicitly. The term "pile" could be used interchangeably with "drilled shaft" in this study; however, each construction method should be considered and evaluated separately using the data and/or assumptions used in its development.

Each method used in practice, therefore, should be restricted in its correct use to the: (a) specific site soil conditions, (b) foundation rigidity and/or geometries tested to verify the method, and (c) loading conditions used in testing and evaluation. The design parameters to be used in any empirical method, therefore, should be the same as those recommended and obtained by the original authors. Any modifications to the original method should be tested and justified against a data base.

2. Lateral Capacity of Drilled Shafts

There exists no clear definition of lateral capacity of drilled shafts. Fig. 1 shows that there are many different interpretations for the response of a laterally loaded deep foundation. Most of these methods suggest that the lateral capacity is to be the load corresponding to specific displacements (either absolute values or a percentage of the shaft diameter, B). One of the methods (Hirany and Kulhawy, 1988, 1989) uses a more general interpretation from the soil/shaft failure mechanism and the apparent depth of rotation (ADOR) versus load level. The definition of ADOR is the ratio of butt displacement to tangent of the butt slope. It should be noted that a different capacity is evaluated for a design depending on the method chosen.

Analytical methods for predicting the ultimate capacity are all limit equilibrium methods that consider the static

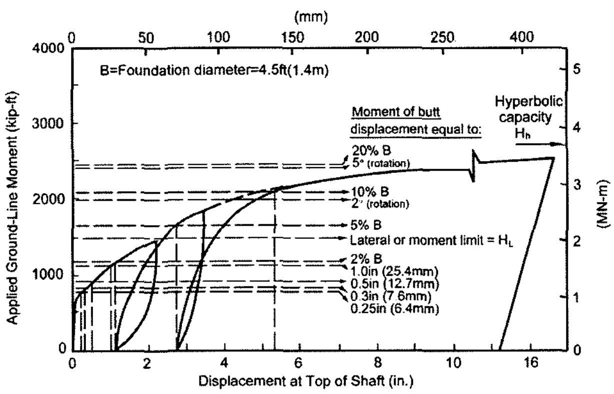


Fig. 1. Various lateral capacity interpretation criteria

equilibrium of the foundation for an assumed soil stress distribution at failure. Failure could be either: (a) complete yielding of the soil along the rigid foundation, or (b) structural failure of the flexible foundation. This study is focused on rigid drilled shafts.

3. Drained Yield Stress

Broms (1964) suggested the drained yield stress (P_u), defined as the maximum average horizontal soil resistance at the foundation-soil interface, to be equal to three times the Rankine passive horizontal stress acting on an infinitely long wall, as given below:

$$p_u = 3\bar{\gamma}zK_p = 3\bar{\gamma}z\tan^2(45° + \bar{\phi}/2) \qquad (1)$$

in which $\bar{\gamma}$ = effective soil unit weight, z = an arbitrary depth, K_p = Rankine passive coefficient of horizontal soil stress, and $\bar{\phi}$ = effective friction angle. If a soil is homogeneous and uniform, the yield stress increases linearly with depth as shown in Fig. 2.

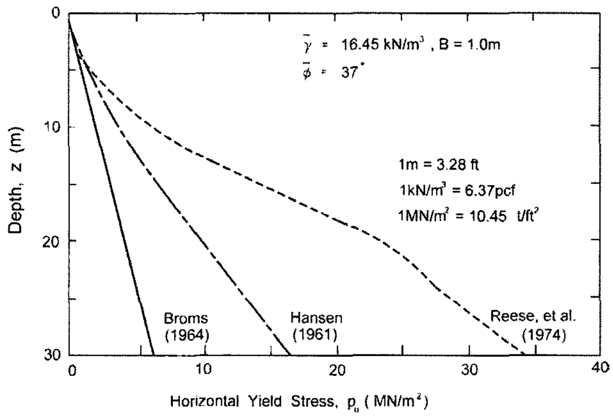


Fig. 2. Yield stress distribution from three different analysis methods [Agaiby et al. (1991)]

Kishida and Nakai (1977) testified Equation (1) for varying effective friction angles from 20° to 45°. Their conclusion is as follows:

$$p_u = (1.8 \text{ to } 8.7)\bar{\gamma}zK_p = (1.8 \text{ to } 8.7)\bar{\gamma}z\tan^2(45° + \bar{\phi}/2) \quad (2)$$

For a typical effective friction angle range, i.e., from 30° to 40°, Kishida and Nakai (1977) recommended using the Broms' formulation for calculating the yield stress.

Hansen (1961) developed an equation by considering the soil yield stress to develop differently at shallow, moderate, and great depths, as given below:

$$p_u = \bar{\sigma}_{vo}K_q + \bar{c}K_c \qquad (3)$$

in which $\bar{\sigma}_{vo}$ = vertical effective stress, $\bar{c}$ = soil cohesion, and K_q and K_c = factors in Fig. 3. For $\bar{c}$ = 0, $P_u = \bar{\sigma}_{vo}K_q$.

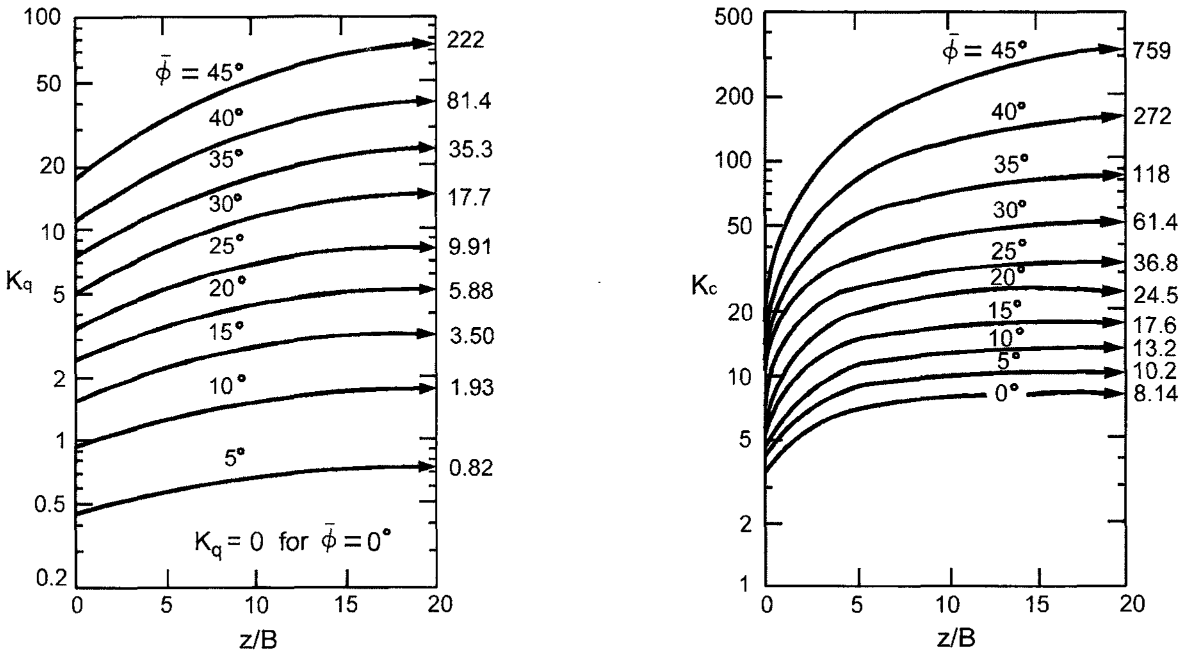


Fig. 3. Hansen's yield stress coefficient [Hansen (1961)]

The soil mechanisms were divided into either shallow or deep failure by Reese at al. (1974). For shallow failure, a three-dimensional passive wedge, shown in Fig. 4 (a), was assumed to form in front of the shaft. By considering the wedge equilibrium and differentiating the resultant passive force with respect to the depth, the horizontal yield stress for shallow depths was obtained as follows:

$$p_u = \bar{\gamma} z [K_0 z (\tan\bar{\phi} \sin\Omega)/\tan(\Omega - \bar{\phi})\cos\psi + \tan\Omega (B + z\tan\Omega \tan\psi)/\tan(\Omega - \bar{\phi}) + K_0 z \tan\Omega(\tan\bar{\phi}\sin\Omega - \tan\psi) - K_a B]/B \quad (4)$$

in which $\Omega = 45° + \bar{\phi}/2$, $\psi = \psi/2$ to $\bar{\phi}/3$, K_o = coefficient of horizontal soil stress at rest, and $K_a = \tan^2(45°-\bar{\phi}/2)$ = Rankine active coefficient of horizontal soil stress. As shown in Fig. 4 (a), the angles Ω and ψ define the shape of the wedge in front of the shaft. Borden and Gabr(1987, 1989) expanded the similar approach to derive the yield stress for shallow depths where the ground surface is inclined.

For deep failure, a plane strain mode of failure was assumed to occur in the horizontal plane, leading to the lateral flow pattern around the foundation shown in Fig. 4 (b). From inter-block stress compatibility, the resultant yield stress was given by:

$$p_u = K_a \bar{\gamma} z(\tan^8\Omega - 1) + K_0 \bar{\gamma} z \tan^4\Omega \quad (5)$$

In the Reese et al. (1974) method, the transition from shallow to deep failure occurs when the yield stresses from the shallow mode exceed those for the deep mode. For an effective stress friction angle equal to 40°, this transition depth is 21.6 times the diameter. Most of the drilled shafts dealt with in the electric utility industry have depth to diameter (D/B) ratios less than ten, where the shallow failure mode controls.

4. Calculation of Ultimate Capacity

Using the yield stress around the shaft, the ultimate capacity can be calculated from static equilibrium of the forces acting on the foundation. For a rigid shaft and a general yield stress distribution such as that in Fig. 5, the ultimate capacity is given by:

$$H_u = \int_0^{z_r} p_u B dz - \int_{z_r}^{D} p_u B dz \quad (6)$$

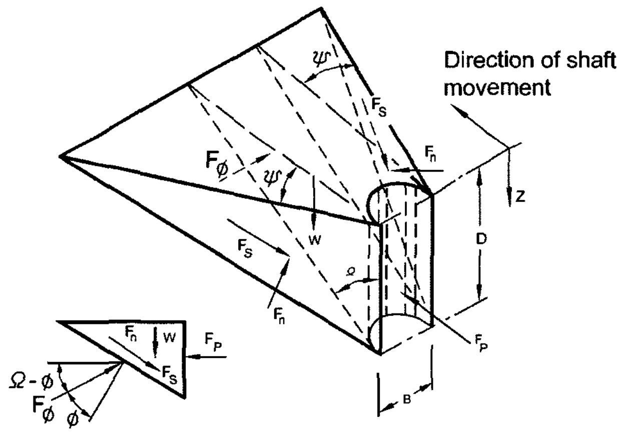
in which z_r = point of zero stress (also the point of rotation) and D = shaft depth.

Broms (1964) replaced the passive stresses that develop below the point of rotation with a concentrated load at the tip. He assumed further that the passive resistance in front of the shaft acts along the entire depth, as shown in Fig. 6. Therefore, for a homogeneous soil with constant $\bar{\phi}$, the ultimate capacity was given by:

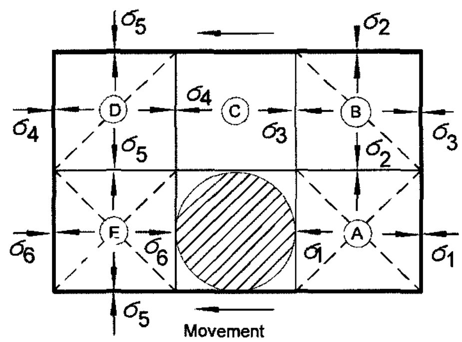
$$H_u = 0.5\bar{\gamma} B D^3 K_p/(e + D) \quad (7)$$

in which e = eccentricity of appliedload.

Davidson et al. (1982) proposed a more complete



(a) Shallow failure



(b) Deep failure

Fig. 4. Failure mechanisms proposed by Reese et al. (1974)

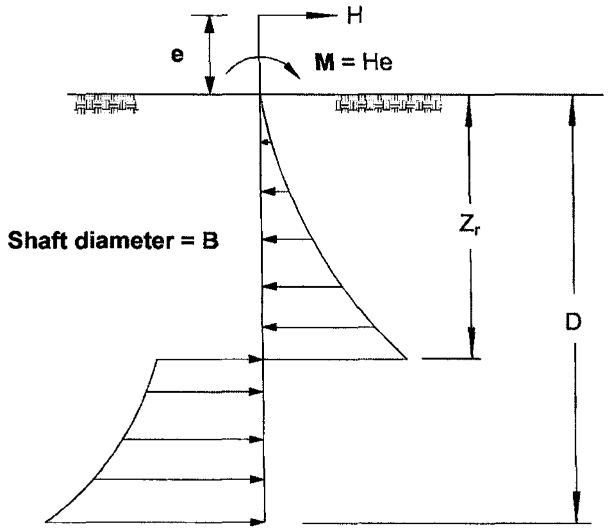


Fig. 5. Illustrative yield stress profile for calculating lateral ultimate capacity

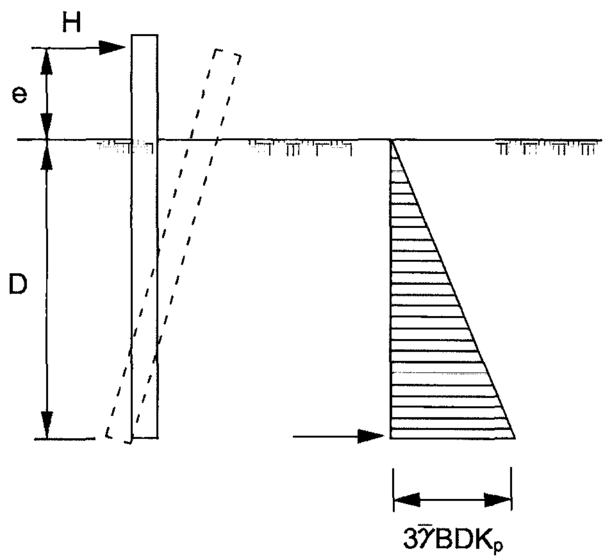


Fig. 6. Broms' simplified yield stress distribution for a rigid shaft

equilibrium system for the lateral or moment loads acting on a rigid drilled shaft, as shown in Fig. 7. With this representation, four modes of load resistance are considered, including: (a) lateral yield stresses above and below the center of rotation, (b) vertical shearing stresses on the perimeter of the shaft above and below the center of rotation, (c) moment at the shaft tip, and (d) shear at the shaft tip. The lateral yield force per unit depth (P_u) is obtained by multiplying Hansen's yield stress by the diameter (B). This yield force can be expressed as normal (F_n) and horizontal shearing components (V_t), as given below:

$$P_u = \int_0^{2\pi} (\sigma_r \cos\psi + \tau_{r\psi} \sin\psi) r \, d\psi = Bp_u \qquad (8)$$

in which σ_r = radial normal stress at shaft surface, $\tau_{r\psi}$ = tangential shear stress at shaft surface, and ψ = circumferential coordinate. The vertical shear stress (τ_{rz}) acting on the perimeter of the shaft produces a net moment (M_z). This M_z per unit depth can be calculated as:

$$M_z = \int_0^{2\pi} (B/2)^2 (\tau_{rz} \cos\psi) d\psi = V_z (B/2) \qquad (9)$$

in which V_z = vertical shear force per unit depth. The tip shearing resistance and moment are considered in calculating the ultimate capacity only if vertical equilibrium is not satisfied after full mobilization of the vertical shear stresses. The base or tip shear force (V_b) can be computed as:

$$V_b = \int_0^{B/2} \int_0^{2\pi} (\tau_{zr} \cos\psi - \tau_{z\psi} \sin\psi) r \, d\psi \, dr \qquad (10)$$

in which τ_{zr} = tip radial shear stress and $\tau_{z\psi}$ = tip circumferential shear stress. Normal stresses (σ_z) acting on the shaft tip produce a net resisting moment (M_b) as:

$$M_b = \int_0^{B/2} \int_0^{2\pi} (\sigma_z \cos\psi) r^2 \, d\psi \, dr \qquad (11)$$

The equilibrium equations for a rigid shaft embedded in a multilayered soil can be also calculated by using the above definition as follows:

$$M_u = H_u \cdot e = \sum_{i=1}^{K} P_{ui} t_i z_i + \sum_{i=K+1}^{N} P_{ui} t_i z_i + \sum_{i=1}^{N} V_{zi} t_i \frac{B}{2} + V_b D + F_b x_b \qquad (12)$$

$$H_u = \sum_{i=1}^{K} P_{ui} t_i - \sum_{i=K+1}^{N} P_{ui} t_i - V_b \qquad (13)$$

$$Q_a = \sum_{i=1}^{K} V_{zi} t_i - \sum_{i=K+1}^{N} V_{zi} t_i - F_b \qquad (14)$$

$$V_b = F_b \tan\bar{\phi} \qquad (15)$$

in which t_i = thickness of layer i, z_i = depth to the center of layer i, F_b = normal force acting on the shaft tip, x_b = moment arm length measured from the center of the

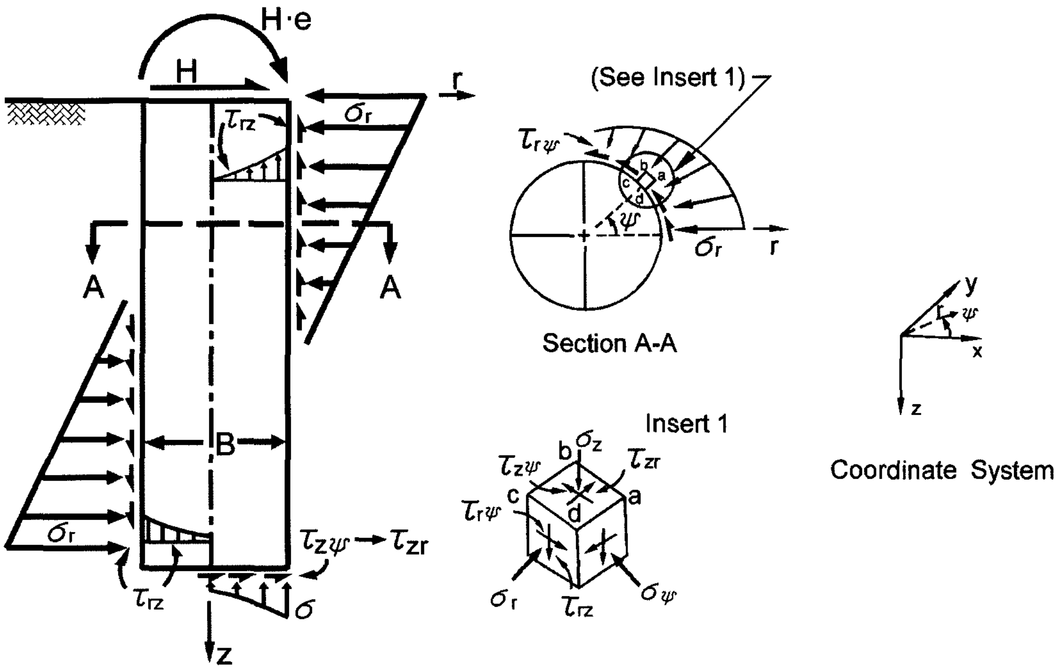


Fig. 7. Assumed stresses at shaft-soil interface [Davidson et al. (1982)]

shaft, and Q_a = axial load induced by the moment. According to Davidson et al. (1982), the lateral soil resistance constitutes 62 to 90 percent of the total resistance, vertical side shear is 5 to 30 percent, base or tip shear is 2 to 7 percent, and tip moment is 2 to 6 percent. It should be noted that this capacity representation can be applied only to drilled shafts with depth/diameter ratios (D/B) of 1 to 10.

Borden and Gabr (1987) distinguished between the ultimate capacity and the capacity corresponding to a displacement criterion. Ultimate capacity was defined as that obtained from a horizontal tangent to the load-displacement curve at large displacements, as shown in Fig. 8. They recommended obtaining the load-displacement curve first and then inferring the ultimate capacity from it.

Manoliu et al. (1985) proposed using a hyperbola to represent the lateral load-displacement response. In this approach, the load is related to the displacement as follows:

$$H = K_i\delta/(1+\delta/\delta_r) \qquad (16)$$

in which H = lateral load, K_i =slope of the initial portion of the load-displacement curve, δ = butt displacement, $\delta_r = H_h / K_i$ as shown in Fig. 9, and H_h = ultimate capacity predicted by the hyperbola, Equation (16) can be re-written as follows:

$$H = \delta/(a+b\delta) \qquad (17)$$

in which $a = 1 / K_i$ and $b = 1 / H_h$. The ultimate hyperbolic capacity (H_h) is obtained by taking the limit of Equation (17) as the displacements approach infinity and is the inverse of the slope of the curve resulting from plotting δ / H versus δ. This interpretation of the response is objective, and it is compatible with the assumptions made when calculating the ultimate capacity using the simple

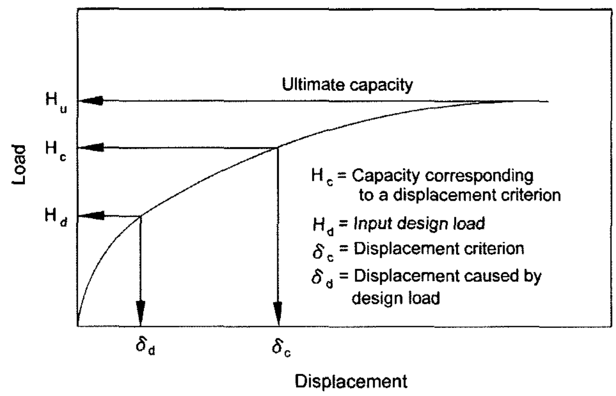


Fig. 8. Ultimate capacity definition [Borden and Gabr (1987)]

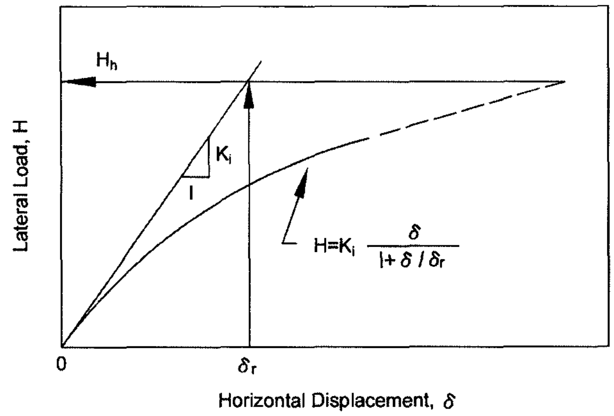


Fig. 9. Hyperbolic projection and initial stiffness for load-displacement curve

two-dimensional models that were reviewed previously. It should be noted that the ultimate capacity is a projected load that is never achieved, much like the stress conditions assumed around a shaft when calculating the ultimate capacity.

Summary of Model Test Program

Nineteen sand deposits have been prepared to conduct thirty three lateral load tests on model drilled shafts in this study. Model drilled shafts with diameters (B) of 76.2 mm and 152.4 mm have been constructed. And the ratios of depth to diameter (D/B) have been varied as 3, 6, and 9. All the drilled shafts tested in this study can be assumed to be rigid. Details on constructing drilled shafts are given elsewhere (e.g., Agaiby et al., 1991).

The sand deposits for tests have been compacted to be classified into three groups of loose, medium dense, anddense sands with meanunit weights and effective friction angles of 15.38 kN/m^3 and 37.9°, 16.42 kN/m^3 and 41.6°, and 17.47 kN/m^3 and 47.4°, respectively. Details of measuring unit weights and effective friction angles are given elsewhere (e.g., Turner and Kulhawy, 1987).

Analysis of Test Results on Lateral Capacity for Drilled Shafts

Table 1 is a summary from thirty three lateral load tests on model drilled shafts in dry sand conducted by research colleagues of the authors. Hyperbolic lateral capacities (H_h) have been interpreted by the hyperbolic transformation of the load-displacement curves obtained from model tests. And the ultimate lateral capacities (H_u) have been calculated by the four methods of Reese et al.(1974), Broms (1964), Hansen (1961), and Davidson et al.(1982). The H_h interpreted from a test is used as a reference value to quantify or verify the accuracy of the ultimate lateral capacities (H_u) predicted by each method. If H_u / H_h = 1, then the calculated capacity is precisely equal to the interpreted capacity from a test. As shown in Table 1, H_u / H_h ratios from Reese's and Hansen's methods are 0.966 and 1.015, respectively, which shows both the two methods yield very closely to the test results. Whereas the H_u predicted by Davidson's method is larger than H_h by about 30%, the coefficient of variation (C.O.V.) of the predicted lateral capacities by Davidson is the smallest among the four. Broms' method, the simplest among the four methods, gives H_u / H_h = 0.896, which estimates the ultimate lateral capacity smaller than the others because some other resisting sources against lateral loading are neglected in his. But his may be one of the most reliable methods with the smallest S.D. in predicting the ultimate lateral capacity.

Summary and Conclusions

A review on the four well known methods for the analysis of laterally loaded rigid drilled shafts has been presented. Broms suggested the yield stress to be equal to three times the Rankine passive horizontal stress acting on an infinitely long wall. Hansen developed an equation

Table 1. Summary of capacities calculated by four methods to hyperbolic

	H_u by various methods / H_h			
	Reese et al.	Broms	Hansen	Davidson
Mean	0.966	0.896	1.015	1.308
S.D.	0.447	0.307	0.317	0.374
C.O.V.	0.464	0.343	0.312	0.286

by considering the soil yield stress to develop differently at shallow, moderate, and great depths. Reese et al. divided soil mechanisms into either shallow or deep failure. Davidson et al. proposed an equilibrium system for the lateral or moment loads acting on a rigid drilled shaft. Four modes of load resistance are considered.

Hyperbolic interpretation of the ultimate lateral capacity is briefly introduced to show the validity of using the hyperbolic capacity as a reference capacity to analyze the ultimate lateral capacity calculated from theory. The predicted capacities from four different methods were compared with hyperbolic capacities obtained from thirty three lateral load tests on model drilled shafts. H_u / H_h ratios from Reese's and Hansen's methods are 0.966 and 1.015, respectively. H_u predicted by Davidson's method is larger than H_h by about 30%. However, C.O.V. of the predicted lateral capacities by Davidson is the smallest among the four available methods. Broms' method results in H_u / H_h = 0.896, which most underestimates the ultimate lateral capacity, but it may be one of the most reliable methods with the smallest S.D. in predicting the ultimate lateral capacity.

None of the four mentioned methods in this study can be superior to the others in the accuracy of predicting the ultimate lateral capacity. Also, more sophisticated or complicated calculating methods are not necessarily to predict the ultimate lateral capacity with more reliability.

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