# Effects of Angular Velocity Components on Head Vibration Measurements 

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#### Abstract

This paper addresses issues encountered in measuring the general, 6 -degree-of-freedom motion of a human head. A complete mathematical description for measuring the head motion using the six-accelerometer configured bite-bar is suggested. The description shows that the six-axis vibration cannot be completely obtained without the roll, pitch and yaw angular velocity components. A new method of estimating the three orthogonal (roll, pitch and yaw) angular velocities from the six accelcration measurements is introduced. The estimated angular velocitics are shown to enable further quantitative error analysis in measuring the translational and angular accelerations at the head. To make this point clear, experimental results are also illustrated in this paper. They show that when the effects of angular velocities are neglected in the head vibration measurement the maximum percentage errors were observed to be more than $3 \%$ for the angular acceleration of the head and to be close to $5 \%$ for its translational acceleration, respectively. It means that the inclusion of all the angular velocity dependent acceleration components gives more accurate measurement of the head vibration.


Keywords: Head vibration, Bite-bar, Whole-body vibration, Human response to vibration

## I. Introduction

The transmission of scat vibration to the head has been extensively investigated by numerous researchers[1] for longer than 50 years. The vibration transmissibility (i.e., the frequency response function from the input vibration on the seat to the output vibration at the head) has been usually measured. The measured transmissibility gives useful information about the biodynamic responses of the whole-body to the various vibration environments and their effects on the loss of comfort, activity interference, and potential injury. It has become apparent that numerous experimental factors, categorized as the "intrinsic" variables (related to the individual subjects) and the "extrinsic" variables (related to the experimental conditions), can bave large

[^0]effects on the results of seat-to-head transmissibility[ 1]. Most early researches were concerned with only vertical scat motion and vertical head vibration. International Standard 7962 (1987) and recent Intemational Standard CD 5982 (1999) gives a 'grand averaged' transmissibility[1-3] to the head of seated subjects exposed to the vertical vibration on the seat. However, it has been reported that even the single-axis vertical or horizontal excitation on the seat gencrates not only the vertical head vibration but also other multi-axis head vibration components, e.g. fore-and-aft motion, lateral motion and rotational motions (i.e., roll, pitch, and yaw) 4,51 .

Under the assumption that the motion of the head satisfies that of a rigid-body within the frequency range of interest, the rotational motions such as roll, pitch and yaw are not affected by the measurement location. However, the translational motion on the head varies with the location of measurement with respect to the center of rotation due to its rotational motion. Consequently,
it is of importance to take account of the influence of measurement positions for analysis of the general motion of head vibration.
For this purpose, some researchers attempted to examine the multi-axis global motion of the head using a limited number of locally measured accelerations[6]. It was shown that the multi-axis global motions of the head are extrapolated from the measurements by using the rigid body kinematics. Although it is known that the nine-axis method[7] or the twelve-axis models [8-10] are necessary to evaluate the global motions of the head completely, six linear accelerometers, which are referred to as "six-axis bite-bar"[6], are more often used to do it. The reason comes from the fact that the former measurement device using nine or twelve axis accelerometers requires the three sets of accelerometer-supporting rods assembled perpendicularly while the latter uses only two right-angled supporting rods fixed on the horizontal plane. Thus, the latter has a simpler structure to use. But, in the latter case, the rotational motions are assumed to be sufficiently small so that the evaluation procedure of the multi-axis global head motion is simplified as in the work by Paddan and Griffin[6]. However, the measurement uncertainty of this simplified evaluation method has rarely been reported.

At the onset of this work, it became apparent that it is not inhibited to estimate the three-axis angular velocity components of the head motion from the measured acceleration signals of the simple-structured six-axis bite-bar. This paper suggests a logical way of their estimation and then proposes a complete form for the measurement of six-degree-of-freedom head motion even by using accelerometers. Detailed mathematical descriptions are presented not only to improve the accuracy of the head motion measurements but also to analyse the measurement uncertainty of the previous method. The seated body vibration test was done in collaboration with Paddan and Griffin in ISVR. The experimental results are illustrated to demonstrate the effectiveness of the proposed method. They show that the previously neglected terms can be of importance in the accurate measurement of the multiaxis head motion depending on the meastrement positions chosen.

## II. Head Vibration Measurement Device

The accelerations at the head were measured using the six-axis bite-bar of weight about 140 g shown in Figure 1. Six translational accelerometers with the very low frequency response
up to DC (Entran EGA-125(F)*-10D) were installed in the differently positioned three blocks. A single accelerometer in block 1 (front right side) is oriented in the $z$-axis. Three accelerometers in block 2 (front left side) are orthogonally set ip in the $\mathrm{x}, \mathrm{y}$, and z -axis directions and two accelerometers in bfock 3 (back left side) are oriented in the $y$ and $z$-axis directions. The distance between front block 1 and 2 was chosen to be $d_{y}=200$ mm such that the paired $z$-axis accelerometers at block 1 and 2 were used to evaluate the angular velocity and acceleration of the x -axis (roll motion component). Similarly, the gap between block 2 and 3 was chosen to be $d_{x}=150 \mathrm{~mm}$ such that the paired $y$ and $z$-axis accelerometers were used to estimate the angular velocities and accelerations of the $z$-axis and $y$-axis, i.e. yaw and pitch motion components, respectively. A T-shaped aluminum rod on which a dental mould was mounted was attached to the center of the lateral rod joining block 1 and 2. In this configuration, the lateral rod was located 35 mm anterior to the subject's front teeth The subject under test was asked to bite the dental mold so that the bite-bar is almost rigidly attached to the subject's head.

## III. Mathematical Derivation of Head Vibration

The use of six accelerations measured from the bite-bar introduced in previous section 2 is attempted to derive a complete mathematical form of the head motion. This attempt consists of three evaluation procedures: evaluations of (1) three-axis angular velocities of the head, (2) three-axis angular accelerations of the head, and (3) three-axis translational


Figure 1. A bite-bar with six transiational accelerometers for measuring vibration in the three transtational and three rotational axes of the head on the moving reference frame $(B)$.
accelerations of the head at the position of interest. The rigid body kinematics is exploited in those evaluation procedures to derive their evaluation formula under the assumption that the head is sufficiently rigid in the frequency range of interest, i.e. 0.5 to 30 Hz .

### 3.1. Evaluation of Three Axes Angular Velocities

Under the assumption that the head is rigid, the translational velocity of the head $\mathbf{v}_{\mathrm{p}}$ at the position $\mathbf{P}$ described in the moving frame $\{\mathrm{B}\}$ (refer to Figure 1), which is attached to the bite-bar, can be written as\{11]
$\mathbf{v}_{\mathrm{p}}=\mathbf{v}_{2}+\boldsymbol{\omega} \times \mathbf{p}$
where $\mathbf{v}_{2}$ is the velocity at block 2, i.e., the origin of the moving reference frame and $\omega$ is the angular velocity of the head. The detailed components of the vectors are
$v_{2}=v_{2 x} \hat{\mathbf{i}}+v_{2 y} \hat{\mathbf{j}}+v_{2 z} \hat{\mathbf{k}}$
$\omega=\omega_{\mathrm{x}} \hat{\mathbf{i}}+\omega_{\mathrm{y}} \hat{\mathbf{j}}+\omega_{\mathrm{z}} \hat{\mathbf{k}}$
$\mathbf{p}=\mathrm{p}_{\mathrm{x}} \hat{\mathbf{i}}+\mathrm{p}_{\mathrm{y}} \hat{\mathbf{j}}+\mathrm{p}_{\mathrm{z}} \hat{\mathbf{k}}$

The position vectors for block 1 and block 3 can be written as

$$
\begin{equation*}
\mathbf{p}_{1}=-\mathrm{d}_{\mathrm{y}} \hat{\mathbf{j}}, \mathbf{p}_{3}=-\mathrm{d}_{\mathrm{x}} \hat{\mathbf{i}} \tag{5}
\end{equation*}
$$

Substituting equation (5) into equation (1), we can obtain algebraic equations for the measured translational velocities at blocks 1 and 3, i.e. $\left\{v_{14}, v_{3,}, v_{3}\right\}$ as follows:
$\mathbf{v}_{1 z}=\mathbf{v}_{2 z}-\omega_{x} \mathbf{d}_{y}$
$v_{3 y}=v_{2 y}-\omega_{z} d_{x}$
$\mathrm{v}_{32}=\mathrm{v}_{2 \mathrm{x}}+\omega_{\mathrm{y}} \mathrm{d}_{\mathrm{x}}$

From equations (6). (7) and (8), the three angular velocity components of the head can be written as follows:

$$
\begin{equation*}
\omega_{\mathrm{x}}=\frac{\mathrm{v}_{2 \mathrm{z}}-\mathrm{v}_{1 \mathrm{z}}}{\mathrm{~d}_{\mathrm{y}}}, \omega_{,}=\frac{\mathrm{v}_{3,}-\mathrm{v}_{2 \mathrm{x}}}{\mathrm{~d}_{\mathrm{y}}}, \omega_{\mathrm{z}}=-\cdot \frac{\mathrm{v}_{2 \mathrm{y}}-\mathrm{v}_{3 y}}{\mathrm{~d}_{\mathrm{x}}} \tag{9}
\end{equation*}
$$

It is interesting that the angular velocities of the head defined in equation (9) are exact and that they are experimentally determined from the five translational velocity components $\left\{\mathrm{v}_{17}, \mathrm{v}_{2 y}, \mathrm{v}_{2 z}, \mathrm{v}_{3_{y}}, \mathrm{v}_{3}\right\}$, which are readily obtained by integrating the corresponding accelerometer signals at block 1,2 , and 3 . This angular velocity estimation procedure will be exploited to analyse their effects on the measurement uncertainty of a general head motion.

### 3.2. Evaluation of Three Axes Angular Accelerations

The translational acceleration $\mathbf{a}_{\mathrm{p}}$ of the head at the position $\mathbf{p}$ can be written from the rigid body kinematics as $[7,11]$

$$
\begin{equation*}
\mathbf{a}_{\mathrm{p}}=\mathbf{a}_{2}+\alpha \times p+\omega \times(\omega \times p) \tag{10}
\end{equation*}
$$

where $\mathbf{a}_{2}$ is the acceleration of block 2 and $\boldsymbol{\alpha}$ is the angular acceleration of the head and $\boldsymbol{\omega}$ is its angular velocity. Detailed vector components in equation (10) are given as
$\mathbf{a}_{2}=a_{2 x} \hat{\mathbf{i}}+\mathrm{a}_{2 \mathrm{y}} \hat{\mathbf{j}}+\mathrm{a}_{2 \mathrm{z}} \hat{\mathbf{k}}$
$\alpha=\alpha_{x} \hat{\mathbf{i}}+\alpha_{y} \hat{\mathbf{j}}+\alpha_{z} \hat{\mathbf{k}}$

Substituting equation (5) into equation (10), we can obtain algebraic equations for the measured translational accelerations at blocks 1 and 3, i.e. $\left\{a_{12}, a_{3 y}, a_{3 z}\right\}$ as follows:
$\mathrm{a}_{1 z}=\mathrm{a}_{2 z}-\alpha_{x} \mathrm{~d}_{y}-\omega_{y} \omega_{z} \mathrm{~d}_{y}$
$a_{3 y}=a_{2 y}-\alpha_{z} d_{x}-\omega_{x} \omega_{y} d_{x}$
$\mathrm{a}_{32}=\mathrm{a}_{2 z}+\alpha_{y} \mathrm{~d}_{x}-\omega_{x} \omega_{z} \mathrm{~d}_{x}$
From equations (13), (14) and (15), we can have the angular accelerations of the head $\boldsymbol{\alpha}$ explicitly as follows:
$\alpha=\boldsymbol{\alpha}_{\mathrm{A}}+\boldsymbol{\alpha}_{\mathrm{R}}$
$=\left(\alpha_{A x} \hat{i}+\alpha_{A y} \hat{\mathbf{j}}+\alpha_{A z} \hat{\mathbf{k}}\right)+\left(\alpha_{R x} \hat{i}+\alpha_{R y} \hat{\mathbf{j}}+\alpha_{R z} \hat{k}\right)$
where
$\alpha_{A x}=\frac{a_{2 z}-\mathbf{a}_{12}}{d_{y}}, \alpha_{A y}=\frac{a_{3 z}-a_{2 z}}{d_{x}}, \alpha_{A x}=\frac{a_{2 y}-a_{3 y}}{d_{x}}$,
$\alpha_{R x}=-\omega_{y} \omega_{z}, \alpha_{R y}=\omega_{x} \omega_{z}, \alpha_{R z}=-\omega_{x} \omega_{y}$.

The angular acceleration $\boldsymbol{\alpha}$ is found to be divided into two parts, such as 'translational acceleration-dependent terms' $\alpha_{A}$ defined in equations (17), and 'angular velocity-dependent terms' $\boldsymbol{\alpha}_{\mathrm{R}}$ defined in equation (18). Equations (16)-(18) indicate that the angular accelerations of the head can be experimentally obtained from both the measured five translational accelerations $\left\{a_{1 z}, a_{2 y}, a_{2 z}, a_{3 y}, a_{3 z}\right\}$ and the three angular velocities $\left\{\omega_{x}, \omega_{y}, \omega_{z}\right\}$. The angular velocity-dependent terms, which consist of second-order products of angular velocities, are in details examined for analysis of their relative significance in the real measurement of head vibration.

### 3.3. Evaluation Of Three Axes Translational Accelerations

The mathematical description for the translational acceleration of the head $\mathbf{a}_{p}$ at position $\mathbf{p}$ can be obtained from equation (10). Substituting equations (16)-(18) into equation (10), we have a complete form of three axes translational accelerations $\mathbf{a}_{p}$ as
$\mathbf{a}_{\mathrm{p}}=\mathbf{a}_{\mathrm{pA}}+\mathbf{a}_{\mathrm{pR}}$
$=\left(a_{P A, x} \hat{\mathbf{i}}+\mathbf{a}_{P A, y} \hat{\mathbf{j}}+a_{P A, Z} \hat{\mathbf{k}}\right)+\left(\mathbf{a}_{P R, x} \hat{\mathbf{i}}+a_{P R, y} \hat{\mathbf{j}}+a_{P R, z} \hat{\mathbf{k}}\right)$

The detailed components of $\mathbf{a}_{\mathrm{pA}}$, which are dependent only on measured translational accelerations $\left\{a_{1 z}, a_{2 x}, a_{2 y}, a_{2 z}, a_{3 y}, a_{3 z}\right\}$, are written as follows:
$a_{p A, x}=a_{2 x}+\frac{a_{3 z}-a_{2 z}}{d_{x}} p_{z}-\frac{a_{2 y}-a_{3 y}}{d_{x}} p_{y}$
(20)
$a_{p A, y}=a_{2 y}+\frac{a_{2 y}-a_{3 y}}{d_{x}} p_{x}-\frac{a_{2 z}-a_{1 z}}{d_{y}} p_{z}$
(20b)
$a_{p A, z}=a_{2 z}+\frac{a_{2 z}-a_{1 z}}{d_{y}} p_{y}-\frac{a_{3 z}-a_{2 z}}{d_{x}} p_{x}$
and those of $\mathbf{a}_{\mathrm{PR}}$, which are dependent only on angular velocity components $\left\{\omega_{x}, \omega_{y}, \omega_{z}\right\}$, are also written as follows:

$$
\begin{align*}
& \mathbf{a}_{\mathrm{pR}, \mathrm{x}}=-\left[\left(\omega_{y}\right)^{2}+\left(\omega_{z}\right)^{2}\right] \mathbf{p}_{x}+2 \omega_{x} \omega_{y} p_{y}+2 \omega_{x} \omega_{z} \mathrm{p}_{z}  \tag{21a}\\
& \mathbf{a}_{\mathrm{pR}, y}=-\left[\left(\omega_{z}\right)^{2}+\left(\omega_{x}\right)^{2}\right]_{y}+2 \omega_{y} \omega_{z} \mathrm{p}_{z} \tag{21b}
\end{align*}
$$

$a_{p R, z}=-\left[\left(\omega_{x}\right)^{2}+\left(\omega_{y}\right)^{2}\right]_{z}$
(21c)

Equation (19) is a complete form of the translational accelerations of the head that is experimentally determined from both measured six translational accelerations and the three angular velocities. Thus in order to get the more rigorous estimation of the translational acceleration of the head, the evaluations of angular velocity of the head should be made in addition to the measurements of six translational accelerations.
It is interesting to note that the number of terms involved in each angular velocity-dependent term in equations (21a)-(21c) is different from each other: three terms for x -axis direction, two terms for $y$-axis direction and one term for $z$-axis direction. The numbers of terms are determined according to the bite-bar configuration, i.e. the locations and orientations of the six accelerometers mounted on the bite-bar.

## IV. Experimental Procedure

An experiment for analysis of the head motion due to vertical seat vibration was conducted in ISVR using an electrohydraulic vibrator with the maximum stroke of 1 m . A male subject (height $=1.83 \mathrm{~m}$ weight $=73 \mathrm{~kg}$ ) was sited on the rigid flat seat in the normal (subject's back erected without backrest) and slouched (siting with a slight stoop and shoulders held forward) postures. Input stimuli were chosen to be bandpass filtered Gaussian random vertical motions with the equal acceleration power spectral density in the frequericy range of 0.5 to 30 Hz . Three vibration amplitudes were chosen to be: $0.5,1.0$, and $2.0 \mathrm{~m} / \mathrm{s}^{2}$ (r.m.s.), each with the duration of 60 seconds. For each experimental condition, the six translational accelerations at the three measuring blocks on the bite-bar were simultaneously measured for analysis of the head motion.

## V. Results and Discussion

In the previous study by Paddan et al.[6], the angular and translational accelerations of the head were calculated only from the six measured accelcrations through the approximate procedure without considering the angular velocity-dependent terms in equations (18) and (21a)-(21c). However, a complete set of head

Table 1. The rm.s. values of the calculated angular velocities for different subject's postures and seat acceferation leveis.

| Posture | Normal |  |  | Slouched |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Axis Level | $0.5 \mathrm{~m} / \mathrm{s}^{2}$ | $1.0 \mathrm{~m} / \mathrm{s}^{2}$ | $2.0 \mathrm{~m} / \mathrm{s}^{2}$ | $0.5 \mathrm{~m} / \mathrm{s}^{2}$ | $1.0 \mathrm{~m} / \mathrm{s}^{2}$ | $2.0 \mathrm{~m} / \mathrm{s}^{2}$ |
| Roll, $\omega_{x}$ ( $\mathrm{rad} / \mathrm{s}$ ) | 0.02 | 0.05 | 0.09 | 0.03 | 0.06 | 0.10 |
| Pitch, ${ }^{(1)}$ y (rad/s) | 0.07 | 0.13 | 0.21 | 0.09 | 0.18 | 0.33 |
| Yaw, $\omega_{2}$ ( $\mathrm{rad} / \mathrm{s}$ ) | 0.02 | 0.03 | 0.05 | 0.04 | 0.04 | 0.07 |

motion formula suggested in this paper is used to analyse the effects of the previously neglected terms. Their relative significance in the calculation of head motion are in details investigated in this chapter as the experimental conditions, e.g. excitation levels and subject's postures, vary.

### 5.1. Estimation of Angular Velocities

The analysis of the head motion starts from the estimation procedure for the angular velocities of the head. Angular velocities $\boldsymbol{\omega}$ can be estimated from the paired translational velocities at the three measuring blocks as shown in equation (9). In this work, the translational velocity at each block is estimated by the spectrally cquivalent integration' method using the Fourier transforms of measured acceleration signals. The method first makes the estimation of Fourier cocfficients for the frequency range of interest from the time series of measured acceleration signals, second integrates the continuous time domain acceleration model constructed by the estimated Founier coefficients to get the continuous time domain velocity model, and finally determine the time series of translational velocity from the obtained continuous velocity model at each sampling interval. The five translational velocities in equation (9), i.e.. $\left\{v_{1,}, v_{2 y}, v_{2 y}, v_{3 y}, v_{3 y}\right\}$ are oblained from the measured five acceleration signals $\left\{a_{1 z}, a_{2 y}, a_{2 \%}, a_{3 y}, a_{3 y}\right\}$ using the Fourier transform-based integration procedure. The angular velocities $\left\{\omega_{x}, \omega_{y}, \omega_{z}\right\}$ in cquations (9) were calculated using the intcgrated five translational velocitics.

Table 1 shows the r.m.s. values of the calculated angular velocities for different subject's postures and seat acceleration amplitudes. The subject's head motion due to the vertical seat vibration in this experiment is observed to have larger pitch ( $\omega_{y}$, rotation with respect to $y$ axis) component than roll ( $\omega_{x}$, rotation with respect to $x$ axis) and yaw ( $\omega_{2}$, rotation with respect to z-axis) components. It is usually observed in the whole body
vibration experiments umdertaken under the vertical seat vibration [12]. It is also shown that angular velocity increases as seat vibration amplitude increases. Subject's postures are seen to effect on the amplitude of head motion: the slouched posture shows larger rotational motion of the head than the normal posture does in this experiment case. These calculated angular velocity components $\left\{\omega_{x}, \omega_{y}, \omega_{z}\right\}$ are used in the following sections to estimate the 'angular velocity-dependent' angular and translational accelerations.

### 5.2. Estimation of Angular Accelerations

To investigate how much the angular velocity-dependent terms in equations (18) effect on the angular acceleration of the head, the translational acceleration-dependent terms $\left\{\alpha_{A x}, \alpha_{A}, \alpha_{A z}\right\}$ in equation (17) and angular velocity-dependent terms $\left\{\alpha_{R_{x}}, \alpha_{R_{y}}, \alpha_{R_{s}}\right\}$ in equation (18) were compared. Table 2 lists the percentage ratios of the r.m.s. values, i.e. $\alpha_{R i} / \alpha_{A i}$ for $i=x, y$, $z$, for different subject's posturcs and seat acceleration amplitudes. It shows that the angular accelerations (both of the translational acceleration dependent terms and angular velocity dependent terms) increase as seat vibration amplitude increases. The percentage ratios of $\alpha_{R i} / \alpha_{A j}$ for $i=x, y, z$. , also increases up to $3.3 \%$ (yaw rotation for the slouched posture) as seat vibration increases. Subject's postures are seen to effect on the amplitude of head motion: the slouched posture yields larger angular accelerations of the head and more relative significance of the angular velocity-dependency than the normal posture does in this experiment case. Results in Table 2 show that if the angular velocity-dependent terms in equations (18) are neglected, at least $3.3^{\prime}$ \% systematic error is produced for angular acceleration estimation in this experiment case. It indicates that the rigorous estimation of the angular accelerations of the head is realized by considering the angular velocity dependent terms suggested in equation (18).

Table 2. The r.ms. values of angular acceleration components $\alpha_{A}$ and the percentage ratios of the angular velocity-dependent terms to the translational velocity-dependent terms, $\alpha_{n_{i} / \alpha_{i j}}(i=x, y, z)$ for different subject's postures and seat acceleration levels.

|  |  | Normal posture |  |  | Slouched posture |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $0.5 \mathrm{~m} / \mathrm{s}^{2}$ | $1.0 \mathrm{~m} / \mathrm{s}^{2}$ | 2.0 m/s ${ }^{2}$ | $0.5 \mathrm{~m} / \mathrm{s}^{2}$ | $1.0 \mathrm{~m} / \mathrm{s}^{2}$ | 2.0 m/ $\mathrm{s}^{2}$ |
| Roll, $\alpha_{\text {A,x }}$ | $\left(\mathrm{rad} / \mathrm{s}^{2}\right)$ | 0.81 | 1.52 | 2.40 | 0.99 | 1.75 | 2.67 |
| Pitch, ${ }^{\boldsymbol{\alpha}}$ A,y | $\left(\mathrm{rad} / \mathrm{s}^{2}\right)$ | 2.55 | 3.97 | 6.03 | 2.91 | 5.70 | 9.59 |
| Yaw, ${ }^{\alpha_{A, z}}$ | $\left(\mathrm{rad} / \mathrm{s}^{2}\right)$ | 0.56 | 1.09 | 1.39 | 0.68 | 1.04 | 1.52 |
| $\alpha_{\text {R, } /} / \alpha_{\text {A, }}$ * | (\%) | 0.2 | 0.3 | 0.4 | 0.5 | 0.5 | 0.9 |
| $\alpha_{\text {R, } / \text { / }} \alpha_{\text {A,y * }}$ | (\%) | 0.0 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| $\alpha_{R, z /} \alpha_{\text {A, } z^{*}}$ | (\%) | 0.4 | 0.8 | 1.8 | 0.6 | 1.5 | 3.3 |

*The percentage ratios are calculated with the r.m.s. values of $\alpha_{R i}$ and those of $\alpha_{A}$ for $\bar{i}=x, y, z$.

Figure 2 shows the spectral distributions of the estimated translational acceleration-dependent terms $\left\{\alpha_{A x}, \alpha_{A y}, \alpha_{A c}\right\}$ and its corresponding angular velocity-dependent terms $\left\{\alpha_{\mathrm{Rx}}, \alpha_{\mathrm{Ry}}, \alpha_{\mathrm{Rz}}\right\}$ for the normal posture and the seat acceleration level of $1 \mathrm{~m} / \mathrm{s}^{2}$ (r.m.s). From the magnitudes of three translational acceleration-dependent terms plotted in Figure 2, larger pitch ( $\alpha_{A y}$ ) acceleration than roll ( $\alpha_{A x}$ ) and yaw ( $\alpha_{A Z}$ ) accelerations are observed. To the contrary, the magnitudes of the angular velocity-dependent components show a different trend: larger yaw ( $\alpha_{R z}$ ) and roli ( $\alpha_{\mathrm{Rx}}$ ) components than pitch component ( $\alpha_{\mathrm{Ry}}$ ) are observed. The reason comes from the fact that the dominant


Figure 2. Spectral distributions of 'translational accelerationdependent' (bold line-typed) and 'angular velocitydependent' angular acceleration components (thin linetyped) for the normal posture and seat acceleration amplitude $1 \mathrm{~m} / \mathrm{s} 2$ ( $\mathrm{r}, \mathrm{m}, \mathrm{s}$ ).
Roll -_ , Yaw.
pitch motion ( $\omega_{y}$ ) shown in Table 1 contributes to the angular velocity dependent roll ( $\alpha_{\mathrm{R}_{\mathrm{x}}}$ ) and yaw ( $\alpha_{\mathrm{Rz}}$ ) components as shown in equation (18). These observations reveal that the relatively larger systematic errors for the estimation of roll and yaw accelerations are expected to exist rather than that of pitch acceleration when the angular velocity-dependant terms are neglected.

### 5.3. Estimation Of Translational Accelerations

The translational acceleration components of the head as already discussed in section 3.3 are determined from the six measured accelerations, three estimated angular velocity components, and the position of interest $\mathbf{p}=p_{x} \hat{\mathbf{i}}+p_{y} \hat{\mathbf{j}}+p_{z} \hat{\mathbf{k}}$. In this section, the relative significance of the angular velocitydependent terms $\left\{a_{\text {PR. }, x}, a_{\text {PR. }, y}, a_{\text {PR. }, ~}\right\}_{\text {in equations (2la)-(21c) are }}$ analysed by comparing them to translational accelerationdependent terms $\left\{a_{p A, x}, a_{p A, y}, a_{p A, z}\right\}_{\text {in equation (20a)-(20c) for }}$ the different calculation positions. For the convenience of this analysis, the calculation position $\mathbf{p}=\mathbf{p}_{x} \hat{\mathbf{i}}+\boldsymbol{p}_{\mathbf{y}} \hat{\mathbf{j}}+\mathbf{p}_{z} \hat{\mathbf{k}}$, which is defined in the reference frame $\{B\}$ shown in Figure 1, was chosen to be on the three biodynamic planes defined in ISO 8727 [13] as shown in Figure 3 as follows:
Mid-coronal plane: $p_{x}=-15.0 \mathrm{~cm},-16.0 \mathrm{~cm} \leq p_{y} \leq-4.0 \mathrm{~cm}$, $-6.0 \mathrm{~cm} \leq p_{2} \leq 15.0 \mathrm{~cm}$;

Mid-sagittal plane: $-26.5 \mathrm{~cm} \leq p_{x} \leq-3.5 \mathrm{~cm}, p_{y}=-10.0 \mathrm{~cm}$, $-6.0 \mathrm{~cm} \leq \mathrm{p}_{\mathrm{z}} \leq 15.0 \mathrm{~cm}$;


Figure 3. Biodynamic axes and biodynamic planes for the human head defined in 1 SO 8717 (by Paddan and Griffin [6]).

Horizontal plane: $-26.5 \mathrm{~cm} \leq \mathrm{p}_{\mathrm{x}} \leq-3.5 \mathrm{~cm}$, $-16.0 \mathrm{~cm} \leq \mathrm{p}_{y} \leq-4.0 \mathrm{~cm}, \mathrm{p}_{\ell}=4.5 \mathrm{~cm}$.

Figure 4 shows the spatial distributions of the percentage ratios of the r.m.s. values, i.e. ${ }^{a_{p R, i} / a_{p A, i}}$ for $\mathrm{i}=\mathrm{x}, \mathrm{y}, z$, on the three biodynamic planes for the slouched posture and seat acceleration amplitude $2.0 \mathrm{~m} / \mathrm{s} 2$ (r.m.s). For the x -axis translational accelerations, the percentage ratios of the angular velocitydependent terms ( ${ }^{a_{p R, x}} /{ }^{a_{p A, x}}$ ) increase up to $5.4 \%$ for the horizontal plane shown in Figures 4(c). For the y-axis accelerations, the percentage ratios of the angular velocitydependent tems ( ${ }^{\mathrm{a}_{\mathrm{PR}, \mathrm{y}} /{ }^{\mathrm{a}}{ }_{\mathrm{PA}, y} \text { ) show small values below } 1.7 \% ~}$ for all three biodynamic planes. For the $z$-axis accelerations, the percentage ratios of the angular velocity-dependent tems ( ${ }^{\mathrm{pR}, \mathrm{x}}$ / $\mathrm{a}_{\mathrm{pA}, \mathrm{z}}$ ) increase up to $2.2 \%$ for the mid-sagittal plane shown in Figure 4 (b). The percentage ratios of x -axis components ( ${ }^{\mathrm{a}}{ }_{\mathrm{pR}, \mathrm{x}} /$

(a)




(b)


(c)

Figure 4. Spatial distributions of the percentage ratios of the angular velocity-dependent terms to the translational velocity dependent terms in translational acceleration of the head, i.e. ... $I_{n-1}(i=x, y, z)$ for the slouched posture and seat acceleration level $2 \mathrm{~m} / \mathrm{s} 2$ (r.m.s) on three bio-dynamic planes. (a) Mid-coronal plane; (b) Mid-sagittal plane; (c) Horizontal plane.

Table 3. The peak percentage ratios of the angular velocity-dependent terms to the translational velocity-dependent terms in the translational acceleration components of the head on the three biodynamic at seat acceleration amplitude $2 \mathrm{~m} / \mathrm{s}^{2}(\mathrm{r} . \mathrm{m} . \mathrm{s})$ planes for different subject's postures.

|  | Mid-coronal plane |  | Mid-sagittal plane |  | Horizontal plane |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Axis | Normal | Slouched | Normal | Slouched | Normal | Slouched |
| $\mathbf{a}_{\mathrm{pR}, \mathrm{x} / \mathrm{a}_{\mathrm{pA}, \mathrm{x}}{ }^{\text {a }} \text { (\%) }}$ | 1.4 | 2.9 | 2.5 | 5.1 | 2.7 | 5.4 |
| $\mathrm{a}_{\mathrm{pR}, y /} \mathrm{a}_{\mathrm{pA}, \mathrm{y}}{ }^{*}$ (\%) | 1.0 | 1.7 | 0.9 | 1.5 | 0.9 | 1.3 |
| $\mathrm{a}_{\mathrm{pR}, z /} \mathrm{a}_{\mathrm{pA}, z} \times$ (\%) | 0.8 | 2.1 | 0.8 | 2.2 | 0.3 | 0.7 |

*The percentage ratios are calculated with the r.m.s. values of ${ }^{a_{p R .}}$ and those of ${ }^{a_{p A, s}}$ for $i=x, y, z$.
$\mathbf{a}_{\mathrm{pA}, \mathrm{x}}$ ) for the mid-sagittal plane (Figure 4b) and horizontal plane (Figure 4 c ) increase as the distance $\left|p_{x}\right|$ increases, and the percentage ratios of the $z$-axis components ( ${ }^{a_{P R, ~} /} /{ }^{a_{p A, z}}$ ) for the mid-coronal plane (Figure 4a) and mid-sqgittal plane (Figure 4b) increase as the distance $\left|p_{x}\right|$ increase. The reason is that the distances $\left|p_{k}\right|$ and $\left|p_{2}\right|$ are muiltiplied by the square of the large angular velocity $\omega_{y}$ (pitch) in the calculations of angular velocity-dependent terms as shown in equations (21a) and (21c). These proctucts, i.e. the centrifugal acceleration components of the head which are mainly produced by the pitch motion, become lager as the calculation position $\mathbf{p}$ is further from the origin of the moving reference frame. It means that when the angular velocity-dependant terms are neglected, the larger systematic errors in estimated x and z -axes translational accelerations should be inevitable as the distance between the calculation position $\mathbf{P}$ and the origin of the moving reference frame (block 2 ) increases.

Table 3 shows the peak peroentage ratios of the angular velocitydependent terms to the translational accelcration-dependent components of the head on the three biodynamic planes for the normal and slouched postures, and seat acceleration amplitude $2 \mathrm{~m} / \mathrm{s}^{2}$ (r.ms). The maximum percentage ratio of $5.4 \%$ was observed in the x -axis acceleration compontent ( $\mathrm{a}_{\mathrm{pR}, \mathrm{x}} / \mathrm{a}_{\mathrm{pA}, \mathrm{x}}$ ) for the slouched posture, which occurs at the back of head and ear-height ( $p_{x}=-26.5 \mathrm{~cm}, p_{x}=4.5 \mathrm{~cm}$ ) on the horizontal plane. The angular velocity-dependent terms are observed to contribute about $2 \%$ to systematic measurement uncertainty on other axes. Relatively larger percentage ratios of the angular velocity-dependent terms are observed in the case of the slouched posture than the normal posture. It reveals that the angular velocity-dependent terms become of more significance as the body posture is more relaxed as in case of the slouched posture
since the magnitude of the pitch motion $\omega_{y}$ increases.
The relatively larger percentage ratio of $x$-directional acceleration component ${ }^{a_{p R, x}} /{ }^{a_{\rho A, x}}$ are observed compared to those of $y$ and $z$-directional acceleration components as shown in Tables 3. The reason is that much number of terms (in this case, three terms) are involved in $x$-directional acceleration component $a_{p R . x}$ in equation (21a) than $y$ and $z$-axes components in equations (21b) and (21c) respectively. As mentioned in section 3.3 , the number of terms in equations (21a)-(21c) is determined by the bite-bar configuration such as the locations and orientations of the six accelerometers mounted on the bite-bar. It reveals that the significances of the angular velocity-dependent acceleration components are dependent upon the sensor configurations as well as the calculation position.

## VI. Conclusions

This work has presented a complete description for measuring the six-degree-of-freedom motion of the head using the six-accelerometer configured bite-bar. The description shows that the six-axis vibration of the head cannot be completely obtained without the three orthogonal (roll, pitch and yaw) angular velocity components since the head acceleration consists of the translational acceleration-dependent, and the angular velocitydependent, components. A new method of estimating the three angular velocities from the Fourier transform-based integration of the six acceleration measurements was introduced. The estimated angular velocities were used for the analysis of the systematic errors induced by neglecting the angular velocity-dependent components.
Experimental results showed that in the estimation of the head angular acceleration, the percentage ratio of the angular
velocity-dependent component to the translational accelerationdependent component increases up to $3.3 \%$ as the seat vibration level increases. From those experimental results, it can be concluded that the previously neglected (hence systematic error) angular velocity related compunents become of more significance as the amplitude of the seat vibration increases. It is interesting to note that the angular velocity-related components contribute mostly to the yaw and roll motions although the pitch motion is dominant.

The translational accelerations of the head were evaluated on the three biodynantic planes in order to examine the effect of the calculation position on the error characteristics. The percentage ratio of the angular velocity-dependent component to the translational acceleration-dependent one increases up to $5.4 \%$ for $x$-axis acceleration. The maximum error is observed to occur at the back of head and ear-height. From those experimental results, it can be concluded that the error terms become of more significance as the distance between the calculation position and the reference sensor goes further, and that the errors in x -axis accelcrations are mostly significant for the bite-bar configuration chosen.

Resultantly, the inclusion of angular velocity dependent terms suggested in this paper gives less systematic errors in measuring the 6 -degree-of-freedom motion of the head.

## Acknowledgement

This work was supported by the Ministry of Science and Technology, the Ministry of Environments and the Korea Research Institute of Standards and Science (Project Code 01-0407-362). The authors gratefully acknowiedge experimental aids and discussions given by Professor M. J. Griffin and Dr. Y. Matsumeto in ISVR. Finally we would like to thank anonymous referees for commenting on the first draft of this paper.

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