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THE STABILITY IN AN INCLINED LAYER OF VISCOELASTIC FLUID FLOW OF HYDROELECTRIC NATURAL CONVECTION

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ABSTRACT The problem of the onset stability in an inclined layer of dielectric viscoelastic fluid (Walter's liquid B') is studied. The analysis is made under the simultaneous action of a normal a.c. electric field and the natural convection flow due to uniformly distributed internal heat sources. The power series method used to obtain the eigen value equation which is then solved numerically to obtain the stable and unstable solutions. Numerical results are given and illustrated graphically.

1. INTRODUCTION

The phenomenal growth of energy requirements in recent years has been attracting considerable attention all over the world. This has resulted in a continuous exploration of new ideas and avenues in harnessing various conventional energy sources, such as tidal waves, wind power, geo-thermal energy, etc. It is obvious that in order to utilize geo-thermal energy to a maximum, one should have a complete and precise knowledge of the amount of perturbations needed to generate convection currents in geo-thermal fluid. Also, knowledge of the quantity of perturbations that are essential to initiate convection currents in mineral fluids found in the earth's crust helps one to utilize the minimal energy to extract the minerals. For example, in the recovery of hydro-carbons from underground petroleum deposits, the use of thermal processes is increasingly gaining importance as it enhances recovery. Heat is being injected into the reservoir in the form of hot water or steam or heat can be generated by burning part of the crude in the reservoir. In all such thermal recovery processes, fluid flow takes place through a dielectric medium and convection currents are detrimental.

In technological fields there exist important class of fluid, called non-Newtonian fluid, are also being studied extensively because of their practical applications, such as fluid film lubrication, analysis of polymers in chemical engineering etc. the micropolar fluid is famous case for non-Newtonian fluid as El-Bary [1]. Also, another example for non-Newtonian fluid is viscoelastic fluid. A detailed theoretical investigation has recently begun for the viscoelestic prototype designated liquid B'Walters [2] and Beard and Walters [3]. Many other authors have contributed to the subject. Sen [4] studied the behavior of unsteady free convection flow of a viscoelastic fluid past an infinite porous plate with constant suction. The effects of suction, free oscillations and free convection currents on flow have been studied by Soundalgeker and Patil [5]. Singh and Singh [6] have studied the magnetohydrodynamic flow of viscoelastic fluid past an accelerated plate. The flow of viscoelastic and electrically conducting fluid past an infinite plate has been studied by Sherief and Ezzat [7]. In most of the above applications, the method of solution due to Lighthil [8] and Stuart [9] is utilized.

The method of the matrix exponential, proposed by Ezzat [10-13], which constitutes the basis of the

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state space approach of modern control theory is applied to the non-dimensional equations of a viscoelastic fluid flow of hydromagnetic free convection flows.

A temperature gradient applied to a dielectric fluid produces a gradient in the dielectric constant and electrical conductivity. The application of a dc electric field the results in the accumulation of free charge in the fluid. The free charge buidup occurs exponentially in tie with a time constant. This constant is known as the electrical relaxation time. If an ac electric field is applied at frequency much higher than the reciprocal of the electrical relaxation, the free charge does not have time to accumulate. The electrical relaxation times of most dielectric fluids appear to be sufficiently long to make free charge effects negligible at standard power line frequencies is so low that it makes no significant contribution to the temperature field. Furthermore, variations in the body force are so rapid that its mean value can be assumed as the effective value in determining fluid motions, except in the case of fluids of extremely low viscosity. Thus, the case of an ac electric field is more tractable than that of a dc electric field. Turnbull and Melcher [14] and Turnbull [15] have examined the ac case.

An important stability problem is the thermal convection in a thin layer of fluid heated from below. A detailed account of thermal convection in a thin layer of Newtonian fluid heated from below, under varying assumptions, has been given by Chandrasekhar [16]. The problem of the onset of convective instability in an inclined fluid layer including heat sources in the presence of a temperature gradient and an a.c. electric field was studied by Mohamed and et al. [17]. They used the power-series method to obtain the eigenvalue equation that is then solved numerically to obtain the stable and unstable solutions. The stability of viscolastic conducting liquid heated from below in the presence of a magnetic field is studied by Othman and Ezzat [18]. Ezzat and Othman [19] are studied the effect of a vertical ac electric field on the onest of convective instability in a dielectric micropolar fluid layer form below confined between two horizontal planes under the simultaneous action of the rotation of the system and the vertical temperature gradient.

The purpose of this work is to study the stability of natural convection in an inclined fluid layer (Walter's liquid B') with internal heat generation in the presence of an ac electric field.

2. Formulation of the Problem

We consider an infinite incompressible and dielectric viscoelastic fluid layer confined between two parallel plates which are separated by a distance and inclined from the vertical by an angleθ. It is assumed that the fluid layer is heated internally by a uniform distribution of heat sources and that the two plates are maintained at constant and equal temperatures T_0. The plate at $x=-\frac{\lambda}{2}$ is maintained at electric potential ($\phi_1 = 0$), whereas the plate at $x=\frac{\lambda}{2}$ is kept at constant and high alternating potential whose root-mean-square value is ϕ_2.

Under the foregoing assumptions, the basic equations can be written as [2]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho\left[\frac{\partial v_i}{\partial t}+v_k\frac{\partial v_i}{\partial x_k}\right]=\rho g-\frac{\partial P}{\partial x_i}+\eta_o\frac{\partial^2 v_i}{\partial x_k\partial x_k}+f_e-K_o\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 v_i}{\partial x_k\partial x_k}\right)\right.$$
$$\left.+v_m\left(\frac{\partial^3 v_i}{\partial x_m\partial x_k\partial x_k}\right)-\left(\frac{\partial v_i}{\partial x_m}\right)\left(\frac{\partial^2 v_m}{\partial x_k\partial x_k}\right)-2\left(\frac{\partial v_m}{\partial x_k}\right)\left(\frac{\partial^2 v_i}{\partial x_m\partial x_k}\right)\right], \tag{2}$$

$$\rho c_v\left[\frac{\partial T}{\partial t} + (v.\nabla)T\right] = k\nabla^2 T + Q \tag{3}$$

$$\text{div}\,(\varepsilon\, E) = 0 \tag{4}$$

and

$$\text{curl}\, E = 0 \qquad \text{or} \qquad E = -\nabla\phi\ , \tag{5}$$

where, v = (u, v, w) is the velocity of the fluid, g = (-g sin θ, 0, -g cos θ) is the gravitational acceleration, ρ is the mass density, P is the pressure, η_0 is the limiting viscosity at small rate of shear, K_0 is the elastic constant of Walters' liquid B', c_v is the specific heat at constant volume, k is the thermal conductivity, T is the temperature of the fluid, Q is the heat generation within the fluid per unit volume per unit time, ε is the dielectric constant, $E \equiv [E_x, 0, 0]$ is the electric field, ϕ is the root-mean-square value of the electric potential and f_e is the force of electrical origin which may be expressed as Landau [20] in the form,

$$f_e = \rho_e E - \frac{1}{2}E^2\nabla\varepsilon + \frac{1}{2}\nabla(\rho\frac{\partial\varepsilon}{\partial\rho}E^2)\,. \tag{6}$$

taking into account the fact the free charge density ρ_e is zero.

If we replace the pressure by

$$P^* = P - \frac{1}{2}\rho\frac{\partial\varepsilon}{\partial\rho}E^2 \tag{7}$$

The electrostriction term disappear from the equation (2), which can be rewritten in the form

$$\rho\left[\frac{\partial v_i}{\partial t} + v_k\frac{\partial v_i}{\partial x_k}\right] = \rho g - \frac{\partial P^*}{\partial x_i} + \eta_o\frac{\partial^2 v_i}{\partial x_k \partial x_k} - \frac{1}{2}E^2\frac{\partial\varepsilon}{\partial x_i} - K_o\left[\frac{\partial}{\partial t}(\frac{\partial^2 v_i}{\partial x_k\partial x_k}) + v_m(\frac{\partial^3 v_i}{\partial x_m\partial x_k\partial x_k}) - (\frac{\partial v_i}{\partial x_m})(\frac{\partial^2 v_m}{\partial x_k\partial x_k}) - 2(\frac{\partial v_m}{\partial x_k})(\frac{\partial^2 v_i}{\partial x_m\partial x_k})\right]. \tag{8}$$

The boundary conditions

$$u = v = w = 0 \qquad \text{at} \qquad x = \pm\frac{\lambda}{2}, \tag{9}$$

$$T = T_0 \qquad \text{at} \qquad x = \pm\frac{\lambda}{2}, \tag{10}$$

$$\phi = 0 \qquad \text{at} \qquad x = -\frac{\lambda}{2}, \tag{11}$$

$$\phi = \phi_2 \qquad \text{at} \qquad x = -\frac{\lambda}{2}, \tag{12}$$

The mass density ρ and the dielectric constant ε are assumed to be linearly dependent on temperature as [18]:

$$\rho = \rho_o[1 - \alpha(T - T_o)], \qquad \alpha > 0 \tag{13}$$

$$\varepsilon = \varepsilon_o[1 - e(T - T_o)], \qquad e > 0 \tag{14}$$

where the subscript "0" refers to values at the midplane x = 0, α is the coefficient of volume expansion and e is the coefficient of relative variation of the dielectric constant with temperature.

We first obtained the following steady solutions (denoted by an overbar).

$$\overline{T} = T_o + \frac{Q}{2k}\left(\frac{\lambda}{4} - x^2\right) \tag{15}$$

$$\bar{\rho} = \rho_o \left[1 - \frac{\alpha Q}{2k}\left(\frac{\lambda}{4} - x^2\right)\right], \tag{16}$$

$$\bar{\varepsilon} = \varepsilon_o\left[1 - \frac{eQ}{2k}\left(\frac{\lambda}{4} - x^2\right)\right] , \tag{17}$$

$$\bar{P}^* = P_o^* - \rho_o g\cos\theta\left[1 - \frac{\alpha Q \lambda^2}{10k}\right]z - \rho_o g\sin\theta\left[\left(1 - \frac{\alpha Q\lambda^2}{8k}\right)x - \frac{\alpha Q}{6k}x^3\right]$$
$$- \frac{1}{2}\int \bar{E}_x^2 \frac{\partial \bar{\varepsilon}}{\partial x}\, dx \tag{18}$$

Since the flow is assumed to be along the z-axis, $\bar{w}$ is obtained in the form

$$\bar{w} = \frac{\alpha g Q}{k \upsilon}\left[\frac{\lambda^4}{1920} - \frac{\lambda^2}{80}x^2 + \frac{1}{24}x^4\right]\cos\theta \quad , \quad \bar{u} = \bar{v} = 0, \tag{19}$$

$$\bar{E}_x = \frac{E_o}{\left[1 - \frac{eQ}{2k}\left(\frac{\lambda^2}{4} - x^2\right)\right]} \quad , \quad \bar{E}_y = 0 \quad , \quad \bar{E}_z = 0 \ , \tag{20}$$

$$\bar{\phi} = -\int \bar{E}_x\, dx \tag{21}$$

where P_o^* is the pressure at z = 0 and x = 0, $\upsilon = \frac{\eta}{\rho_o}$ is the kinematic viscosity, and $\bar{w}$ and $\bar{P}^*$ have been determined under the condition that the total flux of flow across a plane z = constant is zero [17].

Let this initial steady state be slightly perturbed where any physical quantities ψ can be expressed after perturbation by the simple relation $\psi = \bar{\psi} + \psi'$, and prime refers to perturbed quantities. Following the usual steps of linear stability theory we can obtain the following main equations:

$$\left(\frac{\partial}{\partial t} - \upsilon\nabla^2\right)\nabla^2 u' + \bar{w}\frac{\partial}{\partial z}\nabla^2 u' - \frac{d^2\bar{w}}{dx^2}\frac{\partial u'}{\partial z} + \alpha g\cos\theta\frac{\partial}{\partial z}\left(\frac{\partial T'}{\partial x}\right) - \alpha g\sin\theta\nabla_1^2 T'$$
$$- \frac{\bar{E}_x}{\rho_o}\frac{d\bar{\varepsilon}}{dx}\frac{\partial}{\partial x}\nabla_1^2\phi' + \frac{e\varepsilon_o}{\rho_o}\bar{E}_x\frac{\partial \bar{E}_x}{\partial x}\nabla_1^2 T' + K_o\left[\frac{\partial}{\partial t} + \bar{w}\frac{\partial}{\partial z}\right]\nabla^4 u' = 0, \tag{22}$$

$$\left(\frac{\partial}{\partial t} - K\nabla^2\right)T' + \bar{w}\frac{\partial T'}{\partial z} + \frac{d\bar{T}}{dx}u' = 0 \quad , \tag{23}$$

$$\nabla^2\phi' + eE_o\frac{\partial T'}{\partial x} = 0 \quad , \tag{24}$$

where $K = \frac{k}{\rho_o c_v}$ is the thermal diffusivity and $\nabla_1^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the two-dimensional Laplacian.

The associated boundary conditions are given by

$$u' = v' = w' = T' = \phi' = 0 \qquad \text{at} \qquad x = \pm\frac{\lambda}{2} \tag{25}$$

We first rendered Eqs. (22) – (24) and the boundary conditions (25) in a dimensionless from by choosing $\lambda, \frac{\lambda^2}{\upsilon}, \frac{\upsilon}{\lambda}, \frac{Q\lambda^2}{k}$ and $\frac{e E_o Q \lambda^3}{k}$ as the units of length, time, velocity, temperature and electrostatic potential respectively and the equations are then simplified in the usual manner by decomposing the solution in terms of normal modes, so that

$$[u', T', \phi'] = [U(x), \Theta(x), \Phi(x)] \exp[\sigma t + i(a_y y + a_z z)] \tag{26}$$

where "a_y" and "a_z" are the (real) wave numbers in the y and z directions and σ is the complex time constant.

This makes it possible to obtain the following system of equations:

$$[\sigma - (D^2 - \lambda^2)]\,(D^2 - \lambda^2)\,U + ia_z R_H \cos\theta[\overline{w_1}(D^2 - \lambda^2)U - D^2\overline{w_1}\,.\,U + D\Theta]$$
$$+ R_H \lambda^2 \sin\theta\,\Theta + R_A \lambda^2 x[\Theta + D\Phi] + K_o^*[\sigma + R_H \cos\theta\overline{w_1}](D^2 - \lambda^2)^2 U = 0, \tag{27}$$

$$[P_r \sigma - (D^2 - \lambda^2)]\,\Theta + P_r[i a_z R_H \cos\theta\,\overline{w_1}\,\Theta - xU] = 0, \tag{28}$$

and

$$(D^2 - \lambda^2)\,\Phi + D\Theta = 0. \tag{29}$$

The boundary conditions are

$$U = DU = \Theta = \Phi = 0 \qquad \text{at} \qquad x = \pm\frac{1}{2} \tag{30}$$

where,

$$P_r = \frac{\upsilon}{K} \qquad \text{Prandtl number,} \tag{31}$$

$$R_H = \frac{\alpha g Q \lambda^5}{\upsilon^2 k} \qquad \text{heat Rayleigh number,} \tag{32}$$

$$R_A = \frac{\varepsilon_o e^2 E_o^2 Q^2 \lambda^4}{\rho_o k^2 \upsilon^2} \qquad \text{electric Rayleigh number,} \tag{33}$$

$$K_o^* = \frac{K_o}{\lambda^2} \qquad \text{the dimensionless elastic constant} \tag{34}$$

$$\overline{w_1} = \frac{1}{1920} - \frac{1}{80}x^2 + \frac{1}{24}x^4 \qquad \text{the dimensionless velocity,} \tag{35}$$

and D denotes differentiation with respect to x.

To describe three-dimensional disturbances, it is convenient to introduce the parameters

$$\lambda^2 = a_y^2 + a_z^2 \tag{36}$$

and,

$$a = \frac{a_z}{\lambda} \tag{37}$$

then, equations (27) and (28) become

$$(D^2 - \lambda^2 - \sigma)(D^2 - \lambda^2)U - ia\lambda R_H \cos\theta[\overline{w_1}(D^2 - \lambda^2) - D^2\overline{w_1}]\,U - ia\lambda R_H \cos\theta\, D\Theta$$
$$- R_H \lambda^2 \sin\theta\,\Theta - \lambda^2 x R_A[D\Phi + \Theta] - K_o^*[\sigma + R_H \overline{w_1}]\,(D^2 - \lambda^2)^2 U = 0, \tag{38}$$

$$[D^2 - \lambda^2 - P_r\sigma]\Theta - P_r[i a \lambda R_H \cos\theta \overline{w_1} \,.\, \Theta - x\,U] = 0 \quad . \tag{39}$$

It should be noted that the value of the parameter "a" is within the range $0 \le a \le 1$. The case $a = 0$ corresponds to the case $a_z = 0$ (i.e. longitudinal rolls); the case $a = 1$ corresponds to the case $a_y = 0$ (i.e. transverse rolls).

3. SOLUTION

Equations (38) and (39) can be rewritten in the form:

$$\left(b_1 + b_2 x^2 + b_3 x^4\right) D^4U - \left(c_1 - c_2 x^2 + c_3 x^4\right) D^2U + \left(f_1 - f_2 x^2 + f_3 x^4\right) U$$

$$- S\,D\Theta - F\Theta - \Gamma x(\Theta + D\Phi) = 0 \quad , \tag{40}$$

$$D^2\Theta - \left(h_1 + h_2 x^2 + h_3 x^4\right)\Theta + P_r\, x\, U = 0 \quad , \tag{41}$$

where,

$$S = i\, a\, \lambda\, R_H \cos\theta \tag{42}$$

$$F = \lambda^2 R_H \sin\theta \tag{43}$$

$$\Gamma = \lambda^2 R_A \tag{44}$$

$$b_1 = [1 - K_o^*(\sigma + \frac{1}{1920} R_H \cos\theta)] \quad , \quad b_2 = \frac{1}{80} K_o^*\, R_H \cos\theta \quad ,$$

$$b_3 = -\frac{1}{24} K_o^* R_H \cos\theta \quad , \tag{45}$$

$$c_1 = 2\lambda^2 + \sigma + \frac{1}{1920} S - 2\lambda^2 K_o^* (\sigma - \frac{1}{1920} R_H \cos\theta)]$$

$$c_2 = \frac{1}{80} S + \frac{1}{40} \lambda^2 K_o^* R_H \cos\theta \quad , \quad c_3 = \frac{1}{24} S - \frac{1}{12}\lambda^2 K_o^* R_H \cos\theta \quad , \tag{46}$$

$$f_1 = \lambda^2(\lambda^2 + \sigma) + S(\frac{\lambda^2}{1920} - \frac{1}{40}) - \lambda^4 K_o^* (\sigma + \frac{1}{1920} R_H \cos\theta)$$

$$f_2 = S(\frac{\lambda^2}{80} - \frac{1}{2}) - \frac{\lambda^4}{80} K_o^* R_H \cos\theta \quad , \quad f_3 = \frac{\lambda^2}{24} S - \frac{\lambda^4}{24} K_o^* R_H \cos\theta \quad , \tag{47}$$

$$h_1 = \lambda^2 + P_r(\sigma + \frac{1}{1920} S) \quad , \quad h_2 = \frac{1}{80} P_r S \quad , \quad h_3 = \frac{1}{24} P_r S \quad , \tag{48}$$

The power series method is adopted to solve equations (40), (41) and (29), since this method is much less laborious than other various approximate methods and moreover, it enables one to obtain essentially exact values of the stability condition, unless the product $\lambda\, R_H$ is exceedingly large.

Applying this power series method, the general solutions of equations (40), (41) and (29) can be constructed in the form

$$U = \sum_{m=1}^{8} C_m \sum_{n=1}^{\infty} A(m,n) x^{n-1} \tag{49}$$

$$\Theta = \sum_{m=1}^{8} C_m \sum_{n=1}^{\infty} B(m,n) x^{n-1} \tag{50}$$

$$\Phi = \sum_{m=1}^{8} C_m \sum_{n=1}^{\infty} H(m,n) x^{n-1} \tag{51}$$

where C_1 to C_8 are arbitrary constants. The series coefficients A(m, n), B(m, n) and H(m, n) are found from equations (40), (41) and (29) to obey the following recurrence relations:

$$A(m,n) = \delta_{m,n} \quad \text{for } m = 1 \rightarrow 8, \quad n = 1, 2, 3, 4 \tag{52}$$

$$\begin{aligned} A(m,n) = {} & \frac{1}{(n-1)(n-2)(n-3)(n-4)b_1} \{[(n-3)(n-4)C_1 A(m,n-2) - f_1 A(m,n-4)]\Delta_{5,n} \\ & + \Gamma B(m,n-5)\Delta_{6,n} - [(n-3)(n-4)(n-5)(n-6)b_2 A(m,n-2) \\ & + (n-5)(n-6)C_2 A(m,n-4) - f_2 A(m,n-6) - \Gamma B(m,n-6)]\Delta_{7,n} \\ & + (n-7)\Gamma H(m,n-6)\Delta_{8,n} + [(n-7)(n-8)C_3 A(m,n-6) - f_3 A(m,n-8) \\ & + (n-5)(n-6)(n-7)(n-8)b_3 A(m,n-4) - (n-8)S B(m,n-7)]\Delta_{9,n}\}, \\ & \text{for } m = 1 \rightarrow 8, \quad n = 5, 6, 7, \ldots \end{aligned} \tag{53}$$

$$B(m,n) = \delta_{m,n+4} \quad \text{for } m = 1 \rightarrow 8, \quad n = 1, 2 \tag{54}$$

$$\begin{aligned} B(m,n) = {} & \frac{1}{(n-1)(n-2)} \{h_1 B(m,n-2)\Delta_{3,n} - [h_2 B(m,n-4) + P_r A(m,n-3)]\Delta_{4,n} \\ & + h_3 B(m,n-6)\Delta_{7,n}\} \quad \text{for } m = 1 \rightarrow 8, \quad n = 3, 4, 5, \end{aligned} \tag{55}$$

$$H(m,n) = \delta_{m,n+6} \quad \text{for } m = 1 \rightarrow 8, \quad n = 1, 2 \tag{56}$$

$$\begin{aligned} H(m,n) = {} & \frac{1}{(n-1)(n-2)} \{\lambda^2 H(m,n-2) - (n-2)B(m,n-1)\}\Delta_{3,n} \\ & \text{for } m = 1 \rightarrow 8, \quad n = 1, 2 \end{aligned} \tag{57}$$

where

$$\left.\begin{aligned} \delta_{i,j} &= 0 \quad \text{for} \quad i \neq j \\ \delta_{i,j} &= 1 \quad \text{for} \quad i = j \end{aligned}\right\} \tag{58}$$

$$\left.\begin{aligned} \Delta_{i,j} &= 0 \quad \text{for} \quad i > j \\ \Delta_{i,j} &= 1 \quad \text{for} \quad i \leq j \end{aligned}\right\} \tag{59}$$

Let us now impose the boundary conditions (30) to obtain eight homogeneous algebraic equations for eight unknown constants C_1 to C_8. The requirement that the determinant of coefficients of C_1 to C_8 must vanish in order to ensure a nontrivial solution of the form

$$|X(\lambda,m)| = 0 \tag{60}$$

where,

$$\left.\begin{aligned} X(1,m) &= \sum_{n=1}^{\infty} A(m,n)\left(\frac{1}{2}\right)^{n-1}, & X(2,m) &= \sum_{n=1}^{\infty} (n-1)A(m,n)\left(\frac{1}{2}\right)^{n-2} \\ X(3,m) &= \sum_{n=1}^{\infty} B(m,n)\left(\frac{1}{2}\right)^{n-1}, & X(4,m) &= \sum_{n=1}^{\infty} H(m,n)\left(\frac{1}{2}\right)^{n-1} \\ X(5,m) &= \sum_{n=1}^{\infty} A(m,n)\left(-\frac{1}{2}\right)^{n-1}, & X(6,m) &= \sum_{n=1}^{\infty} (n-1)A(m,n)\left(-\frac{1}{2}\right)^{n-2} \\ X(7,m) &= \sum_{n=1}^{\infty} B(m,n)\left(-\frac{1}{2}\right)^{n-1}, & X(8,m) &= \sum_{n=1}^{\infty} H(m,n)\left(-\frac{1}{2}\right)^{n-1} \end{aligned}\right\} \tag{61}$$

4. NUMERICAL RESULTS

The case when $0^\circ < \theta \le 90^\circ$, $E \neq 0$, $K_o^* = 0$ and $a = 0$ has been studied in [17].

In the present paper the case $0^\circ \le \theta \le 90^\circ$, $E \neq 0$, $a = 0$ (i.e. $a_z = 0$ or longitudinal rolls) and $K_o^* \neq 0$ is treated. It is shown that if we fix the values of Prandtl number P_r, wave number λ, heat Rayleigh number R_H, the inclination angle θ, the time constant ($\sigma = 1$) and the elastic constant K_o^*, one obtains a quadratic equation of R_A with coefficients that depends on λ as a parameter. This equation can be solved numerically to obtain the critical values of R_A. The function $R_A(\lambda)$ is illustrated graphically for various fixed values of K_o^*, R_A and θ as parameters. It can seen that the curves obtained pass through a critical minimum values R_{Ac} corresponds to a critical wave number λ_c.

Figures 1, 2 and 3 show the plots of R_A, when $P_r = 5$, $R_H = 1000$ and $0^\circ \le \theta \le 90^\circ$, for the values of $K_o^* = 0$, 0.1, 0.4 and $0 < \lambda < 3$ at the different values of θ. We notice from these figures that as θ increases the critical values of the electric Rayleigh number increases, which indicates that the angle of the inclination has destabilizing effect but the elastic coefficient K_o^* has stabilizing effect.

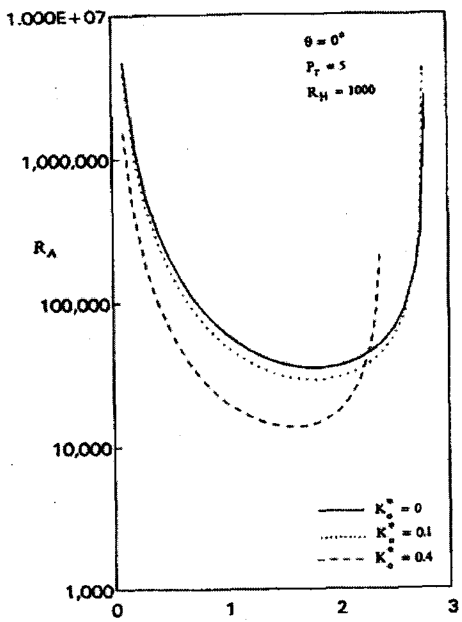

Figure 1
represents the variation of R_A with λ for various fixed values of k_o^* and $R_H = 1000$, $P_r = 5$, $\theta = 0^\circ$.

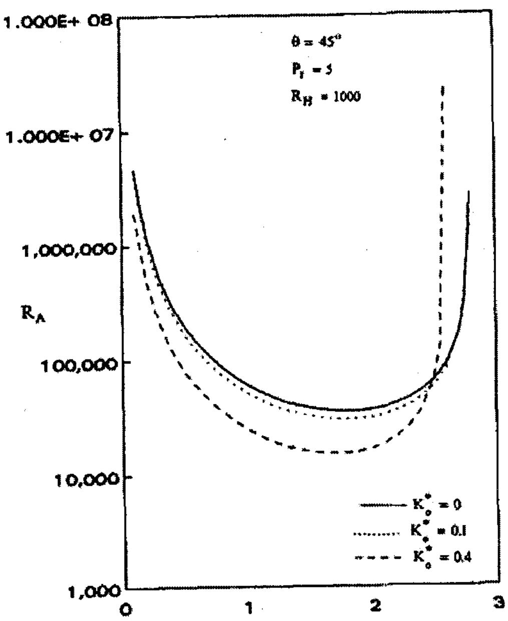

Figure 2
represents the variation of R_A with λ for various fixed values of k_o^* and $R_H = 1000$, $P_r = 5$, $\theta = 45^\circ$.

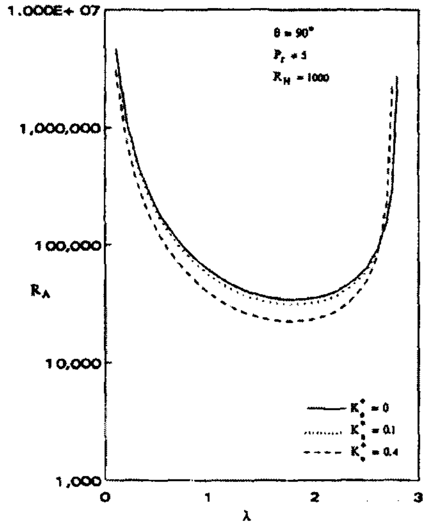

Figure 3
represents the variation of R_A with λ for various fixed values of k_o^* and $R_H = 1000$, $P_r = 5$, $\theta = 90°$.

Figures 4, 5 and 6 show the plots of R_{Ac}, when $P_r = 5$, $\lambda = 1.8$ and $0° \le \theta \le 90°$ for the values of $K_o^* = 0, 0.1, 0.4$ and $100 \le R_H \le 1000$. The critical value of electric Rayleigh number decreases as K_o^* increases.

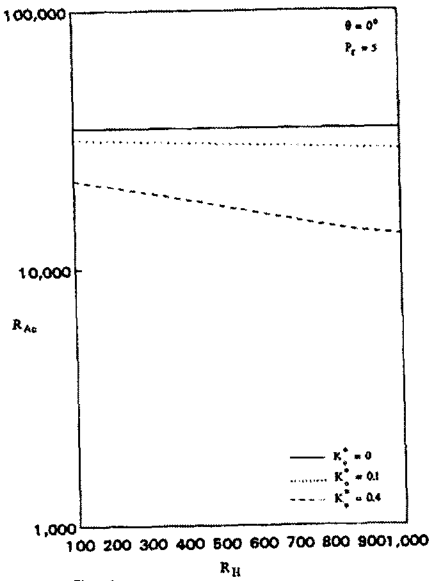

Figure 4
represents the variation of R_{Ac} with R_H for various fixed values of k_o^* and $P_r = 5$, $\theta = 0°$.

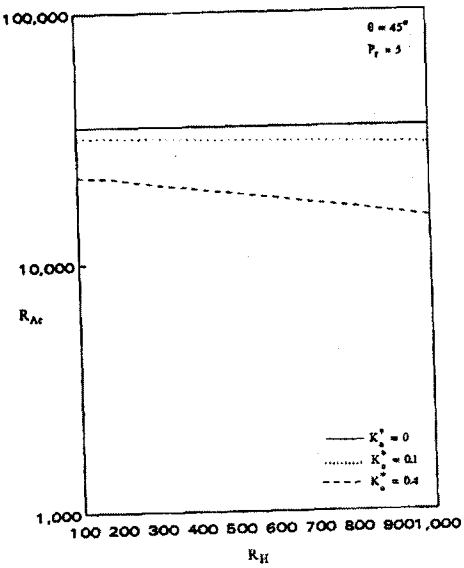

Figure 5
represents the variation of R_{Ac} with R_H for various fixed values of k_o^* and $P_r = 5$, $\theta = 45°$.

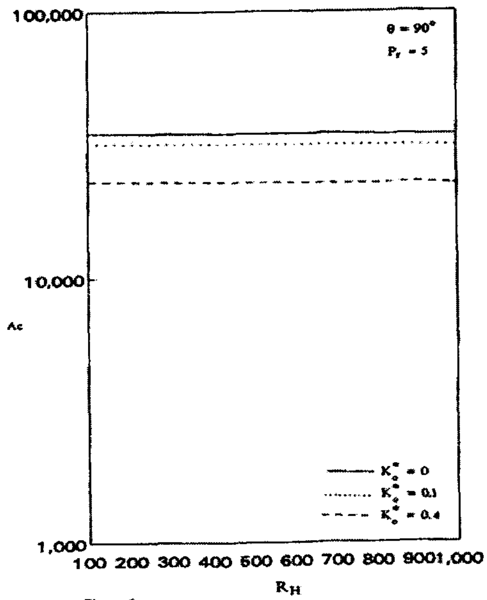

Figure 6
represents the variation of R_{Ac} with R_H for various fixed values of k_o^* and $P_r = 5$, $\theta = 90°$.

Figures 7 and 8 show that for fixed values of K_0^* and P_r, as the heat Rayleigh number increases the critical values of electric Rayleigh number decreases.

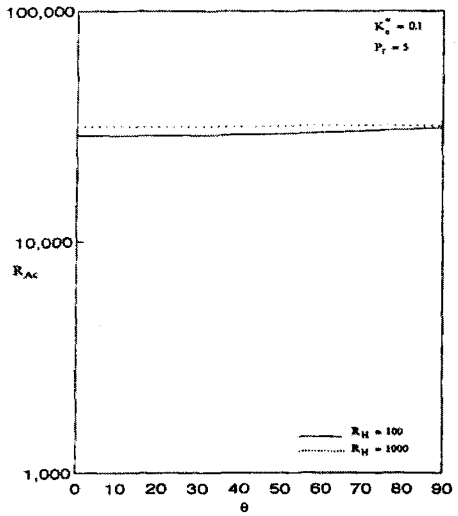

Figure 7
represents the variation of R_{Ac} with θ for various fixed values of R_H and $P_r = 5$, $k_o^* = 0.1$.

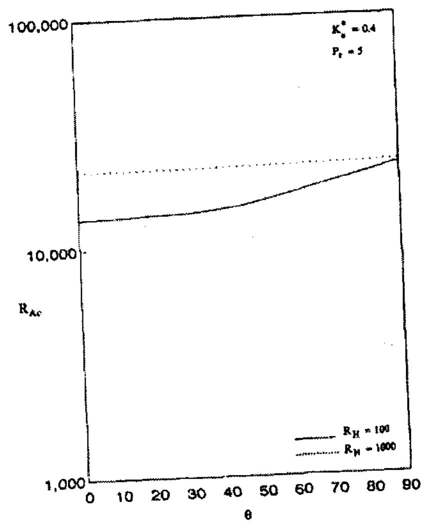

Figure 8
represents the variation of R_{Ac} with θ for various fixed values of R_H and $P_r = 5$, $k_o^* = 0.4$.

5. CONCLUSION

In general one can conclude that:

1. At fixed values of P_r, R_H and for various values of K_o^* it is noticed that the angle of inclination about the vertical axis has instability effect.
2. For fixed P_r, R_H and θ it is shows that the elastic coefficient K_o^* has stabilizing effect.

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