방향정보를 이용한 위치측정의 분석적 방법

A New Analytical Method for Location Estimation Using the Directional Data

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ABSTRACT

This paper presents a new analytical method for estimating the location of a target using directional data. Based on a nonlinear programming (NLP) problem formulated for the line method, which is a well known algorithm for two-dimensional location estimation, we present a method to find an optimal solution for the problem. Then we present a two-stage method for better location estimation based on the NLP problem. In addition, another two-stage method is presented for location estimation problems in which different types of observers are used to obtain directional data based on the analysis of the maximum likelihood estimate of the target location. The performance of the suggested method is evaluated through simulation experiments, and results show that the two-stage method is computationally efficient and highly accurate.

주요기술용어(주제어) : Location Estimation(위치 측정), Directional Data(방향 정보), Nonlinear Programming(비선형 문제)

1. Introduction

The location of a target is vital information for military operations. One way of locating a target is processing directional information pointing to the target. The direction(estimated by the observer) toward the target can be specified by the directional data. Once the directional data is obtained, the location of the target can be computed using a location estimation algorithm.

In this research, we consider an observer that estimates the location of a target by processing the directional data. Direction finder is an example of the observer. An observer estimates the directional data of a radio frequency signal emitted from the target by computing the arrival angle of the signal. Since the directional data can give only one-dimensional information about the location of the target in the two-dimensional location estimation, at least two or more observers are necessary for generating the coordinate of the target.

Several algorithms have been developed for

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location estimation such as the angle method, the line method, the point method and the line-to-point transformation method[2,3]. The angle method is most accurate among these, but it cannot give a closed-form solution because of the difficulty of the problem. The line method, the next accurate one, is originally developed by Stansfield with its closed-form solution and has been widely used[4,5]. Recently, Li and Quek suggest a method of which accuracy is little different from the angle method, but much better computationally efficient[2].

In this paper, we suggest a new analytical method, which is solving a nonlinear programming (NLP) problem formulated for the line method. By the method, the optimal location estimate of the line method is obtained. To make a location estimate as accurate as one by the angle method, we suggest a two-stage method based on the NLP problem. To show the computational efficiency and accuracy of the proposed method, we evaluate it with the angle method through simulation experiments.

2. Preliminaries

We briefly review two well-known algorithms for location estimation. They are the line method and the angle method. Necessary notation is given first. As stated earlier, at least two observers are necessary for location estimation, we denote by n-OB system a system composed of n observers that are used together to estimate the location of a target.

Notation

t location of the target with coordinate(x_t, y_t)

$\mathbf{f}_i$ location of observer i with coordinate(x_i, y_i) for $i = 1,\cdots,n$

$\mathbf{r}_i$ relative location of the target from the viewpoint of observer i with coordinate $(x_i^r, y_i^r) = (x_t - x_i, y_t - y_i)$, i.e., the location of the target when observer i is set to the origin in the coordinate system

e estimated location of the target by a location estimation algorithm with coordinate(x_e, y_e)

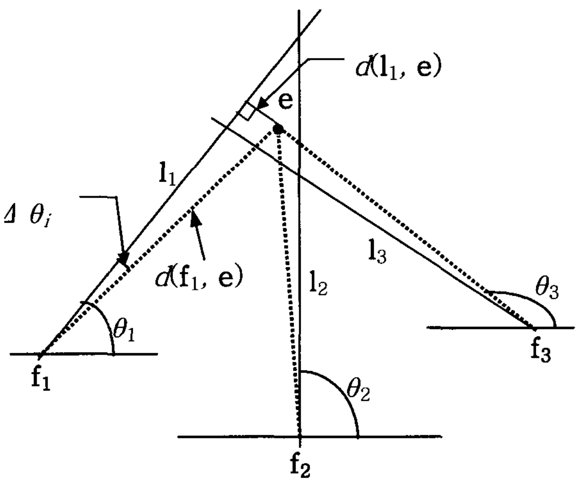
$\mathbf{l}_i$ directional data, line of bearing(LOB), obtained by observer i, i.e., the line starting from observer i toward the observed target

θ_i azimuth angle at which observer i generates $\mathbf{l}_i$, which is measured in counterclockwise from the x-axis

$\Delta\theta_i$ azimuth angular difference between $\mathbf{l}_i$ and the line connecting $\mathbf{f}_i$ and **e**

d(**a**, **b**) shortest distance between points **a** and **b** (or between line **a** and point **b**)

See Figure 1 for a clearer description of some of the above symbols. Here, the origin in the coordinate system can be set to any reference point designated by the system operator.



[Figure 1] Pictorial description of symbols

Line method

When measuring the directional data(LOB) toward the target, the measured direction deviates the target because of errors occurred in the LOB measurement. Thus, the azimuth angle θ_i of the LOB measured by observer i is defined such that $\theta_i = \tan^{-1}\left(x_i^r / y_i^r\right) + \varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_i^2)$, i.e., errors in the LOB measurement follow the normal distribution with mean 0 and variance σ_i^2[6]. Here, σ_i can be regarded as the accuracy of observer i[2].

An optimal location estimate of the target, **e**, is the point that minimizes the sum of squared distances between each LOB and the unknown point itself, i.e., minimizing

$$L(x,y) = \sum_{i=1}^{n} \{d(\mathbf{l}_i, \mathbf{e})\}^2 .$$

The closed-form solution is available. For the solution to find **e**, see Washburn[5].

Angle method

This is known to be the most accurate algorithm using directional data. A location estimate by the angle method is given by the point, which minimizes the sum of squared (azimuth) angular differences between each LOB and the line connecting $\mathbf{f}_i$ and **e**. Note that the angular difference($\Delta\theta_i$), which can be regarded as ε_i, associated with $\mathbf{l}_i$ can be defined(from Figure 1) as

$$\Delta\theta_i = \sin^{-1}\left\{\frac{d(\mathbf{l}_i, \mathbf{e})}{d(\mathbf{f}_i, \mathbf{e})}\right\} .$$

Then estimated location **e** by an n-OB system can be found by minimizing $A(x, y)$ such that

$$A(x,y) = \sum_{i=1}^{n}\left[\sin^{-1}\left\{\frac{d(\mathbf{l}_i, \mathbf{e})}{d(\mathbf{f}_i, \mathbf{e})}\right\}\right]^2 .$$

By assuming $\Delta\theta_i$ is a small value, $\sin(\Delta\theta_i)$ can be approximated with $\Delta\theta_i$, as is done in Li and Quek[2]. Then a location estimate can be obtained alternatively by minimizing the following simplified function,

$$A'(x,y) = \sum_{i=1}^{n}\left\{\frac{d(\mathbf{l}_i, \mathbf{e})}{d(\mathbf{f}_i, \mathbf{e})}\right\}^2 .$$

Since even $A'(x, y)$ is a complicated nonlinear function, a closed-form solution for the angle method is not available[2]. However, an optimal location estimate can be found by using an optimizer in which a search engine is included.

3. The Proposed Method

We first present a new analytical method to derive the optimal location estimate of the line method. In the method, the line method is defined as a nonlinear programming(NLP) problem and a location estimate is obtained by solving the NLP problem.

NLP-based method

The line method is formulated as a nonlinear programming problem. First, the direction vector of $\mathbf{l}_i$, $\mathbf{u}_i \sim (a_i, b_i)$, can be computed such that $a_i = \cos\theta_i$ and $b_i = \sin\theta_i$, where θ_i contains error as described in the line method. Then $\mathbf{l}_i$ of observer i with a direction vector $\mathbf{u}_i$ can be defined as

$$\mathbf{l}_i : y - y_i = \frac{b_i}{a_i}(x - x_i) .$$

Let $\mathbf{h}_i$ be the perpendicular foot from the point $\mathbf{e}$ on $\mathbf{l}_i$. Since $\mathbf{h}_i$ is on $\mathbf{l}_i$, the coordinate of $\mathbf{h}_i$ can be represented as $(a_i/b_i \times k_i + x_i,\ k_i + y_i)$ by introducing a parameter k_i to $\mathbf{h}_i$ specified as

$k_i = y - y_i$ and $k_i = (x - x_i)b_i/a_i$.

Then we obtain a vector

$$\overrightarrow{\mathbf{eh}_i} = \overrightarrow{\mathbf{Oh}_i} - \overrightarrow{\mathbf{Oe}} = \left(a_i/b_i \times k_i + x_i - x_e,\ k_i + y_i - y_e\right).$$

Since $\overrightarrow{\mathbf{eh}_i} \perp \mathbf{l}_i$, the inner product between $\overrightarrow{\mathbf{eh}_i}$ and $\mathbf{u}_i$ is zero. Thus, we obtain following n equations for an n-OB system to determine unknown $\mathbf{e}$ such that

$$\left(\frac{a_i}{b_i} \times k_i + x_i - x_e\right)a_i + \left(k_i + y_i - y_e\right)b_i = 0 \quad \text{for } i = 1, 2, \cdots, n.$$

Thus the line method can be formulated as the following nonlinear programming problem to obtain an optimal location estimate.

[NLP]
Minimize

$$F(\mathbf{x}) = \sum_{i=1}^{n}\left\{\left(a_i/b_i \times k_i + x_i - x\right)^2 + \left(k_i + y_i - y\right)^2\right\}$$

subject to

$$g_i(\mathbf{x}) = \left(a_i/b_i \times k_i + x_i - x\right)a_i + \left(k_i + y_i - y\right)b_i = 0 \quad \forall i$$

The objective function is the same as the line method because $d(\mathbf{l}_i, \mathbf{e})$ is just the distance between $\mathbf{e}$ and $\mathbf{h}_i$, and n equality constraints are derived for an n-OB system to determine k_i. Let $\mathbf{x}^*$ be a local minimum found for [NLP], which can be obtained by applying the Kuhn-Tucker optimality condition[1] for problems having only equality constraints such that

$$\nabla F(\mathbf{x}^*) + \sum_{i=1}^{n} \lambda_i^* \nabla g_i(\mathbf{x}^*) = 0$$

where $\nabla F(\mathbf{x}^*)$ and $\nabla g_i(\mathbf{x}^*)$ are the gradient vector of $F(\mathbf{x})$ and $g_i(\mathbf{x})$ for $i=1,\cdots,n$, respectively, in terms of $\mathbf{x}^* = (x^*, y^*, k_1^*, k_2^*, \ldots, k_n^*)$ such as

$$\nabla F(\mathbf{x}^*) = \left\{\frac{\partial F(\mathbf{x}^*)}{\partial x^*}, \frac{\partial F(\mathbf{x}^*)}{\partial y^*}, \frac{\partial F(\mathbf{x}^*)}{\partial k_1^*}, \ldots, \frac{\partial F(\mathbf{x}^*)}{\partial k_n^*}\right\}$$

$$\nabla g_i(\mathbf{x}^*) = \left\{\frac{\partial g_i(\mathbf{x}^*)}{\partial x^*}, \frac{\partial g_i(\mathbf{x}^*)}{\partial y^*}, \frac{\partial g_i(\mathbf{x}^*)}{\partial k_1^*}, \ldots, \frac{\partial g_i(\mathbf{x}^*)}{\partial k_n^*}\right\}.$$

Note that [NLP] is a convex programming since its objective function is convex and all the constraints are linear. Therefore, the solution for $\mathbf{x}^*$ is global optimal. With obtained $(n+2)$ linear equations resulting from the Kuhn-Tucker optimality condition and n equations from the constraints, total $(2n+2)$ linear equations are available to determine $\mathbf{x}^*$ and a Lagrange multiplier vector $(\lambda_i^*, \cdots, \lambda_n^*)$.

An optimal solution for [NLP] can be easily computed by solving the linear equations. For example, optimal location for an 3-OB system can be computed with the linear system, $\mathbf{Ax}=\mathbf{b}$, such that

$$\begin{bmatrix} 6 & 0 & -2s_1 & -2s_2 & -2s_3 & -a_1 & -a_2 & -a_3 \\ 0 & 6 & -2 & -2 & -2 & -b_1 & -b_2 & -b_3 \\ -2s_1 & -2 & q_1 & 0 & 0 & p_1 & 0 & 0 \\ -2s_2 & -2 & 0 & q_2 & 0 & 0 & p_2 & 0 \\ -2s_3 & -2 & 0 & 0 & q_3 & 0 & 0 & p_3 \\ -a_1 & -b_1 & p_1 & 0 & 0 & 0 & 0 & 0 \\ -a_2 & -b_2 & 0 & p_2 & 0 & 0 & 0 & 0 \\ -a_3 & -b_3 & 0 & 0 & p_3 & 0 & 0 & 0 \end{bmatrix}$$

$$=\begin{bmatrix} x \\ y \\ k_1 \\ k_2 \\ k_3 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \begin{bmatrix} 2(x_1+x_2+x_3) \\ 2(y_1+y_2+y_3) \\ -2s_1x_1-2y_1 \\ -2s_2x_2-2y_2 \\ -2s_3x_3-2y_3 \\ -a_1x_1-b_1y_1 \\ -a_2x_2-b_2y_2 \\ -a_3x_3-b_3y_3 \end{bmatrix} \text{in which } s_i = a_i/b_i,$$

$p_i = s_ia_i + b_i$ and $q_i = 2s_i^2 + 2$.

Two-stage method: homogeneous observers

To enhance the accuracy of the line method, i.e., [NLP], we need to give a weight to $\mathbf{l}_i$ according to its importance. Note that the angle method is the one such that each $\mathbf{l}_i$ is given a weight of $1/d(\mathbf{f}_i, \mathbf{e})$, i.e., a weighted line method, from the description of the angle method, $A'(x, y)$, as stated earlier.

Since $d(\mathbf{f}_i, \mathbf{e})$ contains unknown variable (x_e, y_e), the coordinate of a target to be estimated, an analytical solution cannot be obtained by the NLP-based method because nonlinear equations occur. However, Li and Quek[2] argue that using estimated distances by a location estimation algorithm dost not significantly degrade the accuracy of location estimation. Therefore, if we use the distance between observer i and the estimated location obtained by solving [NLP] as weights, then we can obtain an optimal location estimate by solving the weighted line method, i.e., an approximate one of the angle method, as following.

[WNLP]

Minimize

$$\sum_{i=1}^{n}\left\{\frac{(a_i/b_i \times k_i + x_i - x)^2 + (k_i + y_i - y)^2}{d_i^2}\right\}$$

subject to

$$(a_i/b_i \times k_i + x_i - x)a_i + (k_i + y_i - y)b_i = 0 \qquad \forall i$$

Here, d_i denotes $d(\mathbf{f}_i, \mathbf{e})$ where e is the solution for [NLP] obtained beforehand.

The optimal solution for [WNLP] can be obtained similarly as [NLP] since a constant d_i is added in the objective function. For example, an optimal location estimate for an 3-OB system can be computed with the linear system, **Ax**=**b**, such that

$$\begin{bmatrix} 2\eta & 0 & -2\xi_1s_1 & -2\xi_2s_2 & -2\xi_3s_3 & -a_1 & -a_2 & -a_3 \\ 0 & 2\eta & -2\xi_1 & -2\xi_2 & -2\xi_3 & -b_1 & -b_2 & -b_3 \\ -2\xi_1s_1 & -2\xi_1 & q_1' & 0 & 0 & p_1 & 0 & 0 \\ -2\xi_2s_2 & -2\xi_2 & 0 & q_2' & 0 & 0 & p_2 & 0 \\ -2\xi_3s_3 & -2\xi_3 & 0 & 0 & q_3' & 0 & 0 & p_3 \\ -a_1 & -b_1 & p_1 & 0 & 0 & 0 & 0 & 0 \\ -a_2 & -b_2 & 0 & p_2 & 0 & 0 & 0 & 0 \\ -a_3 & -b_3 & 0 & 0 & p_3 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ k_1 \\ k_2 \\ k_3 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} 2(\xi_1x_1+\xi_2x_2+\xi_3x_3) \\ 2(\xi_1y_1+\xi_2y_2+\xi_3y_3) \\ -2\xi_1s_1x_1-2\xi_1y_1 \\ -2\xi_2s_2x_2-2\xi_2y_2 \\ -2\xi_3s_3x_3-2\xi_3y_3 \\ -a_1x_1-b_1y_1 \\ -a_2x_2-b_2y_2 \\ -a_3x_3-b_3y_3 \end{bmatrix} \text{where } \xi_i = 1/d_i^2,$$

$\eta = \xi_1 + \xi_2 + \xi_3$ and $q_i' = 2\xi_is_i^2 + 2\xi_i$.

The solution can be used directly to the direction finder system in Korean Army in which three direction finders are operated together to locate a target.

Two-stage method: heterogeneous observers

When heterogeneous observers are operated together, it is necessary to consider not only distances between the observer and target but also the accuracies of the observers to reflect the importance of the directional data in the proposed two-stage method. The idea of how to take account of the accuracy for weighting can be obtained by analyzing the maximum likelihood estimate of the location of a target.

Directional data obtained by observer i can be defined as a probability density function with an unknown parameter **t**, the location of a target with coordinate (x_t, y_t), such as

$$f(\theta_i, \mathbf{t}) = \frac{1}{\sqrt{2\pi}\sigma_i}\exp\left\{\frac{-(\theta_i-\mu_i)^2}{2\sigma_i^2}\right\},$$

where θ_i is the (azimuth) angular direction toward the target measured by observer i and μ_i is the mean direction that can be defined as $\tan^{-1}\{(x_t - x_i)/(y_t - y_i)\}$ and σ_i is the accuracy of observer i. Then the likelihood of a set of n directional data measured by n observers is therefore

$$Q(\theta_1, \ldots, \theta_n, \mathbf{t}) = \prod_{i=1}^{n} f(\theta_i, \mathbf{t}) =$$
$$\left(\frac{1}{\sqrt{2\pi}\sigma_1}\right)\cdots\left(\frac{1}{\sqrt{2\pi}\sigma_n}\right)\exp\left\{-\sum_{i=1}^{n}\frac{(\theta_i-\mu_i)^2}{2\sigma_i^2}\right\}.$$

So, the log-likelihood is

$$\ln(Q) = -\sum_{i=1}^{n}\ln\left(\sqrt{2\pi}\sigma_i\right) - \frac{1}{2}\sum_{i=1}^{n}\frac{(\theta_i-\mu_i)^2}{\sigma_i^2}.$$

Since the first term in $\ln(Q)$ is a constant, to maximize $\ln(Q)$ is equal to minimize the second term

$$\sum_{i=1}^{n}\frac{(\theta_i-\mu_i)^2}{\sigma_i^2}.$$

Note that the numerator $|\theta_i - \mu_i|$ can be regarded as the angular difference, $\Delta\theta_i$, between the directional data of observer i and the line connecting $\mathbf{f}_i$ and the unknown target's location, thus $|\theta_i - \mu_i|$ can be defined as

$$\sin^{-1}\left\{\frac{d(\mathbf{l}_i, \mathbf{e})}{d(\mathbf{f}_i, \mathbf{e})}\right\}.$$

Therefore, the log-likelihood is maximized by minimizing

$$\sum_{i=1}^{n}\left\{\frac{d(\mathbf{l}_i, \mathbf{e})}{d(\mathbf{f}_i, \mathbf{e})\sigma_i}\right\}^2$$

by approximating $\sin(\Delta\theta_i)$ with $\Delta\theta_i$(as described earlier in this paper). In result, the accuracy should be treated as the same manner as $d(\mathbf{f}_i, \mathbf{e}) \equiv d_i$ to enhance the performance of the two-stage method when heterogeneous observers are used together.

The solution of the two-stage method for heterogeneous observers can be developed by simply changing the weight d_i^2 to $(\sigma_i d_i)^2$ in the objective function in [WNLP] as following.

[HNLP]
Minimize

$$\sum_{i=1}^{n}\left\{\frac{(a_i/b_i \times k_i + x_i - x)^2 + (k_i + y_i - y)^2}{(\sigma_i d_i)^2}\right\}$$

subject to

$$(a_i/b_i \times k_i + x_i - x)a_i + (k_i + y_i - y)b_i = 0 \qquad \forall i$$

Then a location estimate is the solution of the linear system, i.e., the matrix **A** and **b** in which the element of d_i^2 replaced by $(\sigma_i d_i)^2$.

4. Performance Evaluation

The proposed two-stage method is evaluated through simulation experiments. As a performance measure, we use $P(r)$, the ratio of location estimates that are enclosed in a circle of radius r (centered on the target location) among a large number of times such location estimation, i.e., the number of replications. Since the value of $P(r)$ shows the approximate probability that the target exists in a specified circular area centered on a location estimate, algorithms can be compared more practically for a certain sized area than using the mean distance-deviation of estimates of commonly used.

Three methods for location estimation are compared. They are the line method, the two-stage method and the angle method. A location estimate of the line method is obtained by solving [NLP]; a location estimate of the two-stage method is obtained by solving [WNLP] and/or [HNLP]. To obtain a location estimate of the angle method, we use a commercial optimizer, the Solver embedded in Microsoft Excel.

The angle method can be formulated as following.

Minimize

$$\sum_{i=1}^{n}\left[\sin^{-1}\left\{\frac{\sqrt{(a_i/b_i\times k_i+x_i-x)^2+(k_i+y_i-y)^2}}{\sqrt{(x_i-x)^2+(y_i-y)^2}}\right\}\right]^2$$

subject to

$$(a_i/b_i\times k_i+x_i-x)a_i+(k_i+y_i-y)b_i=0 \qquad \forall i$$

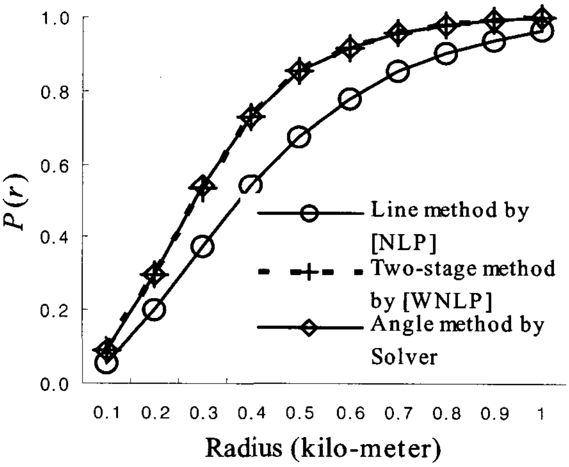
The objective function is just the original form of the angle method as described earlier. When heterogeneous observers are used together, the accuracy should be considered like a weight by modifying the term $\sqrt{(x_i-x)^2+(y_i-y)^2}$ to $\sigma_i\sqrt{(x_i-x)^2+(y_i-y)^2}$. To speed up the search procedure by Solver, the resulted location estimate of the line method, i.e., [NLP], is initialized.

We coded the algorithms for solving [NLP], [WNLP] and [HNLP] in Visual Basic and executed on a personal computer with a Pentium 4 processor operating at 2.4GHz clock speed.

The simulation experiments were performed on two cases of a 3-OB system, one for homogeneous observers and the other for heterogeneous observers. For each case, $P(r)$ was computed for each method with results from 100,000 replications of location estimation.

In the first case (Case 1), accuracies of observers were set to one degree for three homogeneous observers and the locations of the observers were set to (18, 18), (60, 8), (30, 8) in kilo-meters, respectively, and the location of the target was set to (25, 25).

Results of the simulation are given in Figure 2, which shows the values of $P(r)$ obtained for different radii for each method. The two-stage

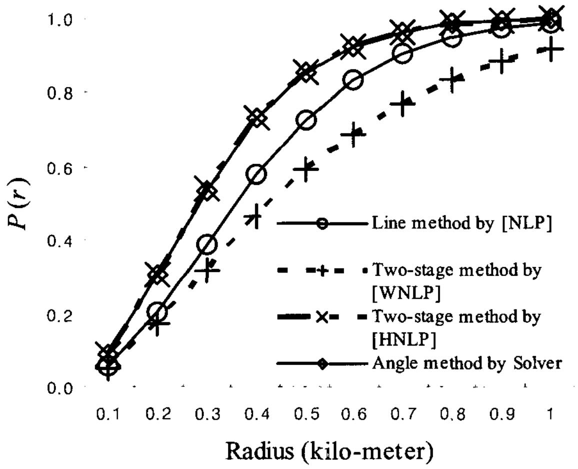


[Figure 2] Simulation results(Case 1)

method using [WNLP] gave location estimates that were as accurate as those from the angle method within much shorter computation time. Computation times required for computing 100,000 location estimates were 72 seconds for the line method using [NLP], 144 seconds for the two-stage method using [WNLP] and 17,350 seconds for the angle method using Solver.

In the second case (Case 2) in which there are three heterogeneous observers, accuracies of the observers, i.e., σ_1, σ_2 and σ_3, were set to 1, 0.5 and 2 degrees, respectively. The locations of the observers and target are the same as those of the first case. To see the effect of proper weighting in the two-stage method, we compared four methods, [NLP], [WNLP], [HNLP] and the angle method. In the angle method, accuracies of the observers were properly considered in the objective function formulated in Solver.

Results of the simulation with 100,000 location estimates are shown in Figure 3. The method [WNLP], which was falsely weighted in this case by considering $1/d(\mathbf{f}_i, \mathbf{e})$ only to each directional data and not considering the accuracy of the observer, was outperformed by the line method.



[Figure 3] Simulation results(Case 2)

In other words, the angle method in which the accuracy of observers is not considered does not work well and work worse than the line method in certain cases if heterogeneous observers are used together. This suggests that it would be better not to give weights at all rather than to give inappropriate or inaccurate weights to directional data.

The two-stage method using [HNLP], which was properly weighted by both $1/d(\mathbf{f}_i, \mathbf{e})$ and σ_i, gave as accurate location estimates as those from the angle method (with reflecting the accuracy in its formulation). Again, the former required much shorter time than the latter. Computation times required for obtaining 100,000 location estimates using [NLP], [WNLP], [HNLP] and the angle method (using Solver) were 72, 144, 144 and 17,350 seconds, respectively.

For detailer comparison among the considered methods in the simulation experiments in Case 2, it is shown in Table 1 that the value of $P(r)$ and its percentage deviation for each method from that of the best one among the four methods for given r. Again, it can be checked there is little difference in the percentage deviation between [HNLP] and the angle method.

5. Conclusion

We presented a new analytical method for a location estimation problem. The line method is formulated as a nonlinear programming (NLP) problem and then an optimal location estimate is obtained by applying the Kuhn-Tucker optimality condition on the problem.

By regarding the angle method as a weighted line method as done by Li and Quek, we suggested a two-stage method for location estimation based on the NLP problem. From the

[Table 1] Comparison result by $P(r)$(Case 2)

r	[NLP]		[WNLP]†		[HNLP]		Angle method	
	$P(r)$	PD††	$P(r)$	PD	$P(r)$	PD	$P(r)$	PD
0.1	0.055	37.00	0.050	42.96	0.087	0.11	0.087	0.00
0.2	0.203	33.85	0.172	43.81	0.306	0.07	0.306	0.00
0.3	0.391	27.44	0.322	40.27	0.538	0.00	0.538	0.04
0.4	0.578	20.83	0.465	36.28	0.730	0.00	0.730	0.04
0.5	0.727	14.62	0.591	30.59	0.852	0.00	0.852	0.02
0.6	0.835	9.76	0.690	25.50	0.925	0.01	0.926	0.00
0.7	0.906	6.08	0.768	20.40	0.965	0.00	0.964	0.04
0.8	0.949	3.66	0.833	15.47	0.985	0.00	0.985	0.00
0.9	0.972	2.10	0.885	10.94	0.993	0.00	0.993	0.00
1.0	0.985	1.19	0.920	7.80	0.997	0.01	0.997	0.00
Avg		15.65		27.40		0.02		0.01

† can be regarded as the angle method in which the accuracy of observers is not considered
†† percentage deviation of $P(r)$ obtained for each location estimation method from that of the best one among four methods for given r

maximum likelihood estimate of the location of the target, the accuracies of observers are reflected as weights to directional data for better location estimation. Experimental results show that the proposed two-stage method is as accurate as the angle method, and much better computationally efficient.

As for future research, a three-dimensional location estimate and its related solution form can be obtained by extending the proposed NLP-based method into the three-dimensional case.

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