할당문제 해법을 이용한 부분적 선후관계가 있는 군사훈련 일정 수립에 관한 연구

Scheduling for Military Training of Serials with Partial Precedence Relationships based on the Assignment Problem

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ABSTRACT

This paper focuses on a scheduling problem of military training. Repetitive and identical training over multiple serials is a common type of military education. A simple but systematic method is suggested to determine a training schedule for small groups divided from each serial. A satisfactory training schedule and the number of such small groups can be determined by iteratively solving assignment problems with additional constraints. With this method, loads of instructors can also be balanced without violating constraints associated with precedence and continuity relationships among lectures.

주요기술용어(주제어) : Training Scheduling(훈련 일정), Assignment Problem(할당 문제), Partial Precedence Relationship(부분적 선후관계), Serial Scheduling(기수 일정)

1. Introduction

About one hundred thousands of recruits are trained by the army every year in Korea. A serial composed of 800 to 900 recruits enters into the Army Training Center(ATC) every two or three weeks. To teach the recruits basic military knowledge necessary for the remaining period of military service, a recruit-training program has been designed and is being executed in the training center.

Differently from other military trainings, the recruit-training focuses on personal education, i.e., providing each recruit with a grounding to be a soldier. Therefore, recruits are randomly assigned to a trainer(meaning a training unit) with no organizational concept for executing any tactical plan. Some recruits trained by the ATC finally go to a tactical military unit by which a tactical training or exercise is carried out under its own preplanned operational concept.

Constructing training schedules for the serials is a basis for executing a training program, which is composed of several subjects specified

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by the Army. For a training schedule, it is necessary to consider some peculiar conditions, which are different from other common education programs. First, the order of training subjects is not fixed, but may be changeable only if relationships for both between subjects and between lectures in a subject are met. The relationships specify that some subjects should be completed earlier prior to a certain subject; and lectures of a subject should be given continuously without interruption by any other lecture. Second, there exists flexibility in assigning instructors to subjects since an instructor is capable of teaching various subjects. Finally, the class size of trainees can be adjustable considering available number of instructors so that the effectiveness of the recruit-training be enhanced by putting recruits into smaller groups(or classes) if a feasible schedule can be found within a given training period.

A subject in a military training program consists of one or more lectures. The training program is carried out by a training schedule by which a lecture is assigned to a certain time slot and an instructor. Currently, a training program for a serial is completed in six weeks. Each serial is assigned to a training battalion and goes under a training schedule that is identical to all the members of the serial. By the army's education guidelines, the training program is divided into three stages and there exists partial precedence relationships among subjects for each stage.

Currently, recruits of the same battalion, i.e., members of a serial, are trained under one training schedule, which is a simple ordering of subjects. Planners of a regiment confirm the training schedule for each battalion by coordinating conflicts or overlaps between training schedules planned by battalions since some training fields cannot be used in common. Then all training activities are conducted by the companies of a battalion. Instructors of each company have to train about 200 recruits assigned to the company according to the training schedule of the battalion. So far, there has been no serious scheduling problem in the current training system.

However, the current training system may result in unequal qualities of trained personnel because of three reasons. First, the quality of training is different for different companies since the recruits of a company are trained by the company's own instructors. Secondly, it is not easy to find specialized instructors because each instructor in a company is responsible for three or more subjects. Finally, effectiveness of training may not be very high since a large number of trainees, say two hundreds recruits, are trained together at the same time.

To produce equally well trained recruits for their better performance in future military service, a specialized training system is required. Since an instructor can teach a subject more effectively by giving a lecture to a smaller group, say that of the size of a platoon, each serial needs to be divided into multiple subgroups, i.e., sections. Then the sections are trained by instructors who are under the direction of a centralized control of a higher echelon, battalion for example.

In this paper, we suggest a training system of training a small group of a reasonable number of trainees with specialized instructors under a well constructed training schedule. The number of sections can be determined by solving scheduling problems iteratively using a solution approach to 0-1 assignment problems with constraints related to the length of planning horizon, the number of available instructors and precedence relationships among subjects.

2. Military training scheduling problem

The military training scheduling problem (MTSP) is the problem of constructing a schedule for military training for a serial with certain relationships among subjects or lectures. There are two types of such relationships, *precedence* relationships and *continuity* relationships. Precedence relationships specify the order of education between two subjects. On the other hand, when two lectures needs to be trained continuously without any time break, the continuity relationship occurs. The MTSP is formulated as a 0-1 assignment problem with additional constraints. The objective is to minimize the time required to complete a given training program.

To formulate the MTSP as an assignment problem, a subject containing two or more lectures is decomposed into unit-time lectures and each lecture is assigned to an *imaginary* instructor, who is responsible for only one lecture of a certain subject. Then an MTSP is formulated for given numbers of instructors, sections and a length of the planning horizon. First, we give notation needed.

Notation

H length of the planning horizon

X_{ijt} is equal to 1 if imaginary instructor i gives a lecture of a unit time to section j at time t, $1 \le t \le H$.

n total number of imaginary instructors

m total number of sections

l_i lecture given by imaginary instructor i

$\bar{l}_k$ lectures of subject k

y_{ij} time slot when lecture l_i is completed by section j, $y_{ij} = \sum_{t=1}^{H} X_{ijt} t$ for $1 \le i \le n$ and $1 \le j \le m$

c_j time slot when the training program is completed by section j, $c_j = \max_i (y_{ij})$ for $1 \le j \le m$

P set of precedence relationships between two subjects (or lectures) such that $P = \{(k', k'') : \bar{l}_{k'}$ precedes $\bar{l}_{k''}\}$ if k' and k'' are indexes of subjects; $P = \{(i', i'') : l_{i'}$ precedes $l_{i''}\}$ if i' and i'' are indexes of imaginary instructors

C set of continuity relationships between two lectures, $C = \{(i^*, i^{**}) : l_{i^{**}}$ starts right after completing $l_{i^*}\}$

Now we give an integer programming formulation for the MTSP.

Formulation

Minimize Y

subject to

$$c_j \le Y \tag{1}$$

$$\sum_{j=1}^{m} X_{ijt} \le 1 \quad \text{for } \forall i,t \tag{2}$$

$$\sum_{i=1}^{n} X_{ijt} \le 1 \quad \text{for } \forall j,t \tag{3}$$

$$\sum_{t=1}^{H} X_{ijt} \le 1 \quad \text{for } \forall i,j \tag{4}$$

$$\sum_{t=1}^{H} X_{i''jt} t - \sum_{t=1}^{H} X_{i'jt} t > 0 \quad \text{for } \forall (i',i'') \in P \tag{5}$$

$$\sum_{t=1}^{H} X_{i^{**}jt} t - \sum_{t=1}^{H} X_{i^{*}jt} t = 1 \quad \text{for } \forall (i^*,i^{**}) \in C \tag{6}$$

$$X_{ijt} \in \{0, 1\} \quad (7)$$

Constraint (1) guarantees that the training program for all sections are completed earlier than or equal to time Y and constraints (2), (3) and (4) make assignment decisions feasible by letting only one lecture be assigned to a section at each time slot for the entire planning horizon. Precedence relationships and continuity relationships among the lectures are considered in (5) and (6), respectively.

Since a subject containing two or more lectures is decomposed into unit-time lectures and each lecture is assigned to an imaginary instructor, schedules for the imaginary instructors associated with the subject need to be combined together into a schedule for one actual instructor to obtain a feasible schedule. Let S_k denote the set of imaginary instructors associated with subject k. Then the following constraint is needed to combine imaginary instructors of subject k into one instructor.

$$\sum_{j=1}^{m} X_{i'jt} - \sum_{j=1}^{m} X_{i''jt} \leq 1 \quad \text{for } \forall (i', i'') \in S_k, t \quad (8)$$

The constraint prevents any time overlap among lectures given by imaginary instructors in S_k because only one imaginary instructor can give a lecture at any time slot.

In addition, by using constraint (8) with constraints (5) and/or (6), *load balancing* among instructors can be achieved, which means that lectures of a subject of an overloaded instructor are reallocated to another instructor by designating him/her as a secondary instructor for the subject.

Since the length of the training program can be flexibly determined within an acceptable range by planners, we do not need to shorten the time required to complete the training program. Rather, we try to enhance the effectiveness of the training system by maximizing the number of sections so that the number of recruits in a section becomes as small as possible close to the size of a platoon, which is suitable for most military training. We suggest an iterative procedure by which an MTSP is solved in a limited computation time (T) for various levels of tightness of the constraints.

An iterative procedure for the MTSP

Step 1 : Formulate an MTSP with given n, m and H.

Step 2 : Solve the MTSP with a (commercial) integer programming solver. If a feasible solution with a training period of $H' \leq H$ is obtained in time T, go to Step 4. Otherwise, go to Step 3.

Step 3 : Relax the level of tightness of the constraints by increasing n by one with reallocating lectures to the added instructor and/or lengthen H by one, go to Step 1.

Step 4 : Add a constraint, $H' \leq Y$, and increase m by one, then go to Step 1.

3. A numerical example of a real application

An example for a real application of the iterative procedure is presented using actual data obtained in the ATC at Non-san city. The current training program consists of three stages and each stage lasts about two weeks. Since the training program for a stage is constructed independently by the army's education guidelines, the whole training schedule can be made by combining three schedules of the three stages.

This reduces the problem size of the MTSP significantly. One unit time in the MTSP represents four hours of actual training in the example.

We try to find a training schedule by which the first stage of the training program can be carried out. In the example problem, there are seven subjects and each subject composes of one or more lectures. For partial precedence relationships, subject 2 precedes subject 3, subject 5 precedes subject 1 and subject 5 precedes subject 3. Thus, the set of precedence relationships is P = {(2,3,(5,1),(5,3)}. Since the instructor of subject 1 has six lectures, a new instructor is added to the training program to take charge of later three lectures of subject 1, which generates a new subject, subject 8, with precedence relationship, (1,8). For load balancing, two lectures of subject 3 are reassigned to the instructor of subject 6 because he/she has only one lecture. The instructor of subject 6 should give two lectures of subject 3 after lectures of subject 2 because of precedence relationship, (2, 3).

Table 1 shows the numbers of lectures of seven subjects assigned to instructors before and after load balancing. Twenty imaginary instructors are generated by decomposing subjects into twenty unit-length lectures and they are indexed from 1 to 20 as shown in the last column of the table. As a result, the problem has 20 imaginary instructors and four precedence relationships, P = {(2,3),(5,1),(5,3),(2,6)} in which (2,6) partially constrains that only two lectures of subject 6 corresponding to imaginary instructor 9 and 10 should be given after the lectures of subject 2. The continuity constraint between lectures is not considered in the example.

The problem with a planning horizon (H) of 26 was solved initially using CPLEX, a commercial software package for integer programming problems, starting from four sections (m = 4). The iterative procedure was run for approximately ten minutes on a personal computer with a Pentium IV 2400 processor. After the first iteration, we obtained a training schedule of 20 training period for four sections in 150 seconds.

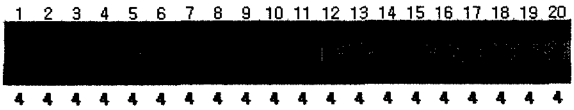
[Table 1] Lecture assignment before and after load balancing

No. of subject	Number of lectures		No. of imaginary instructor
	Before load balancing	After load balancing	
1	6	3	1, 2, 3
2	3	3	4, 5, 6
3	4	2	7, 8
4	2	2	11, 12
5	2	2	13, 14
6	1	3	9†, 10†, 15
7	2	2	16, 17
8‡		3	18, 19, 20

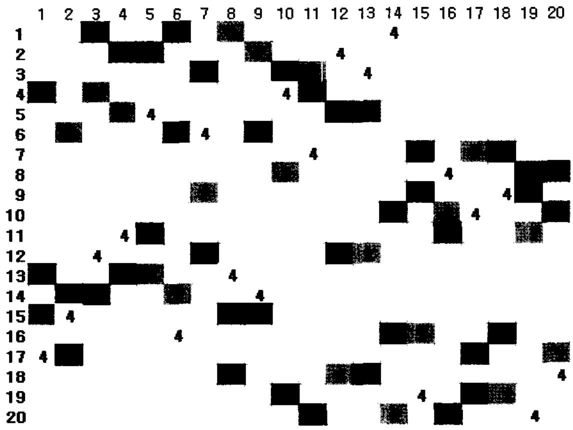
† Imaginary instructors 9 and 10 are assigned to two lectures of subject 3 for load balancing.

‡ Some lectures of subject 1 are regarded as subject 8, which is a part of subject 1.

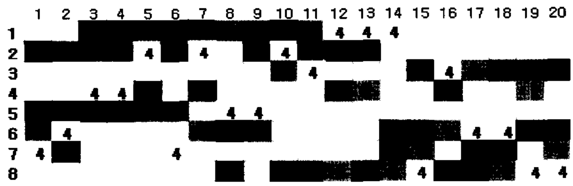
Figure 1-(a) shows how lectures are given during the 20 training period for each section. Note that the schedule is optimal since there is no break in the schedule for all sections. Figure 1-(b) shows how an imaginary instructor gives lectures to sections. Schedules for (actual) instructors were constructed by combining the schedules of imaginary instructors of the same subject as shown in Figure 1-(c). For example, the first three rows of Figure 1-(b) were combined into the first row of Figure 1-(c) since imaginary instructors 1, 2 and 3 were corresponding to subject 1 as shown in table 1.



[Figure 1-(a)] A schedule of four sections shown for each section



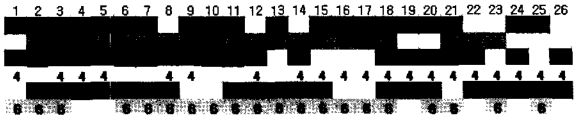
[Figure 1-(b)] A schedule of four sections shown for each imaginary instructor



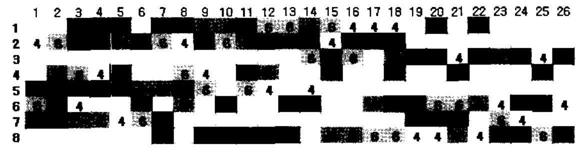
[Figure 1-(c)] A schedule of four sections shown for subject

After the second iteration, a schedule of 25 training period was obtained for five sections. A final schedule of 26 training period for six sections was obtained after the third iteration. Figure 2-(a) shows the schedule for each section, and Figure 2-(b) shows when each (actual) instructor gives a lecture to which section.

As expected, a longer training period was required if recruits were divided into a greater number of sections when the number of instructors was fixed. Note that about 200



[Figure 2-(a)] A schedule of six sections shown for each section



[Figure 2-(b)] A schedule of six sections shown for each subject

recruits as a class, which can be viewed as one section, are currently trained by the training company according to the training schedule given by the training battalion. Based on the result in Figure 2-(c), the recruits in a company, however, can be actually trained with six sections using its own instructors only if it is allowed for a company to have its own schedule.

By the schedule, recruits can be equally trained by specialized eight instructors because each instructor is in charge of only one subject of load balanced. Moreover, the quality of training may be enhanced since each instructor deals with only 34 (200divided by 6) recruits rather than 200 recruits during a lecture while the current company-based training system in the ATC is preserved. With a larger pool of instructors, a battalion-based training system can be designed as well by dividing a serial into a number of sections, say 15~20 sections. In this case, the company may be responsible only for taking care of recruits in barrack life not training.

4. Concluding remarks

We presented a simple procedure to solve the military training scheduling problem(MTSP) with partial precedence relationships among subjects. Solutions of the problem provide planners with relatively good schedules by solving the MTSP iteratively. Educating a military subject can become more effective by giving lectures to a small group with a specialized instructor for the lectures.

To implement the training schedules obtained with the method suggested in this study, it is recommended that the current training system be well prepared for the specialization of its educational function, i.e., managing a group of specialized instructors for educating multiple sections partitioned from a group of recruits. The proposed method can be an alternative to cope with the current changes in military situation, for example, the reduction in the period of service of enlisted men.

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