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Application of Analytic Hierarchy Process for the Selection of Cotton Fibers

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Abstract: In many engineering applications, the final decision is based on the evaluation of a number of alternatives in terms of a number of criteria. This problem may become very intricate when the selection criteria are expressed in terms of different units or the pertinent data are difficult to be quantified. The Analytic Hierarchy Process (AHP) is an effective way in dealing with such kind of complicated problems. Cotton fiber is selected or graded, in the spinning industries, based on several quality criteria. However, the existing selection or grading method based on Fiber quality Index (FqI) is rather crude and ambiguous. This paper presents a novel approach of cotton fiber selection using the AHP methodology of Multi Criteria Decision Making.

Keywords: Analytic Hierarchy Process, Cotton fiber, Elongation, Micronaire, Tenacity, Uniformity index, Upper half mean length

Introduction

Cotton is a natural fiber having variability galore in its properties. Most of these properties play a decisive role in determining the tensile and evenness characteristics of spun yarns. For example, yarn strength, which is considered to be the most important property of spun yarns, is largely influenced by the tenacity, elongation, length, length uniformity, short fiber content and fineness (micronaire or microgram/inch) of constituent cotton. For ring spun yarns, the contribution of fiber tenacity, elongation, length, length uniformity and micronaire to yarn tenacity is 20 %, 5 %, 22 %, 20 % and 15 %, respectively [1]. Therefore, the selection of suitable cotton fibers gives rise to a situation, which involves Multi Criteria Decision Making (MCDM). A cotton fiber may have an edge over the other alternatives with respect to a particular criterion. However, the relative dominance may be toppled when another criterion is taken into consideration. Under these circumstances, our aim will be to search a cotton fiber, which will have an overall preponderance over the other alternatives.

In a simple MCDM situation, all the criteria are expressed in terms of same unit (e.g., kg or dollars). However, in many real life MCDM problems, different criteria may be expressed in different units. Examples of such dimensions encompass gm, cm, rupees etc. It is this issue of multiple dimensions, which makes the typical MCDM problem to be a complex one and the Analytic Hierarchy Process (AHP) or its variants may offer great assistance in solving this type of problem [2-5]. Generally, the overall quality of cotton fiber is measured by a dimensionless parameter known as Fiber Quality Index (FQI). To obtain the FQI value of a cotton fiber the product of 50 % span length, bundle strength and maturity is divided by the fineness (micronaire) value [6]. Therefore, in FQI length, strength and maturity of cotton fiber receives equal importance irrespective of their influence on the final yarn quality. Moreover, the contribution of various fiber parameters to yarn quality varies with the type of yarn spinning technology. For example, the influence of fiber length to yarn tenacity is more predominant in ring spun yarns as compared to rotor spun yarns [7,8]. In a stark contrast to FQI system, the decision maker in AHP could assign different weights to the fiber parameters depending on their importance and separate models could be developed for ring and rotor spun yarns. In this work, an attempt has been made to select the best cotton fiber for ring yarns using the AHP. The ranking of cotton fibers has also been done from the global priority of alternatives.

The Analytic Hierarchy Process (AHP)

AHP is a MCDM approach and was introduced by Saaty [9-14]. It uses multilevel hierarchical structure of objectives or goal, criteria, sub-criteria and alternatives. The pertinent data are derived by using a set of pair-wise comparison. These comparisons are used to obtain the weights of importance of the decision criteria and the relative performance measures (scores) of the alternatives in terms of each decision criterion. Although some researchers [15-18] have raised concerns over the theoretical basis of AHP, it has proven to be an extremely useful method for decision-making.

Four steps of AHP methodology are as follows:

1) Build a decision "hierarchy" by breaking or decomposing the problem into various components i.e., objective or goal, criteria, sub-criteria and alternatives.
2) Gather relational data for the decision criteria and alter-

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natives and encode them using the AHP relational scale.

3) Estimate the relative priorities (weights) of the decision criteria and alternatives.
4) Perform the composition (synthesis) of priorities of criteria and alternatives, which gives the rank of the alternatives with respect to the objective of the problem.

Details of AHP methodology is presented below:

Step 1: Develop the hierarchical representation of the problem. The overall objective of the problem is positioned at the top of the hierarchy and the decision alternatives are placed at the bottom. Between the top and bottom levels there are the relevant attributes of the decision problem such as criteria and sub-criteria. The number of levels in the hierarchy depends on the complexity of the problem and the decision model of the problem hierarchy.

Step 2: Generate relational data for comparing the alternatives. This requires the analyst to make pair-wise comparisons of elements at each level in the hierarchy relative to each activity at the next higher level. In AHP relational scale of real numbers from 1 to 9 and their reciprocals are used to assign preferences in a systematic manner. When comparing two attributes (or alternatives) with respect an attribute in a higher level, the relational scale proposed by Saaty [9-11] is used. The scale is shown in Table 1.

When elements being compared are closer together than indicated by the scale, one can use the values 1.1, 1.2, ..., 1.9. If still finer, one can use the appropriate percentage refinement.

Step 3: Utilizing the pair-wise comparisons of step 2, an eigenvalue method (mathematical approach used in AHP) is used to determine the relative priority or weight of each attribute with respect to each attribute, one level up in the hierarchy. In addition a consistency parameter is also calculated. According to Saaty [10], small Consistency Ratio (<0.10) does not drastically affect the ratings. The user has the option to reevaluate the comparison matrix and generally it is done if the value of Consistency Ratio (C.R.) is higher than 0.10.

Step 4: In this step, the priorities of the lowest level (alternatives) relative to the top most level (objective or goal) are determined.

In AHP, if a problem has M alternatives and N criteria, then the decision maker has to construct N judgment matrices (one for each criteria) of alternatives of order M × M and one judgment matrix of criteria of order N × N (for N criteria). Finally, the decision matrix of order M × N is formed by using the relative weights (scores) of the alternatives with respect to each criterion. The entry a_{ij} in the M × N matrix represents the relative weight (score) of the i th alternative when it is considered in terms of criterion j. According to AHP the best alternative (in the maximization case) is indicated by the following relationship:

$$A_{AHP} = \max \sum_{j=1}^{N} a_{ij} \cdot w_j \quad \text{for } i = 1, 2, 3, \ldots M$$

where w_j is the relative weight of the j th criterion.

The Revised Analytic Hierarchy Process

Belton and Gear [19] proposed a revised version of AHP model. They demonstrated that an inconsistency in the relative ranking could occur when an alternative identical to one of the already existing alternatives is introduced in the fray. Some other researchers have also expatiated on the aspect of rank reversal in AHP [20, 21]. According to Belton and Gear, the root for this inconsistency lies in the fact that the sum of the relative weights (scores) of alternatives for each criterion sum up to one, i.e., $\sum_{i=1}^{M} a_{ij} = 1$. In order to overcome this deficiency, they proposed that the each column of the AHP decision matrix to be divided by the maximum entry of that column. Thus they introduced a variant of original AHP and it is called as Revised AHP or Ideal Mode AHP. In a study concerning the effectiveness of various MCDM methodologies, Triantaphyllou and Mann [22, 23] found the Revised AHP to

Table 1. The fundamental relational scale for pair-wise comparisons

Intensity of importance on an absolute scale	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective.
3	Moderate importance of one over another	Experience and judgment slightly favour one activity over another.
5	Essential or strong importance	Experience and judgment strongly favour one activity over another.
7	Very strong importance	An activity is strongly favoured and its dominance is demonstrated in practice.
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation.
2, 4, 6, 8	Intermediate values between two adjacent judgment	When compromise is needed.
Reciprocals	If activity p has one of the above numbers assigned to it when compared with activity q, then q has the reciprocal value when compared with p.	

be the best decision-making method.

The Naive and Structured Analytic Hierarchy Process

When a decision making problem is decomposed into a number of components, the structure of the hierarchy may be different. In Naive AHP, pair-wise comparison is done for each criterion with every other criterion without considering the general forms of the criterion. However, in Structured AHP, clustering or grouping of criteria is done into a hierarchy based on qualitative similarity. Generally, this qualitative clustering is done in a 'top down' manner after the main differentiating attributes between the alternatives have been discovered. In Structured AHP the comparison at any level should be between things of a corresponding level of generality. Hence, a possible benefit of the structured approach would be that the general categories might subsume other important but unarticulated feelings. The Naive AHP causes a problem when there are many criteria and alternatives. The total number of pair-wise comparisons in a decision problem having M alternatives and N criteria is expressed by the following equation:

$$\frac{N(N-1)}{2} + N \cdot \frac{M(M-1)}{2}$$

This leads to an excessive number of pair-wise comparison questions and a drop of interest towards the end of the questioning process resulting in higher inconsistencies. For example, if there are 21 criteria, there will be $N(N-1)/2 = 210$ pair-wise comparisons between criteria. However, if the criteria are clustered equally under three broader classes then the number of pair-wise comparisons will be reduced to $3 \times 1/2 \times 7 \times 6 = 63$. There will be another 3 pair-wise comparisons between the three general categories making the total number of pair-wise comparisons to 66. Structured AHP gives better consistency scores. However, it requires greater understanding of the issues in order to do the structuring.

Measurement of Consistency Parameters

One of the useful contributions of AHP is that the degree of inconsistency of the judgments could be evaluated easily as shown below:

1) After normalizing the columns of pair-wise comparison matrix, the row averages are calculated to get the principal eigenvector.
2) The original pair-wise comparison matrix is then multiplied by the eigenvector to get the product.
3) The product is then divided by the eigenvector and averages are calculated to get the maximum eigenvalue (λ_{max}) of the original matrix.

C.I. and C.R. are then calculated using the following formula:

$$\text{C.I.} = \frac{(\lambda_{max} - N)}{(N-1)} \quad \text{and} \quad \text{C.R.} = \frac{\text{C.I.}}{\text{R.C.I.}}$$

Table 2. R. C. I. values for different values of N

N	1	2	3	4	5	6	7	8	9
R.C.I.	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

where R. C. I. is Random Consistency Index and N is number of elements in the pair-wise comparison matrix. The values of R. C. I. are given in Table 2. If the C.R. value is greater than 0.10, then it is better to reevaluate the pair-wise comparison.

Selection of Cotton Fiber

Hierarchy Formulation for the Cotton Fiber Selection Problem

The objective of this particular hierarchy is directed towards the selection of a cotton fiber from the available alternatives to maximize the ring yarn strength. This objective acquires position at the top (Level 1) of the hierarchy. Now, it has been an established perception in the spinning industries that the strength of ring spun yarns, for a given set of process conditions, is decisively influenced by the tensile, length and fineness properties of cotton fiber. Therefore, these three general attributes or criteria of cotton fiber form the second level of the hierarchy. Among the tensile properties of cotton fibers, bundle tenacity and elongation are the most commonly evaluated and consequently they become the sub-criteria (Level 3) of tensile properties. Likewise, Upper Half Mean Length (UHML), Length Uniformity Index (UI) and Short Fiber Content (SFC) are the sub-criteria of length properties. Fineness properties do not have any sub-criteria and it is almost solely represented by the micronaire value of cotton fiber. At the lowest level (Level 4) of the hierarchy, we have the eight cotton fiber alternatives. Some of the other cotton properties like surface characteristics and natural convolutions are also having significant influence on yarn tenacity. However, these properties cannot be measured with commercial fiber testing instruments like HVI or AFIS. Besides, the relative contribution of surface characteristics and natural twist of cotton to yarn tenacity is also unknown in the absence of proper published work. Therefore, these properties could also be

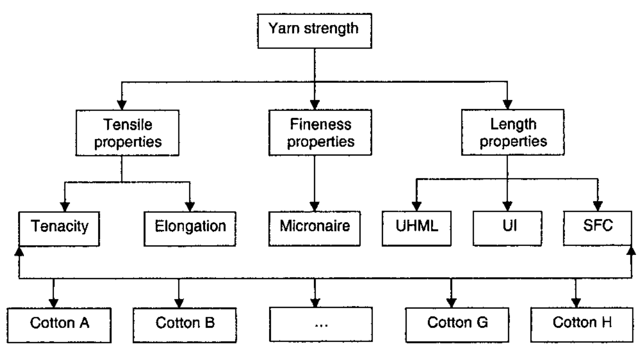

Figure 1. Hierarchical structure of cotton fiber selection problem.

Table 3. Pair-wise comparison matrix of criteria with respect to objective

Criteria	Tensile properties	Length properties	Fineness properties
Tensile properties	1	1/2	3
Length properties	2	1	5
Fineness properties	1/3	1/5	1

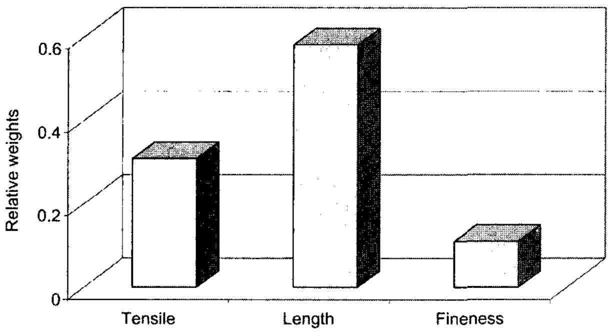

Figure 2. Relative weights of criteria with respect to objective.

incorporated in the decision hierarchy if quick measurement of them is possible and the decision maker possesses knowledge about their impact on yarn tenacity. The schematic diagram of the hierarchical structure is depicted in Figure 1.

Pair-wise Comparison and Determination of Weights of Criteria and Sub-criteria

The pair-wise comparison matrix of three criteria with respect the overall objective of the present problem is given in Table 3. Here the elements of Level 2 are arranged into a matrix and judgments are made according to the Saatys scale given in Table 1.

It can be inferred from the matrix that tensile properties are having moderate dominance over the fineness properties. However, length properties demonstrate strong preponderance over the fineness properties. The dominance of length properties over the tensile properties is in between equal to moderate importance. After normalization of matrix columns and taking the row averages, we get the priority vector (relative weights) of criteria with respect to the objective as shown in Table 4 and depicted in Figure 2.

Figure 2 and the priority column in Table 4 indicate that the length properties of cotton fibers have the most dominant influence on ring yarn strength with a priority of 0.581. The priority of tensile and fineness properties are 0.309 and 0.110, respectively. For the measurement of consistency of judgment the original matrix is multiplied by the priority vector to get the product as shown below:

$$\begin{bmatrix} 1 & 1/2 & 3 \\ 2 & 1 & 5 \\ 1/3 & 1/5 & 1 \end{bmatrix} \times \begin{bmatrix} 0.309 \\ 0.581 \\ 0.110 \end{bmatrix} = \begin{bmatrix} 0.930 \\ 1.749 \\ 0.329 \end{bmatrix}$$

$$\text{Now, } \lambda_{max} = \left(\frac{0.930}{0.309} + \frac{1.749}{0.581} + \frac{0.329}{0.110}\right)/3 = 3.004$$

Therefore, C.I. = (3.004 − 3)/(3 − 1) = 0.002

$$\text{And C.R.} = \frac{\text{C.I.}}{\text{R.C.I}} = 0.002/0.58 = 0.003$$

As the value of C. R. is well below the critical value of 0.10, therefore there is no need to reconsider the pair-wise comparison matrix.

The next step is concerned with finding the priorities of various sub-criteria (Level 3) with respect to the corresponding criteria (Level 2), which are situated one level up in the hierarchy. The pair-wise comparison between the sub-criteria of tensile and length properties and the derived priority vectors are shown in Tables 5 and 6, respectively. Then the global weights of sub-criteria are calculated by multiplying the relative weight of a sub-criterion with respect to the corresponding criterion and the relative weight of that criterion with respect to the objective. For example, global

Table 5. Pair-wise comparison of sub-criteria with respect to tensile properties

Tensile properties	Tenacity	Elongation	Priority vector
Tenacity	1	7	0.875
Elongation	1/7	1	0.125

C.R. = 0.

Table 6. Pair-wise comparison of sub-criteria with respect to length properties

Length properties	UHML	UI	SFC	Priority vector
UHML	1	2	2	0.500
UI	1/2	1	1	0.250
SFC	1/2	1	1	0.250

C.R. = 0.

Table 4. Normalized matrix and priority vector of criteria

Criteria	Tensile properties	Length properties	Fineness properties	Row total	Priority vector
Tensile	0.3	0.294	0.333	0.927	0.309
Length	0.6	0.588	0.556	1.744	0.581
Fineness	0.1	0.118	0.111	0.329	0.110

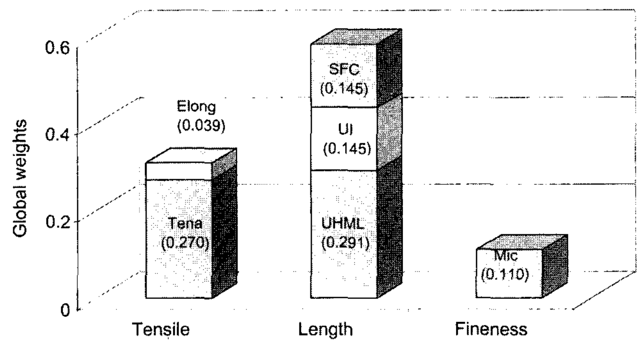

Figure 3. Global weights of sub-criteria with respect to objective.

weight of tenacity is 0.875 × 0.309 = 0.270. Figure 3 depicts the global weights of various sub-criteria with respect to the overall objective of the problem. For tenacity, elongation, UHML, UI, SFI and Micronaire the values of global weights are 0.270, 0.039, 0.291, 0.145, 0.145 and 0.110, respectively.

Determination of Alternative Scores and Synthesis

Now, the eight alternative cottons are compared against each other in terms of each of the sub-criterion and individual priority vectors are determined. Here Liberatore's [24] five point rating scale is used to rate the alternative cotton fibers in order to facilitate and reduce the time and effort in making pair-wise comparisons. The ratings received by eight cottons are shown in Table 7. The pair-wise comparison matrix of five point rating scale is shown in Table 8. This matrix is then normalized to obtain the relative weights of each rating scale. The relative weights of excellent, good, average, fair and poor are found to be 0.513, 0.261, 0.129, 0.063, 0.034, respectively. Table 9 shows the ultimate decision matrix of the problem. The priority vector of eight cottons with respect to a sub-criterion is indicated against its column. The column at the extreme right shows the final priority of the alternative cotton fibers obtained by normalizing the sum of the products of alternative score in a particular sub-criterion and the global

Table 7. Ratings of cotton fiber alternatives according to five-point rating scale

Cotton	Criterion					
	Tenacity	Elongation	UHML	UI	SFC	Micronaire
A	Average	Average	Average	Poor	Poor	Good
B	Excellent	Excellent	Average	Excellent	Average	Fair
C	Average	Excellent	Average	Fair	Fair	Excellent
D	Excellent	Excellent	Excellent	Good	Good	Good
E	Poor	Fair	Poor	Fair	Good	Poor
F	Excellent	Good	Good	Good	Average	Fair
G	Good	Fair	Poor	Poor	Poor	Poor
H	Average	Excellent	Good	Fair	Fair	Excellent

Table 8. Pair-wise comparison judgment matrix for five-point rating scale

Rating scale	Excellent	Good	Average	Fair	Poor	Relative weights
Excellent	1	3	5	7	9	0.513
Good	1/3	1	3	5	7	0.261
Average	1/5	1/3	1	3	5	0.129
Fair	1/7	1/5	1/3	1	3	0.063
Poor	1/9	1/7	1/5	1/3	1	0.034

Table 9. Decision matrix and final priority of alternatives

Cotton	Criterion						Final priority (normalized)
	Tenacity (0.270)	Elongation (0.039)	UHML (0.291)	UI (0.145)	SFC (0.145)	Micronaire (0.110)	
A	0.129	0.129	0.129	0.034	0.034	0.261	0.070
B	0.513	0.513	0.129	0.513	0.129	0.063	0.179
C	0.129	0.513	0.129	0.063	0.063	0.513	0.101
D	0.513	0.513	0.513	0.261	0.261	0.261	0.249
E	0.034	0.063	0.034	0.063	0.261	0.034	0.044
F	0.513	0.261	0.261	0.261	0.129	0.063	0.174
G	0.261	0.063	0.034	0.034	0.034	0.034	0.058
H	0.129	0.513	0.261	0.063	0.063	0.513	0.125

weight of that sub-criterion.

Conclusions

From the values of the final priority, shown in Table 9, it is evident that cotton D is the best alternative from the viewpoint of maximizing the ring yarn strength with a final priority value (normalized) of 0.249. Cotton B and cotton F hold the second and third positions in the preference list of alternatives with the final priority values of 0.179 and 0.174, respectively. Cotton E is the least preferred alternative as it exhibits the lowest normalized value (0.044) of the final priority. This study has proposed an AHP based model in order to help the spinning technologists in evaluating the overall quality of cotton fiber with respect to ring yarn strength. This approach could facilitate and systematize the selection and grading process of cotton fiber. Moreover, the method is very transparent for any type of further analysis. Not only this model can make tradeoffs between both qualitative and quantitative factors, but it also enables decision maker to deal with inconsistent judgment systematically. The pair-wise comparison procedure is able to capture relative judgments of two elements in a trustworthy manner and ensure consistency of these values. Similar studies could be initiated for rotor spun yarns also.

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