라그란제 보간을 사용한 비선형 클라인 고든 미분방정식의 수치해

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요 약

비선형 클라인 고든 방정식의 수치해를 구하기 위해 라그란제 보간을 사용하는데 비선형 항을 계산하기위해 보간식의 차이가 거의 없는 변형된 식을 사용하여 해의 ,안정성과 해의 수렴성을 밝히고 오차를 분석하였다. 즉 $I(x)^3$ 대신에 $f(x_i)^3 I_i(x)$을 사용하였으며 오차는 $C\left(\frac{1}{N}\right)^{N-1} hN(N-1)\left(\frac{N}{2}\right)^{N-1} / \left(\frac{N}{2}\right)!$ 이하임을 보였고 여기서 N은 다항식의 차수이다.

Numerical Solution for Nonlinear Klein-Gordon Equation by Using Lagrange Polynomial Interpolation with a Trick

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ABSTRACT

In this paper, by using Lagrange polynomial interpolation with a trick such that for $f(x)^3$ we shall use $f(x_i)^3 I_i(x)$ instead of $I(x)^3$ where $I(x) = \sum_i f(x_i) I_i(x)$. We show the convergence and stability and calculate errors. These errors are approximately less than $C\left(\frac{1}{N}\right)^{N-1} hN(N-1)\left(\frac{N}{2}\right)^{N-1} / \left(\frac{N}{2}\right)!$ where N is a polynomial degree.

키워드 : 비선형 클라인고든 미분방정식(Non-linear Klein-Gordon Equation), 라그란제 보간(Lagrange Interpolation)

1. Introduction

The nonlinear Klein Gordon equation

$$\frac{\partial^2 u}{\partial t^2} - \Delta u + V_u(u) = f \qquad (1)$$

where Δ is the Laplacian operator in $R^d(d=1,2,3)$, $V_u(u)$ is the derivative of the "Newtonian potential function" V, and f is a source term independent of the solution u, in various areas of mathematical physics. Among the particular cases which are the practical relevance, we take $V_u(u) = |u|^{\alpha} u$ with $\alpha > 0$ (quantum mechanics), refer to[5].

The convergence of the Galerkin finite element method for second order hyperbolic equations has been studied by many authors : cf. among others Dupont[3], who obtained error estimates for time-discrete and time continuous approximations of linear problems, and Dendy[2], who examined nonlinear problems as well as various modified Galerkin methods. To compute the nonlinear term, the product approximation is used by Yves Tourigny[6]. This approximation is a technique which consists of replacing the nonlinear term by its interpolant in the finite-dimensional subspaces. This provides an interesting alternative to numerical quadrature and greatly eases the implementation of the Galerkin method.

In this paper, by using Lagrange polynomial interpolation with a trick such that let $I(x)$ be an interpolation function with n-node x_i of an arbitrary function $f(x)$, if we need an interpolation for $f(x)^3$, then we shall use $f(x_i)^3 I_i(x)$ instead of $I(x)^3$ where $I(x) = \sum_i f(x_i) I_i(x)$, we get the numerical solutions of (1) when $\Delta u = \frac{\partial^2 u}{\partial x^2}$.

※ 이 논문은 2004년도 호서대학교 학술연구조성비에 의하여 연구되었음.
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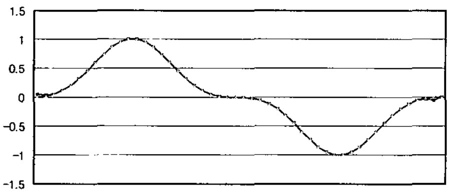
논문접수 : 2004년 6월 2일, 심사완료 : 2004년 11월 9일

We show an example about $f(x)^3$, $f(x_i)^3 I_i(x)$, $I(x)^3$, where $f(x) = \sin x$ has 15 nodes in (Figure 1).



(Figure 1) $f(x)^3$ and $I(x)^3$ are same for every point but $f(x)^3 I_i(x)$ has some difference at near the boundary

To show the stability and convergence, we set as many distinct points

$$x_k \ k \in J \text{ (a set of indices)}$$

in the domain Ω or in its boundary $\partial\Omega$, as the dimension of the space $Pol_N(\Omega)$. At the number of these points, located on $\partial\Omega$, the boundary conditions are imposed. The remaining points are used to enforce the differential equation.

We assume that for any $k \in J$, there exists a polynomial $\phi_k \in Pol_N(\Omega)$, necessarily unique, such that

$$\phi_k(x_m) = \begin{cases} 1 & \text{if} \quad k = m, \\ 0 & \text{if} \quad k \neq m. \end{cases}$$

The ϕ_k's form a basis for the polynomials of degree N, since $v(x) = \sum_{k \in J} v(x_k)\phi_k(x)$ for all $v \in Pol_N(\Omega)$. Let J be divided into two disjoint subsets J_e and J_b, such that if $k \in J_b$, the x_k's are on the part $\partial\Omega$ of the boundary. Moreover, let L_N be an approximation to the operator L in which derivatives are taken at the points x_k's. The polynomial $u^N \in Pol_N(\Omega)$ is a solution that satisfies the equations

$$\begin{cases} L_N u^N(x_k) = f(x_k) & \textit{for all } k \in J_e, \\ Bu^N(x_k) = 0 & \textit{for all } k \in J_b. \end{cases}$$

The unknowns in this method are the values of u^N at the points x_k's, i.e., the coefficients of u^N with respect to the Lagrange polynomial. We consider a bilinear form $(u, v)_N$ on the space $C^0(\Omega)$ of the functions continuous up to the boundary of Ω by fixing a family of weights w_k and setting

$$(u, v) = \sum_{k \in J} u(x_k)\overline{v(x_k)}\, w_k.$$

The existence of the basis ensures that $(u, v)_N$ is an inner product on $Pol_N(\Omega)$. Consequently, we define a discrete norm on $Pol_N(\Omega)$ as

$$\| u \|_N = \{(u, u)_N\}^{\frac{1}{2}} \text{ for } u \in Pol_N(\Omega).$$

The basis of ϕ_k's is orthogonal under the discrete inner product. We make the assumption that the nodes $\{x_k\}$ and the weights $\{w_k\}$ are such that

$$(u, v)_N = (u, v) \text{ for all } u, v \text{ such that } uv \in Pol_{2N-1}(\Omega).$$

In all the applications, this assumption is fulfilled since the x_k's are the knots of quadrature formulas of Gaussian type.

Let X_N be the space of the polynomials of degree less than or equal to which satisfy the boundary conditions, i.e.,

$$X_N = \{v \in Pol_N(\Omega) \mid Bv(x_k) = 0 \quad \text{for } \textit{all } k \in J_b\}.$$

Then this method is equivalently written as

$$\begin{cases} u^N \in X_N \\ (L_N u^N, \phi_k) = (f, \phi_k)_N \text{ for } \textit{all } k \in J_e. \end{cases}$$

If Y_N is the space spanned by the ϕ_k's with $k \in J_e$, i.e.,

$$Y_N = \{v \in Pol_N(\Omega) \mid v(x_k) = 0 \quad \text{for } \textit{all } k \in J_b\},$$

then can be written as

$$\begin{cases} u^N \in X_N \\ (L_N u^N, v) = (f, v)_N \text{ for } \textit{all } v \in Y_N. \end{cases}$$

2. Stability

Let Ω be an interval [-1, 1]. We would like to approximate the solution of the following problem

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + |u|^\alpha u = f \quad \text{in } Q = \Omega \times [0, T]$$

$$u(.,0) = u_0 \text{ and } \left(\frac{\partial u}{\partial t}\right)(.,0) = u_1 \text{ in } \Omega$$

$$u(-1, t) = u(1, t) = 0 \text{ for } t \in [0, T]$$

where $u_0 \in H_0(\Omega)$, $u_1 \in L(Q)$ and $f \in L(Q)$ are given functions, a small $T > 0$.

The solution $u^N(x, t)$ of the Legendre Tau approximation of this problem is for all $t > 0$ a polynomial of degree N in x, which is zero at $x = \pm 1$ and satisfies the equations,

$$\int_{-1}^{1}[u_{tt}^{N}(x,t) - u_{xx}^{N}(x,t) + |u^{N}(x,t)|^{\alpha}u^{N}(x,t)]v(x)dx$$
$$= \int_{-1}^{1} f(x,t)v(x)dx \quad t > 0, \text{ for all } t \in P_{N-2}$$
$$\int_{-1}^{1}[u^{N}(x,0) - u_0(x)]v(x)dx = 0$$
$$\int_{-1}^{1}[u_t^{N}(x,0) - u_1(x)]v(x)dx = 0 \tag{3}$$

Let we set $X_N = \{u \in P_N \mid u(-1) = u(1) = 0\}$, $Y_N = P_{N-2}$, and $(u, v) = \int_{-1}^{1} u(x)v(x)dx$. For all $u \in X_N$ we have $\int_{-1}^{1} u_{xx}P_{N-2}udx = -\int_{-1}^{1} u_{xx}^{N}udx = \int_{-1}^{1}(u_x)^2dx$

But, we know that the degree of $|u^N|^{\alpha}u^N$ is greater than 2N-1. Here, we shall use the approximation of $|u^N|^{\alpha}u^N$ in (3). We substitute $I_N|u^N|^{\alpha}u^N$ instead of $|u^N|^{\alpha}u^N$ where $I_N : C(\Omega) \to X_N$ is the interpolation operator.

We shall find the approximate solution $u_N \in X_N$ such that

$$\int_{-1}^{1}[u_{tt}^{N}(x,t) - u_{xx}^{N}(x,t) + I_N|u^{N}(x,t)|^{\alpha}u^{N}(x,t)]v(x)dx$$
$$= \int_{-1}^{1} f(x,t)v(x)dx \quad t > 0, \text{ for all } t \in P_{N-2},$$
$$\int_{-1}^{1}[u^{N}(x,0) - u_0(x)]v(x)dx = 0$$
$$\int_{-1}^{1}[u_t^{N}(x,0) - u_1(x)]v(x)dx = 0. \tag{4}$$

Theorem 1. For some $T > 0$,

$$\|P_{N-2}u_t^N(t)\|^2_{L_w(-1,1)} + \|u_x^N(t)\|^2 + (2\beta/P)\|u^N(t)\|^p_{L^p(-1,1)}$$
$$\le \{P_{N-2}u_t^N(0)\|^2_{L_w(-1,1)} + \|u_x^N(0)\|^2 + (2\beta/P)\|u^N(0)\|^p_{L^p(-1,1)}$$
$$+ \int_0^T \|f(s)\|^2_{L_w(-1,1)}ds\}e^T$$

proof. Take $v \in Pol_{N-2}u_t^N$, from the left hand side first term in (4)

$$\int_{-1}^{1} u_{tt}(x,t)P_{N-2}u_t^N(x,t)dx$$
$$= \int_{-1}^{1} P_{N-2}u_{tt}(x,t)P_{N-2}u_t^N(x,t)(1-x^2)dx$$
$$= (1/2)\frac{d}{dt}\|P_{N-2}u_t^N(t)\|^2_{L_w(-1,1)},$$

and the second term,

$$-\int_{-1}^{1} u_{xx}^N(x,t)P_{N-2}u_t^N(x,t)dx$$
$$= \int_{-1}^{1} u_{xx}^N(x,t)P_{N-2}\frac{d}{dt}u^N(x,t)dx$$
$$= \int_{-1}^{1} u_x^N(x,t)\frac{d}{dt}u_x^N(x,t)dx$$
$$= (1/2)\frac{2}{dt}\|u_x^T(t)\|^2.$$

Now, for $p = \alpha + 2$, refer to [4],

$$(1/p)\frac{d}{dt}\|u^N(x,t)\|^p_{L^p(-1,1)}$$
$$= \int_{-1}^{1}|u^N(x,t)|^{\alpha}u^N(x,t)u_t^N(x,t)dx$$
$$= \int_{-1}^{1}|u^N(x,t)|^{\alpha}u^N(x,t)(1-x^2)P_{N-2}u_t^N(x,t)dx$$

We can choose the β which satisfies $\|u^N - I_Nu^N\|_w^2 = \|u^N - P_Nu^N\|_w^2 + \|R_Nu^N\|_w^2$.

$$(1/p)\frac{d}{dt}\|u^N(x,t)\|^p_{L^p(-1,1)}$$
$$\le \beta\int_{-1}^{1} I_N\{|u^N(x,t)|^{\alpha}u^N(x,t)\}P_{N-2}u_t^N(x,t)dx$$

Therefore, from the equation

$$\int_{-1}^{1}[u_{tt}^N(x,t) - u_{xx}^N(x,t) + I_N|u^N(x,t)|^{\alpha}u^N(x,t)]$$
$$P_{N-2}u_t^N(x,t)dx$$
$$= \int_{-1}^{1} f(x,t)P_{N-2}u_t^N(x,t)dx$$

we obtain

$$(1/2)\frac{d}{dt}\|P_{N-2}u_t^N(t)\|^2_{L_w(-1,1)} + (1/2)\frac{d}{dt}\|u_x^N(t)\|^2$$
$$+ (1/p)\beta\frac{d}{dt}\|u^N(t)\|^N_{L^p(-1,1)}$$
$$\le \int_{-1}^{1}[u_{tt}^N(x,t) - u_{xx}^N(x,t) + I_N|u^N(x,t)]P_{N-2}u_t^N(x,t)dx$$
$$\le (1/2)\|f(t)\|^2_{L_w(-1,1)} + (1/2)\|P_{N-2}u_t^N(x)\|^2_{L_w(-1,1)}$$
$$\|P_{N-2}u_t^N(t)\|^2_{L_w(-1,1)} + \|u_x^N(t)\|^2 + (2/p\beta)\|u^N(t)\|^N_{L_p(-1,1)}$$
$$\le \|P_{N-2}u_t^N(0)\|^2_{L_w(-1,1)} + \|u_x^N(0)\|^2 + (2/p\beta)\|u^N(0)\|^N_{L^p(-1,1)}$$
$$+ \int_0^t \|f(s)\|^2_{L_w(-1,1)}ds + \int_0^t \|P_{N-2}u_t^N(s)\|^2ds$$

Applying Gronwall's inequality we complete the proof.

This theorem shows the stability of the approximate solution of u^N for

$$0 = \int_{-1}^{1}(u^N(x,0)-u_0(x))\,u^N{}_{0_{xx}}dx$$
$$= -\int_{-1}^{1}(u_x^N(x,0)-u_{0_x}(x))\,u^N{}_{0_x}dx$$
$$\int_{-1}^{1}u_x^N(x,0)^N_{u_{0_x}}dx = \int_{-1}^{1}u_{0_x}(x)\,u_{0_x}(x)u^N_{0_x}dx$$
$$\le c\int_{-1}^{1}u_{0_x}(x)\,u_{0_x}(x)\,dx \le c\|u_0\|^2_{H_0^1(\Omega)}$$

3. Convergence

Let R_N be a projection operator from a dense subspace W of D_B upon X_N, where D_B is a set which satisfies the boundary condition of (2). For each $u \in W$, we further require $R_N u$ to satisfy the exact boundary conditions, i.e.,

$$R_N : W \to X_N \cap D_B.$$

We define the norm $\|g\|_{E^*} = \sup_{u\in E,\,u\neq 0}\frac{(g,u)}{\|u\|_E}$ for all $g \in E^*$ that is dual of E.

Let $e(x,t) = u^N(x,t) - R_N u$. We obtain the following theorem.

Theorem 2. Assume that $|u|^\alpha u \in H^1(-1,1)$.

$$\|P_{N-2}e_t(t)\|_{L_w(-1,1)} + \|e_x(t)\|^2$$
$$\le \{\|P_{N-2}e_t(0)\|^2_{L_w(-1,1)} + \|e_x(0)\|^2 + M^2T\}e^T$$
$$\le \{\|P_{N-2}e_t(0)\|^2_{L_w(-1,1)} + c\|e_0\|^2_{H_0^1(\Omega)} + M^2T\}e^T$$

proof. From (3), we have

$$\int_{-1}^{1}[u_{tt}^N(x,t) - u_{xx}^N(x,t) + I_N|u^N(x,t)|^\alpha u^N(x,t)]v(x)\,dx = 0$$

$t > 0$, for all $v \in P_{N-2}$

Take $v = e_t(x,t)$

$$0 = \int_{-1}^{1}[u_{tt}^N - u_{xx}^N + I_N|u^N|^\alpha u^N] - (u_{tt} - u_{xx} + |u|^\alpha u)e_t dx$$
$$= \int_{-1}^{1}(u_{tt}^N - R_N u_{tt}^N + R_N u_{tt}^N - u_{tt})e_t dx$$
$$- \int_{-1}^{1}(u_{xx}^N - R_N u_{xx}^N + R_N u_{xx}^N - u_{xx})e_t dx$$
$$+ \int_{-1}^{1}(I_N|u^N|^\alpha u^N - |u|^\alpha u)e_t dx$$

We get

$$\frac{1}{2}\frac{d}{dt}\|P_{N-2}e_t(t)\|^2_{L_w(-1,1)} + \frac{1}{2}\frac{d}{dt}\|e_x(t)\|^2$$
$$= \int_{-1}^{1}(u_{tt} - R_N u_{tt}^N)e_t + (R_N u_{tt}^N - u_{xx})e_t$$
$$+ (|u|^\alpha u - I_N|u^N|^\alpha u^N)e_t dx$$

We refer to [1] : For each $v \in H_0^1(-1,1)$

$$(P_{N-2}(u_{tt} - R_N u_{tt}), v)$$
$$= (u_{tt} - R_N u_{tt}, v) - (u_{tt} - R_N u_{tt}, v - P_{N-2}v)$$
$$= ((u_{tt} - R_N u_{tt})_x, (\phi - R_N\phi)_x) - (u_{tt} - R_N u_{tt}, v - P_{N-2}v)$$

where ϕ is the only function in $H_0^1(-1,1)$ satisfying $-\phi_{xx} = v$,

then we obtain,

$$\|P_{N-2}(u_{tt} - R_N u_{tt})\|_{E^*} \le CN^{1-m}\|u_{tt}\|_{H^{m-2}(-1,1)}.$$

For each $v \in H_0^1(-1,1)$

$$(P_{N-2}(u - R_N u)_{xx}, v)$$
$$= -((u - R_N u)_x, v_x) - ((u - R_N u)_{xx}, v - P_{N-2}v)$$
$$= -((u - R_N u)_x, v_x) - (u_{xx} - P_{N-2}u_{xx}, v - P_{N-2}v)$$

here we have used the fact that both $P_{N-2}u_{xx}$ and $(R_N u)_{xx}$ are orthogonal to $v - P_{N-2}v$. Using the same approximation results as before, we deduce

$$\|P_{N-2}(u - R_N u)_{xx}\|_{E^*} \le CN^{1-m}\|u\|_{H^{m-2}(-1,1)}.$$

In Legendre approximations, for all $u \in H^m(-1,1)$

$$\|u - I_N u\|_{H^l(-1,1)} \le CN^{2l+\frac{1}{2}-m}\|u\|_{H^m(-1,1)}$$

for $0 \le l \le m$ with $m > \frac{1}{2}$.

Assume that $|u|^\alpha u \in H^1(-1,1)$, let $l = 0$. We get

$$\||u|^\alpha u - I_N|u^N|u^N\|_{L^2(-1,1)} = \||u|^\alpha u - I_n|u|^\alpha u\|_{L^2(-1,1)}$$
$$\le CN^{\frac{1}{2}-m}\||u|^\alpha u\|_{H^m(-1,1)}$$

We may assume that $m > 2$.

$$\text{Let } M = CN^{1-m}\|u_{tt}\|_{H^{m-2}(-1,1)} + CN^{1-m}\|u\|_{H^m(-1,1)}$$
$$+ CN^{-\frac{1}{2}}\||u|^\alpha u\|_{H^{m-2}(-1,1)}$$

clearly $M \to 0$ as $N \to \infty$. From (5)

$$\frac{1}{2}\frac{d}{dt}\|P_{N-2}e_t(t)\|^2_{L_w(-1,1)} + \frac{1}{2}\frac{d}{dt}\|e_x(t)\|^2$$
$$\le \frac{1}{2}M^2 + \frac{1}{2}\|P_{N-2}e_t(t)\|^2_{L_w(-1,1)}$$
$$\|P_{N-2}e_t(t)\|^2_{L_w(-1,1)} + \|e_x(t)\|^2$$
$$\le \|P_{N-2}e_t(0)\|^2_{L_w(-1,1)} + \|e_x(0)\|^2$$
$$+ \int_0^t M^2 ds + \int\|P_{N-2}e_t(x,s)\|^2_{L_w(-1,1)}ds$$

We know that $\|e_x(0)\|^2 \le c\| e_0\|^2_{H^2_0(\Omega)}$ and applying Gronwall's inequality we conclude the proof.

4. Numerical results

Set $u^N(x,t) = \sum_{i=0}^{N} a_i(t)\, l_i(x)$ where $l_i(x)$ is a N-degree Lagrange polynomial with $N+1$ nodes as $-1 = x_0 < x_1 < x_2 < \cdots < x_N = 1$, $\alpha = 2$. We substitute $u^N(x,t)$ into (2), we get

$$\frac{d^2 a_i(t)}{dt^2} - (l_0''(x_i)a_0(t) + \cdots + l_N''(x_i)a_N(t)) - |a_i(t)|^2 a_i(t) = f(x_i, t)$$
$$i = 0, 1, 2, \cdots, N.$$

in here we use the trick at $|a_i(t)|^2 a_i(t)$.
Applying the boundry condition and the difference equation with

$$\frac{d^2 a_i(t)}{dt^2} = \frac{a_i(t_{j+1}) - 2a_i(t_j) + a_i(t_{j-1})}{h^2}$$
$$a_i(t_0) = 0$$
$$a_i(t_1) = hu_1(x_i)$$

where h is a mesh size and $t_j = jh$. For one example, let

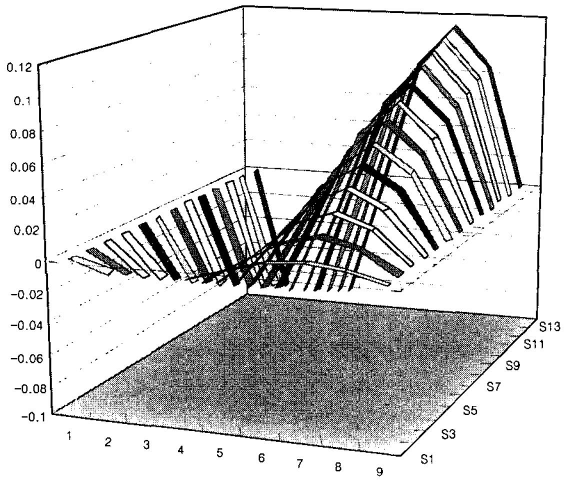
$$f(x,t) = -2\sin(\pi x) + |(t-t^2)\sin(\pi x)|^2 (t-t^2)\sin(\pi x) + \pi^2 (t-t^2)\sin(\pi x)$$
$$\frac{\partial u}{\partial t}(x, 0) = \sin(\pi x),$$

then we obtain the numerical solution as follows.

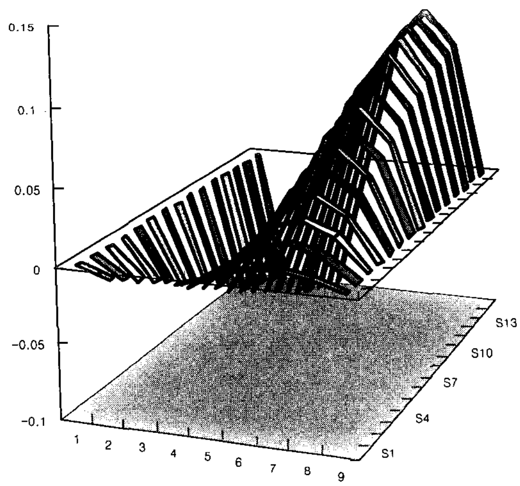
Practically, this example has exact solution such that $(t-t^2)\sin(\pi x)$. We can calculate errors. These errors are approximately less than $C\left(\frac{1}{N}\right)^{N-1} hN(N-1)\left(\frac{N}{2}\right)^{N-1} / \left(\frac{N}{2}\right)!$ in that $C\left(\frac{1}{N}\right)^{N-1}$ is estimated from M which is in theorem 2., and $N(N-1)\left(\frac{N}{2}\right)^{N-1} / \left(\frac{N}{2}\right)!$ is calculated by a second order differentiation of N-degree Lagrange polynomial in <Table. 1>. Briefly, the errors are less than $\left(\frac{1}{2}\right)^{N-1} hN(N-1)\left(\frac{N}{2}\right)^{N-1} / \left(\frac{N}{2}\right)!$ and are independent of α.

〈Table 1〉 The numerical estimation of u(1/2, 0.01), error and error bound. For time value t we show results by 10th iteration, where $K = \left(\frac{1}{2}\right)^{N-1} hN(N-1)\left(\frac{N}{2}\right)^{N-1} / \left(\frac{N}{2}\right)!$ h=0.001

Node	Numerical	Exact	Error	K
N=8	9.913096E-3	9.9E-3	1.3096E-5	1.8229E-5
N=12	9.900053E-3	9.9E-3	5.3E-8	8.9518E-8



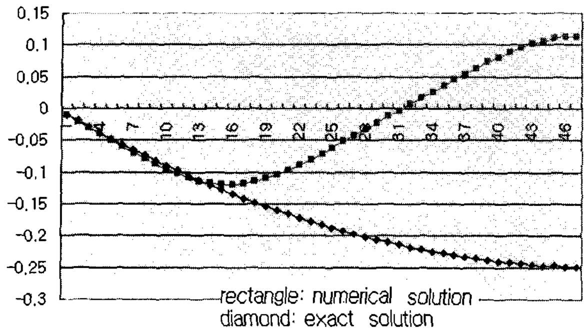
(a) Exact Solution $t \in [0, 0.15]$



(b) Numerical Solution $t \in [0, 0.15]$

(Figure 2) Exact solution and numerical solution when t is from 0 to 0.15 mesh h=0.001

Until the time variable t is small, the numerical solution is stable in (Figure 2), but if t is greater than 0.15, this numerical solution is unstable. In (Figure 3), we show that the numerical solution is not stable when t is greater than 0.15. Nevertheless, this numerical solution is not bad for some small t.



(Figure 3) The difference between numerical solution and exact solution is big when t is greater than 0.15

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