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Basic Study of Glimm's Algorithm for Green Water Simulation

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Abstract : *Experiments revealed that green water phenomena resemble dam-break, in which flow over deck edge forms a vertical wall of water and suddenly falls down into deck. In this paper the dam breaking problems were formulated using Glimm's algorithm, so-called, Random Choice method and, several validations were presented.*

Key words : *Glimm's Algorithm, Random Choice Method, Green Water*

1. Introduction

Stoker (1957) described extensively the one dimensional dam-breaking problem, where it is assumed that at the time zero there is a vertical wall of water on the side of a vertical dam. At that moment the dam is removed and the water flows into the empty region. One approach to the problem is to model the flow using the shallow water equations. They can be treated by the method of characteristics which involves the applications of the Riemann solutions comprised of standard forms such as rarefaction waves, shock, etc. Since water on deck is assumed to be shallow, it is necessary to employ a numerical method to solve the shallow water equations expressed as nonlinear hyperbolic partial differential equations. The solutions of the equations have in general discontinuities. These discontinuities may be produced by non-smooth initial data or may be very well developed spontaneously in the domain, even in the case of smooth initial data. The main attention is drawn in this paper to the application of Glimm's algorithm (Random Choice Method, which is denoted as RCM) introduced by Glimm (1965) to the shallow water equations. The advantages of RCM are (1) discontinuities as shocks or contact surfaces are computed without numerical diffusion and dispersion (2) there are no numerical oscillations behind discontinuities (3) boundary conditions are readily handled. The disadvantages are (1) due to the randomness the profile of a rarefaction wave is not computed smoothly but the average is very close to the exact solution (2) the locations of discontinuities at any time are not exact, however, their average positions are exact. This paper is primarily devoted to the explanation of RCM and a possibility to be applied to the green water problems.

2. Glimm's algorithm

2.1 One dimensional RCM

When one dimensional shallow water equations are considered, the equations with respect to the x direction are:

$$H_t + uH_x + Hu_x = 0 \tag{1}$$

$$u_t + uu_x + gH_x = 0 \tag{2}$$

- When 0 sec is taken as a starting time

We divide the computational length (L) into intervals of length (Δx) in a domain [0, L] and approximate the water height (H) and velocity (u) by piecewise function of position (x) and time (t) (see Fig. 1). For example, in a space (i-1/2, i+1/2), the water height is approximated as Hi and the velocity is approximated as ui, and in a space (i+1/2, i+3/2), the water height is approximated as Hi+1 and the velocity is approximated as ui+1. If we divide L into very small spatial intervals, the approximation of H and u in a space can be more exact up to a certain point. For easy explanation the length (L) is divided only into two intervals of Δx.

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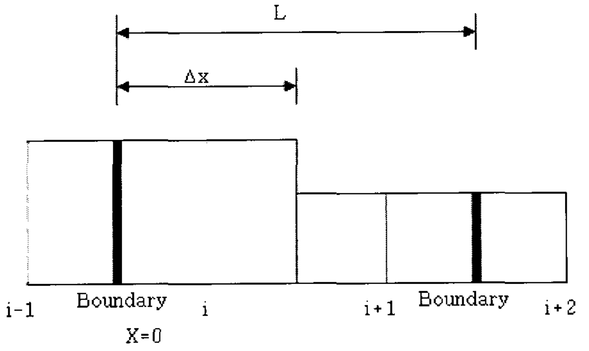

Fig. 1 Initial conditions for the first half time step in one dimension

We need wall boundary conditions at x=0 and x=L. The boundary conditions are assumed to be fixed and impermeable walls. In a space (i-1, i-1/2), the water height is approximated as H_i, which is same as in the space (i-1/2, i+1/2) and the velocity is approximated as $-u_i$, which is opposite to that in the space (i-1/2, i+1/2). In a space (i+2/3, i+2), the water height is approximated as H_{i+1}, which is the same as that in the space (i+1/2, i+3/2) and the velocity is approximated as $-u_{i+1}$, which is opposite to that in space (i+1/2, i+3/2). The time step (Δt) is chosen so that the Riemann solutions at adjacent points do not overlap. This can be guaranteed by application of the Courant-Friedrichs-Lewy (CFL) condition:

$$\Delta t = \frac{C_{cfl}\Delta x}{\max(ABS(V_w) + C)} \tag{3}$$

C_{cfl} is the CFL coefficient which satisfies $0 < C_{cfl} < \frac{1}{2}$. Hence the RCM has a stability limit of $\frac{1}{2}$. max $(ABS(V_w)+C)$ is the maximum wave velocity present through the domain at any given time. V_w and C denotes the water particle velocity and the wave speed ($\sqrt{gH}$).

• first half time step from 0 to Δt/2

There are three steps to advance time from 0 to Δt/2 sec. The first step is to apply the local Riemann problems (see Toro (2001) for Riemann problems). Each Riemann problem is solved in the spaces (i-1, i), (i, i+1) and (i+1, i+2) in Fig. 1. See Fig. 2 for the possible water pattern after the imaginary dam breaks, in which the dams exist at the locations of i-1/2, i+1/2 and i+3/2.

The second step is to find randomly sampled points in the same spaces as used in the solutions of the first step. The random number, which is between -0.5 and 0.5, is positive in the first half time step. The randomly chosen points (where water heights and velocities are obtained from the solutions of the local Riemann problems) are obtained by multiplying the positive random number by Δx (see Fig. 2 for the positions of the randomly sampled points). The randomly sampled water heights and velocities are assumed to represent each space. The third step is to treat the boundary conditions. The sampled point in the space (i-1, i) is located in the real domain. The sampled point in the space (i+1, i+2) is outside the boundary. In that case we move both of the wall boundaries by a distance of Δx/2 to the left.

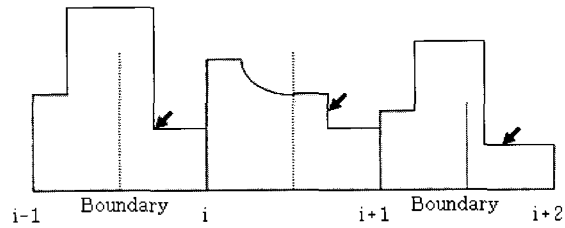

Fig. 2 The local Riemann problems are implemented for the first half time step (Δt/2) and the small arrows represent the randomly sampled points at each designated space.

• second half time step from Δt/2 to Δt

There are the same three steps as in the first half time step to advance time from Δt/2 to Δt. The local Riemann problems are solved. The randomly sampled points are obtained using the negative random numbers so that they can represent the water height and velocity at each space. The wall boundaries move half-spatial interval (Δx/2) to the right and the domain returns to the original one.

We repeat the above procedure for each Δt obtained by the CFL conditions. When the accumulation of Δt reaches to a pre-defined time limit, the computer program stops.

2.2 Two dimensional RCM

Two dimensional shallow water equations are written for computing green water flow as follows:

$$H_t + uH_x + Hu_x + vH_y + Hv_y = 0 \tag{4}$$

$$u_t + uu_x + vu_y + gH_x = 0 \tag{5}$$

$$v_t + uv_x + vv_y + gH_y = 0 \tag{6}$$

H: water height

u: water velocity in the x direction

v: water velocity in the y direction

g: acceleration of gravity
Subscripts represent partial differentiation with respect to t(time), x or y.

The basic procedure of RCM is the use of Glimm's algorithm as a building block in a fractional step method. At each time step four quarter-steps are performed; each quarter-step represents a sweep in either the x or y direction. (1), (2) and (3) are rewritten for y=constant (x sweep) as;

$$u_t + uu_x + gH_x = 0 \tag{7}$$

$$v_t + uv_x = 0 \tag{8}$$

$$H_t + uH_x + Hu_x = 0 \tag{9}$$

And one can find a similar way for x=constant (x sweep) as;

$$v_t + vv_y + gH_y = 0 \tag{10}$$

$$u_t + vu_y = 0 \tag{11}$$

$$H_t + vH_y + Hv_y = 0 \tag{12}$$

(8) and (11) say that v and u are transported as passive scalars in the x sweep and y sweep, respectively.

The solutions of (7)-(12) are obtained using two dimensional RCM, of which the procedures are:

1. (4)-(6) are written only in the x direction and only in the y direction resulting in (7)-(9) and (10)-(12), respectively.
2. Δt_1 is found through the CFL conditions in (3).
3. During the first half time step ($\Delta t1/2$), the same procedure as when time advances from 0 sec to $\Delta t/2$ sec in the 1D solution is applied only in the x direction (y is constant) using (7)-(9). This procedure is named as the first x sweep.
4. The passive scalar is determined for the second half time step ($\Delta t_1/2$) according to the sign of random number.
5. During the second half time step ($\Delta t_1/2$), the same procedure as when time advances from 0 sec to $\Delta t/2$ sec in the 1D solution is applied only in the y direction (x is constant) using (10)-(12). This procedure is named as the first y sweep.
6. The passive scalar is determined for the third half time step ($\Delta t_2/2$)
7. Δt_2 is found through the CFL conditions.
8. During the third half time step ($\Delta t_2/2$), the same procedure as when time advances from $\Delta t/2$ sec to Δt sec in the 1D solution is applied only in the x direction (y is constant) using (7)-(9). This procedure is named as the second x sweep.
9. The passive scalar is determined for the fourth half time step ($\Delta t2/2$)
10. During the fourth half time step ($\Delta t_2/2$), the same procedure as when time advances from $\Delta t/2$ sec to Δt sec in the 1D solution is applied only in the y direction (x is constant) using (10)-(12). This procedure is named as the second y sweep.

3. Validation

3.1 Presentation of diagonal dam-break in the square box

Let us consider a square box. The dimensions of the square box are 10 m × 10 m and are divided into intervals of length (0.2 m) in the both directions. The box is filled with different states of water separated by the initial discontinuity (see Fig. 3 for the initial conditions). There is no experimental data for this test. However, when along the diagonal line (which is instructed by an arrow in the box) the problem can be treated as one dimensional, the effect of some techniques (e.g. x-y-x-y- sweeps and splitting schemes in two dimensional RCM) may be found for the first few seconds after the dam-break along the diagonal (see Fig. 4 to 5).

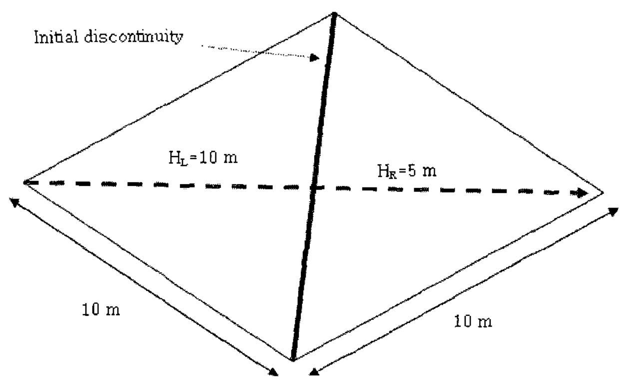

Fig. 3(a) Initial conditions for the diagonal dam-break

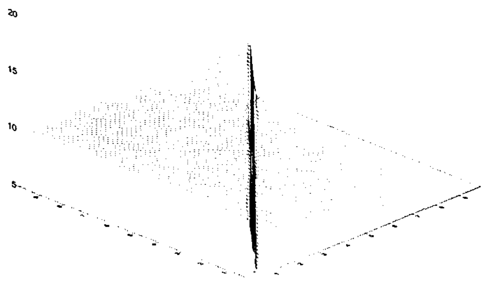

Fig. 3(b) Water Simulation of initial conditions for the diagonal dam-break (time = 0 sec)

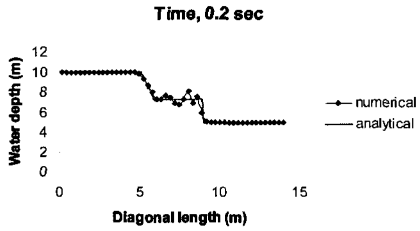

Fig. 4(a) comparison between numerical solution of two dimensional RCM and analytical solution .

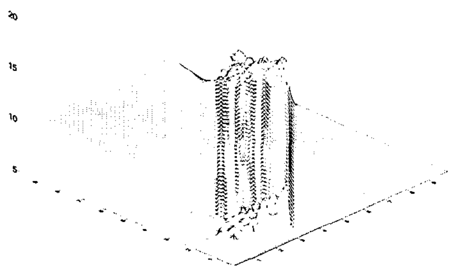

Fig. 4(b) Water Simulation for the diagonal dam-break (time = 0.2 sec)

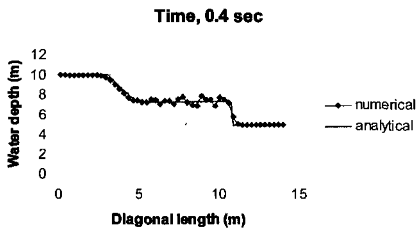

Fig. 5(a) comparison between numerical solution of two dimensional RCM and analytical solution .

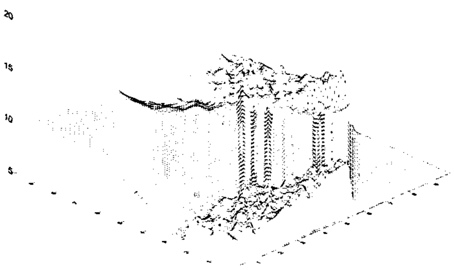

Fig. 5(b) Water Simulation for the diagonal dam-break (time = 0.4 sec)

3.2 Presentation of water flow onto a deck-shaped dry space

Fekken (1999) mentioned that a reasonable approximation for the model test of Green water seemed to be the dam breaking problem with a vertical wall of water height at the most forward part of the bow, linearly decreasing to a certain point. In his simulation using Navier-stokes equations, the appearance of the high-velocity water 'tongue' is very well visible in a movie of simulation. A similar approach for the initial conditions around the deck is used to show the validation of RCM by simulating water flow on the deck. See the initial water shape in Fig. 6 and the sequence of the dam-break events on deck in Fig. 7.

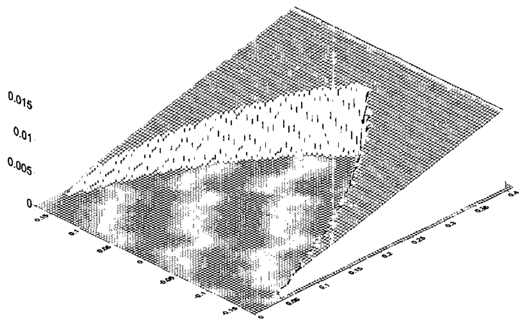

Fig. 6 Initial conditions; the vertical water is piled up around the bow from 0 cm to 10 cm and the velocity is assumed 0 m/sec.

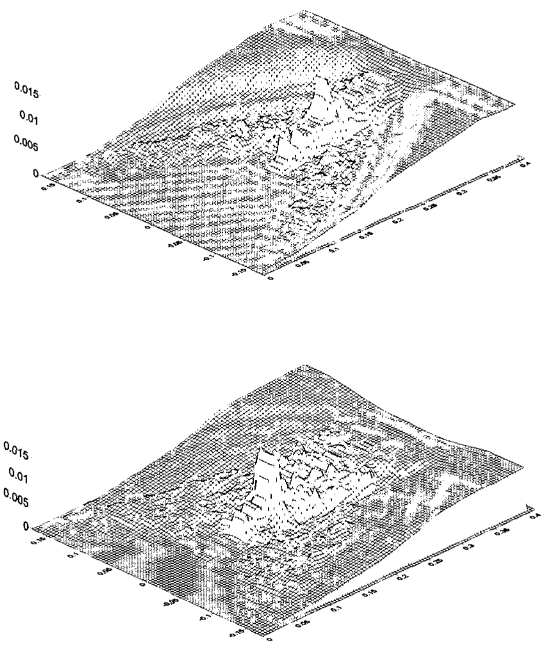

Fig. 7 The evident appearance of the water tongue and the water pattern similar to Fekken's work, in which the time step is 0.2 sec

4. Conclusions

This paper showed a possibility of the use of Glimm's algorithm, so-called Random Choice method to be applied to the matter of green water. The next stage of this research will be carried on with an effort to reveal the effect of green water on ship oscillations considered.

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