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Robustness of 2nd-order Iterative Learning Control for a Class of Discrete-Time Dynamic Systems

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Abstract

In this paper, the robustness property of 2nd-order iterative learning control(ILC) method for a class of linear and nonlinear discrete-time dynamic systems is studied. 2nd-order ILC method has the PD-type learning algorithm based on both time-domain performance and iteration-domain performance. It is proved that the 2nd-order ILC method has robustness in the presence of state disturbances, measurement noise and initial state error. In the absence of state disturbances, measurement noise and initialization error, the convergence of the 2nd-order ILC algorithm is guaranteed. A numerical example is given to show the robustness and convergence property according to the learning parameters.

Key words : Robustness, 2nd-order Iterative Learning Control, PD-type Learning Algorithm, Discrete-time, Dynamic Systems.

1. Introduction

Ever since Arimoto suggested ILC methodology[1], there have been a number of efforts to improve and apply ILC method. In fact, ILC can be easily applied to the repetitive tasks that is in many robotic industrial operations since it requires less a priori knowledge about the controlled system in the controller design phase and it has the capability of modifying an unsatisfactory control input signal based on the knowledge of previous operations of the same task[2-10]. Also, ILC is known to guarantee an eventual uniform tracking performance as the algorithm repetitively applies.

External disturbances such as state disturbances, measurement noise and initial state error are inevitable in the real control systems. This disturbances can have an bad effect on the ILC system and make the system diverge by its iterative property. Therefore, the robustness problem of ILC has been studied by many researchers[11-20]. Lee and Bien[11] reported the possibility of divergence of control input due to the initial state error. Lee and Bien[12, 13] showed that the trajectory errors can be estimated in terms of initial state error and parameters of ILC algorithm. Heinzinger et. al. have studied the robustness properties of a class of learning control algorithm for the nonlinear system[14]. Saab proved the convergence and the robustness of both P-type learning control for the nonlinear time varying system and D-type learning control for the linear discrete-time system[16, 17]. Park et. al. investigated the effect of initial state error in the PID-type ILC systems[19].

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Bien and Hur[10] proposed the higher-order ILC method that utilize more than one past error history contained in the trajectories generated at prior iterations. It was shown that the higher-order ILC can improve the convergence performance and the robustness to the disturbances by using the multiple past-history data pairs at the expense of additional storage. However, this ILC method can be applied to the dynamic system that has the direct linkage between the input and the output and there may arise some difficulty in finding the suitable weighting matrices satisfying the convergence conditions, especially when the number of past-history data pairs is large[10,18].

Kim and Bien[20] proposed 2nd-order PD-type ILC algorithm based on both time-domain performance and iteration-domain performance for linear continuous-time and discrete-time dynamic systems. The convergence of the 2nd-order PD-type ILC algorithm was proved[20]. In this paper, we study the robustness property of 2nd-order PD-type ILC method for linear discrete-time dynamic systems. Then 2nd-order ILC method for nonlinear discrete-time dynamic systems is proposed and the robustness of the proposed ILC algorithm is investigated. A numerical example is given to show the robustness and the convergence property according to parameters change.

In this paper, the following notational convention is adopted : k is the iteration number; $x(i)$ is state vectors, $u(i)$ is control input vector and $y(i)$ is output vector for discrete-time systems; I_r is $r \times r$ identity matrix; $\| x \|$ denotes the Euclidean norm of a vector x; $\| A \|$ denotes the induced matrix norm of a matrix A; $\| x \|_\infty$ denotes the infinity norm of a vector x [21]. The λ_d norm for a time function $g : [0, N] \to R^n$ is defined as follows[20].

$$\| g(\cdot) \|_{\lambda_d} = sup_{i \in [0,N]} e^{\lambda i} \| g(i) \|$$

where $\lambda > 0$ if $a > 1$ and $\lambda < 0$ if $a < 1$. From the λ_d norm definition, it is obvious that $\| f \|_{\lambda_d} \le \| f \|_\infty \le e^{\lambda T} \| f \|_{\lambda_d}$, implying that the λ_d norm and infinity norm are equivalent[21]. In this paper, the robustness of the 2nd-order ILC is proved by employing the λ_d norm.

2. Robustness of 2nd-order ILC for linear discrete-time systems

In this section, a robust ILC algorithm for linear discrete-time dynamic systems is proposed. Consider the linear discrete-time dynamical system described by

$$\begin{aligned} x_k(i+1) &= A x_k(i) + B u_k(i) + w_k(i) \\ y_k(i) &= C x_k(i) + v_k(i) \end{aligned} \tag{1}$$

where $x_k \in R^n$, $u_k \in R^r$ and $y_k \in R^r$ denote the state vector, input vector and output vector respectively. $w_k \in R^n$ and $v_k \in R^r$ denote state disturbance and output measurement noise. A, B and C are constant matrices with appropriate dimensions. We assume the following properties.

A1 $w_k(i)$ and $v_k(i)$ are bounded by b_w and b_v, $\forall k$, $i \in [0,N]$. i.e., $\forall k, \sup_{i \in [0,N]} \| w_k(i) \| \le b_w$ and $\forall k$, $\sup_{i \in [0,N]} \| v_k(i) \| \le b_v$.

A2 Initial state error is bounded by b_{x0}, $\forall k$ in $[0,N]$. i.e., $\| x_0 - x_k(0) \| \le b_{x0}$ for $\forall k$.

Let $y_d(i)$ be the desired output trajectory on $i \in [0,N]$. Then the 2nd-order PD-type ILC learning law for the system (1) can be described as follows[20].

$$\begin{aligned} u_{k+1}(i) = \; & u_k(i) + \Gamma[\delta y_k(i+1) + \Lambda \delta y_k(i) \\ & + \Phi(\delta y_k(i) - \Theta \delta y_{k-1}(i))] \end{aligned} \tag{2}$$

where $\delta y_k(i) = y_d(i) - y_k(i)$, $i = 0, 1, \cdots, N$ and Γ, Λ, Φ, Θ are the parameters of learning law.

Theorem 1 Let the system described by (1) satisfy the assumptions A1-A2 and use the learning law (2). For a desired initial state $x_d(0)$ and a desired output trajectory $y_d(i)$, $i \in [0,N]$ which are achievable[14], if $\| I - \Gamma CB \| \le \rho < 1$, then input error between u_d and u_k is bounded as $k \to \infty$. Also, state error and output error are bounded. These bounds depend on the bounds of initial state error, state disturbance and measurement noise. Moreover, whenever b_w, b_v and b_{x0} tend to zero, state error, output error, and input error converge uniformly to zero as $k \to \infty$.

Proof

From (1) and (2), the input error can be written as

$$\begin{aligned} u_d(i) - u_{k+1}(i) = \; & u_d(i) - u_k(i) + \Gamma[C(x_k(i+1) \\ & - x_d(i+1)) + v_k(i+1) \\ & + (\Lambda + \Phi)(Cx_k(i) - Cx_d(i) + v_k(i)) \\ & - \Phi\Theta(Cx_{k-1}(i) - Cx_d(i)) + v_{k-1}(i))] \\ = \; & (I - \Gamma CB)(u_d(i) - u_k(i)) \\ & - \Gamma(CA + \Lambda C + \Phi C)(x_d(i) - x_k(i)) \\ & + \Gamma\Phi\Theta C(x_d(i) - x_{k-1}(i)) + \Gamma C w_k(i) \\ & + \Gamma v_k(i+1) + \Gamma(\Lambda + \Phi) v_k(i) \\ & - \Gamma\Phi\Theta v_{k-1}(i) \end{aligned} \tag{3}$$

Taking the norm, and by using $\delta u_k \triangleq u_d - u_k$ and the bounds, we have

$$\begin{aligned} \| \delta u_{k+1}(i) \| \le \; & \| I - \Gamma CB \| \, \| \delta u_k(i) \| \\ & + \| \Gamma(CA + \Lambda C + \Phi C) \| \, \| x_d(i) - x_k(i) \| \\ & + \| \Gamma\Phi\Theta C \| \, \| x_d(i) - x_{k-1}(i) \| \\ & + \| \Gamma C \| \, \| w_k(i) \| + \| \Gamma \| \, \| v_k(i+1) \| \\ & + \| \Gamma(\Lambda + \Phi) \| \, \| v_k(i) \| \\ & + \| \Gamma\Phi\Theta \| \, \| v_{k-1}(i) \| \\ \le \; & \rho \| \delta u_k(i) \| + h_0 \| x_d(i) - x_k(i) \| \\ & + h_1 \| x_d(i) - x_{k-1}(i) \| \\ & + b_{\Gamma C} b_w + (b_\Gamma + b_{\Gamma\Lambda\Phi} + b_{\Gamma\Phi\Theta}) b_v \end{aligned} \tag{4}$$

where $\rho \triangleq \| I - \Gamma CB \|$, $h_0 \triangleq \| \Gamma(CA + \Lambda C + \Phi C) \|$, $h_1 \triangleq \| \Gamma\Phi\Theta C \|$, $b_\Gamma \triangleq \| \Gamma \|$, $b_{\Gamma C} \triangleq \| \Gamma C \|$, $b_{\Gamma\Lambda\Phi} \triangleq \| \Gamma(\Lambda + \Phi) \|$, and $b_{\Gamma\Phi\Theta} \triangleq \| \Gamma\Phi\Theta \|$.

Now writing a summation expression for $x_d(i) - x_k(i)$, we have

$$\begin{aligned} x_d(i) - x_{k+1}(i) = \; & A^i(x_d(0) - x_k(0)) + \sum_{m=0}^{i-1} A^{i-m-1} \\ & \cdot [B(u_d(m) - u_k(m)) + w(m)]. \end{aligned} \tag{5}$$

Taking norms and using the bounds, we obtain

$$\begin{aligned} \| x_d(i) - x_k(i) \| \le \; & a^i \| x_d(0) - x_k(0) \| + \sum_{m=0}^{i-1} a^{i-m-1} \\ & \cdot [b_B \| u_d(m) - u_k(m) \| + b_w], \end{aligned} \tag{6}$$

where $a = \| A \|$ and $b_B = \| B \|$.

Inserting (6) in (4), we can obtain

$$\begin{aligned} \| \delta u_{k+1}(i) \| \le \; & \rho \| \delta u_k(i) \| + h_0 b_B \sum_{m=0}^{i-1} a^{i-m-1} \| \delta u_k(m) \| \\ & + h_1 b_B \sum_{m=0}^{i-1} a^{i-m-1} \| \delta u_{k-1}(m) \| \\ & + (h_0 + h_1) \, b_w \sum_{m=1}^{i-1} a^{i-m-1} \\ & + (h_0 + h_1) \, a^i \, b_{x0} + b_{\Gamma C} \, b_w \\ & + (b_\Gamma + b_{\Gamma\Lambda\Phi} + b_{\Gamma\Phi\Theta}) \, b_v. \end{aligned} \tag{7}$$

By multiplying both side of (7) by $a^{-\lambda i}$ and taking the λ_d norm,

$$\begin{aligned}\|\delta u_{k+1}(i)\|_{\lambda_d} \le\ & \rho\|\delta u_k(i)\|_{\lambda_d} + h_0 b_B \sup_{i\in[0,N]} a^{-(\lambda-1)i} \sum_{m=0}^{i-1} a^{(\lambda-1)m} \sup_{m\in[0,N]} a^{-\lambda m}\|\delta u_k(m)\| \\ &+ h_1 b_B \sup_{i\in[0,N]} a^{-(\lambda-1)i}\sum_{m=0}^{i-1} a^{(\lambda-1)m} \cdot \sup_{m\in[0,N]} a^{-\lambda m}\|\delta u_{k-1}(m)\| \\ &+ (h_0+h_1) b_{x0} \sup_{i\in[0,N]} a^{-\lambda i} a^i \\ &+ (h_0+h_1) b_w \sup_{i\in[0,N]} a^{-\lambda i}\sum_{m=0}^{i-1} a^{i-m-1} \\ &+ (b_{\Gamma C} b_w + (b_\Gamma + b_{\Gamma\Lambda\Phi} + b_{\Gamma\Phi\Theta})\, b_v) \cdot \sup_{i\in[0,N]} a^{-\lambda i} \\ \le\ & (\rho + k_1 a^{-1}\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1})\ \|\delta u_k(i)\|_{\lambda_d} \\ &+ k_2 a^{-1}(\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1})\|\delta u_{k-1}(i)\|_{\lambda_d} \\ &+ k_3\, b_{x0} + (k_3\, k_4 + b_{\Gamma C})\, b_w \\ &+ (b_\Gamma + b_{\Gamma\Lambda\Phi} + b_{\Gamma\Phi\Theta})b_v\ ,\end{aligned} \tag{8}$$

where $k_1 = h_0 b_B$, $k_2 = h_1 b_B$, $k_3 = h_0 + h_1$, and $k_4 = \sup_{i\in[0,N]} a^{-\lambda i}\sum_{m=0}^{i-1} a^{i-m-1}$. Define $\rho_1 = \rho + k_1 a^{-1} \cdot (\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1})$, $\rho_2 = k_2 a^{-1}(\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1})$, and $\varepsilon = k_3 b_{x0} + (k_3 k_4 + b_{\Gamma C}) b_w + (b_\Gamma + b_{\Gamma\Lambda\Phi} + b_{\Gamma\Phi\Theta}) b_v$.

Then the inequality (8) yields

$$\|\delta u_{k+1}(i)\|_{\lambda_d} \le \rho_1\|\delta u_k(i)\|_{\lambda_d} + \rho_2\|\delta u_{k-1}(i)\|_{\lambda_d} + \varepsilon. \tag{9}$$

Since $0 \le \rho < 1$ by assumption, it is possible to choose λ sufficiently large so that

$$\begin{aligned}\rho_1 + \rho_2 =\ & \rho + k_1 a^{-1}(\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1}) \\ &+ k_2 a^{-1}(\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1}) < 1.\end{aligned} \tag{10}$$

Thus, (9) implies

$$\lim_{k\to\infty}\|\delta u_k(i)\|_{\lambda_d} \le \frac{\varepsilon}{1-(\rho_1+\rho_2)}\ . \tag{11}$$

Since ε is bounded, (11) implies that the input error is bounded $\forall k$ in $[0,N]$. Also, we can easily show that $\lim_{k\to\infty}\|\delta u_k(i)\|_{\lambda_d} = 0$, whenever $\varepsilon \to 0$. Using (6) and (11), the state error bound is obtained as

$$\lim_{k\to\infty}\|\delta x_k(i)\|_{\lambda_d} \le b_{x0} + b_B a^{-1}(\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1}) \cdot \frac{\epsilon}{1-(\rho_1+\rho_2)} + b_w\, k_4 \tag{12}$$

and the output error bound is also obtained as

$$\lim_{k\to\infty}\|\delta y_k(i)\|_{\lambda_d} \le b_c[b_{x0} + b_B a^{-1}(\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1}) \cdot \frac{\epsilon}{1-(\rho_1+\rho_2)} + b_w k_4] + b_v. \tag{13}$$

Therefore, (12) and (13) imply that the state and output error are bounded. This completes the proof.

3. Robustness of 2nd-order ILC for a Class of nonlinear discrete-time systems

Consider a class of nonlinear discrete-time dynamic system described by

$$\begin{aligned} x_k(i+1) &= f(x_k(i),i) + B(x_k(i),i)u_k(i) + w_k(i), \\ y_k(i) &= Cx_k + v_k(i)\end{aligned} \tag{14}$$

where $x_k(i)\in R^n, u_k(i)\in R^r, y_k(i)\in R^r, w_k(i)\in R^n$, and $v_k(i)\in R^r$ denote the state, control input, system output, state disturbance, and output noise, respectively. We assume the following properties.

B1 For each fixed $x_k(0)$ with $w_k = 0$ and $v_k = 0$, the state mapping S and the output mapping R are one-to-one. That is, $x_k(\cdot) = S(x_k(0), u_k(\cdot))$ and $y_k(\cdot) = R(x_k(0), u_k(\cdot))$.

B2 The functions $f(x_k, i)$ and $B(x_k, i)$ are uniformly globally Lipschitz in x on the interval $[0,N]$.

B3 The function $B(x_k, i)$ is bounded on $R^n \times [0,N]$.

B4 $w_k(i)$ and $v_k(i)$ are bounded by b_w and b_v, $\forall k$, $i\in[0,N]$. i.e., $\forall k$, $\sup_{i\in[0,N]}\|w_k(i)\| \le b_w$ and $\forall k$, $\sup_{i\in[0,N]}\|v_k(i)\| \le b_v$.

B5 Initialization error is bounded by b_{x0}, $\forall k\in[0,N]$. i.e., $\|x_0 - x_k(0)\| \le b_{x0}$.

Then the 2nd-order PD-type ILC learning law for the nonlinear system (14) is proposed as follows.

$$\begin{aligned} u_{k+1}(i) =\ & u_k(i) + \Gamma(y_k(i), i)[\delta y_k(i+1) + \Lambda\delta y_k(i) \\ &+ \Phi(\delta y_k(i) - \Theta\delta y_{k-1}(i))]\end{aligned} \tag{15}$$

where $\delta y_k(i) = y_d(i) - y_k(i)$, $i = 0, 1, \cdots, N$ and Γ, Λ, Φ, Θ are the learning parameters.

For proof clarification, function parameters will be shown in subscript notation as: $f_k \triangleq f(x_k(i), i)$, $f_d \triangleq f(x_d(i), i)$, $u_k \triangleq u_k(i)$, $u_d \triangleq u_d(i)$, $w_k \triangleq w_k(i)$, $v_k \triangleq v_k(i)$, $B_k \triangleq B(x_{(i)}, i)$, $B_d \triangleq B(x_d(i), i)$, $y_k \triangleq y_k(i)$, $y_d \triangleq y_d(i)$, $\Gamma_k \triangleq \Gamma(y_k(i), i)$, and k_f, k_B are the Lipschitz constants for f_k and B_k, respectively.

Theorem 2 Let the nonlinear system described by (14) satisfy the assumptions B1-B5 and use the learning law (15). For a desired initial state $x_d(0)$ and a desired output trajectory y_d, which are achievable [14], if $\| I-\Gamma_k CB_k \| \le \rho < 1$, then input error between u_d and u_k is bounded as $k \to \infty$. In addition, state error and output error are bounded. The bounds of input error, state error and output error depend on the bounds on initial state error, state disturbance, and measurement noise. Moreover, whenever b_w, b_v, and b_{x0} tend to zero, state error, output error, and input error converge uniformly to zero as $k \to \infty$.

Proof

From (14) and (15), the input error can be written as

$$\begin{aligned} u_d - u_{k+1} &= u_d - u_k - \Gamma_k [C(f_d + B_d u_d) \\ &\quad - C(f_k + B_k u_k + w_k) - v_k(i+1) \\ &\quad + (\Lambda + \Phi)(Cx_d - Cx_k + v_k) \\ &\quad - \Phi\Theta(Cx_d - Cx_{k-1}) + v_{k-1})] \\ &= (I - \Gamma_k CB_k)(u_d - u_k) - \Gamma_k (C(f_d - f_k) \\ &\quad + C(B_d - B_k)u_d) + \Gamma_k \Phi\Theta C(x_d - x_{k-1}) \\ &\quad - \Gamma_k(\Lambda C + \Phi C)(x_d - x_k) + \Gamma_k C w_k \\ &\quad + \Gamma_k v_k(i+1) + \Gamma_k(\Lambda+\Phi)v_k \\ &\quad + \Gamma_k \Phi\Theta v_{k-1} \end{aligned} \tag{16}$$

Taking the norm, and by using $\delta u_k \triangleq u_d - u_k$ and $b_{ud} \triangleq sup_{i\in[0,N]} \| u_d \|$, we can obtain

$$\begin{aligned} \| \delta u_{k+1} \| &\le \| I - \Gamma_k CB_k \| \, \| \delta u_k \| \\ &\quad + \| \Gamma_k \| \, (k_f + k_B b_{ud}) \| C \| \, \| x_d - x_k \| \\ &\quad + \| \Gamma_k (\Lambda + \Phi) C \| \, \| x_d - x_k \| \\ &\quad + \| \Gamma_k \Phi\Theta C \| \, \| x_d - x_{k-1} \| \\ &\quad + \| \Gamma_k C \| \, \| w_k \| + \| \Gamma_k \| \, \| v_k(i+1) \| \\ &\quad + \| \Gamma_k (\Lambda+\Phi) \| \, \| v_k \| + \| \Gamma_k \Phi\Theta \| \, \| v_{k-1} \| \\ &\le \rho \| \delta u_k \| + h_0 \| x_d - x_k \| \\ &\quad + h_1 \| x_d - x_{k-1} \| + b_\Gamma b_C b_w \\ &\quad + (b_\Gamma + b_{\Gamma_k} b_{\Lambda\Phi} + b_\Gamma b_{\Phi\Theta}) b_v \end{aligned} \tag{17}$$

where $\rho = \| I - \Gamma_k CB_k \|$, $b_\Gamma = \| \Gamma_k \|$, $b_C = \| C \|$, $b_{\Lambda\Phi} = \| \Lambda + \Phi \|$, $b_{\Phi\Theta} = \| \Phi\Theta \|$, $h_0 = b_\Gamma (k_f + k_B b_{ud} b_{\Lambda\Phi}) b_C$, and $h_1 = b_\Gamma b_{\Phi\Theta} b_C$.

Taking the norm and using the Lipschitz conditions, we have

$$\begin{aligned} \| x_d(i+1) - x_k(i+1) \| &= \| f_d + B_d u_d - (f_k + B_k u_k + w_k) \|, \\ &\le k_f \| x_d - x_k \| + k_B \| x_d - x_k \| \, b_{ud} \\ &\quad + b_B \| u_d - u_k \| + \| w_k \|, \\ &\le (k_f + k_B b_{ud}) \| x_d - x_k \| \\ &\quad + b_B \| u_d - u_k \| + b_w. \end{aligned} \tag{18}$$

Now writing a summation expression for $\| x_d(i) - x_{k+1}(i) \|$,

$$\| x_d - x_{k+1} \| \le a^i \| x_d(0) - x_k(0) \| + \sum_{m=0}^{i-1} a^{i-m-1} [b_B \| u_d(m) - u_k(m) \| + b_w] \tag{19}$$

where $a = k_f + k_B b_{ud}$.

Inserting (19) in (17), we obtain

$$\begin{aligned} \| \delta u_{k+1} \| &\le \rho \| \delta u_k \| + h_0 b_B \sum_{m=0}^{i-1} a^{i-m-1} \| \delta u_k(m) \| \\ &\quad + h_1 b_B \sum_{m=0}^{i-1} a^{i-m-1} \| \delta u_{k-1}(m) \| \\ &\quad + (h_0 + h_1) b_w \sum_{m=1}^{i-1} a_{i-m-1} \\ &\quad + (h_0 + h_1) a^i b_{x0} + b_\Gamma b_C b_w \\ &\quad + b_\Gamma (1 + b_{\Lambda\Phi} + b_{\Phi\Theta}) b_v. \end{aligned} \tag{20}$$

By multiplying both side of (20) by $a^{-\lambda i}$ and taking the λ_d norm,

$$\begin{aligned} \| \delta u_{k+1}(i) \|_{\lambda_d} &\le \rho \| \delta u_k(i) \|_{\lambda_d} + h_0 b_B \sup_{i\in[0,N]} a^{-(\lambda-1)i} \\ &\quad \cdot \sum_{m=0}^{i-1} a^{(\lambda-1)m} \sup_{m\in[0,N]} a^{-\lambda m} \| \delta u_k(m) \| \\ &\quad + h_1 b_B \sup_{i\in[0,N]} a^{-(\lambda-1)i} \sum_{m=0}^{i-1} a^{(\lambda-1)m} \\ &\quad \cdot \sup_{m\in[0,N]} a^{-\lambda m} \| \delta u_{k-1}(m) \| \\ &\quad + (h_0 + h_1) b_{x0} \sup_{i\in[0,N]} a^{-\lambda i} a^i \\ &\quad + (h_0 + h_1) b_w \sup_{i\in[0,N]} a^{-\lambda i} \sum_{m=0}^{i-1} a^{i-m-1} \\ &\quad + b_\Gamma (b_C b_w + (1 + b_{\Lambda\Phi} + b_{\Phi\Theta}) b_v) \\ &\quad \cdot \sup_{i\in[0,N]} a^{-\lambda i} \\ &\le (\rho + h_0 b_B a^{-1} \frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1}) \| \delta u_k(i) \|_{\lambda_d} \\ &\quad + h_1 b_B a^{-1} (\frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1}) \| \delta u_{k-1}(i) \|_{\lambda_d} \\ &\quad + (h_0 + h_1) b_{x0} + ((h_0 + h_1) k_1 + b_\Gamma b_C) b_w \\ &\quad + b_\Gamma (1 + b_{\Lambda\Phi} + b_{\Phi\Theta}) b_v \\ &= \rho_1 \| \delta u_k \|_{\lambda_d} + \rho_2 \| \delta u_{k-1} \|_{\lambda_d} + \varepsilon \end{aligned} \tag{21}$$

where $k_1 = \sup_{i\in[0,N]} a^{-\lambda i} \sum_{m=0}^{i-1} a^{i-m-1}$, $\rho_1 = \rho + h_0 b_B a^{-1} (\frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1})$, $\rho_2 = h_1 b_B a^{-1} (\frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1})$, and $\varepsilon = (h_0 + h_1) b_{x0} + ((h_0 + h_1) k_1 + b_\Gamma b_C) b_w + b_\Gamma (1 + b_{\Lambda\Phi} + b_{\Phi\Theta}) b_v$.

Since $0 \le \rho < 1$ by assumption, it is possible to choose λ sufficiently large so that

$$\rho_1 + \rho_2 = \rho + k_1 a^{-1} (\frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1}) + k_2 a^{-1} (\frac{1 - a^{-(\lambda-1)N}}{a^{\lambda-1} - 1}) < 1 . \tag{22}$$

Thus, we can easily show that, whenever all disturbances tend to zero, i.e., $\varepsilon \to 0$, $\lim_{k\to\infty} \| \delta u_k(i) \|_{\lambda_d}$

$=0$ [15]. The equation (21) implies

$$\lim_{k\to\infty} \| \delta u_k(i) \|_{\lambda_d} \le \frac{\varepsilon}{1-(\rho_1+\rho_2)}. \quad (23)$$

Since ε is bounded, (23) implies that the input error is bounded $\forall k$ in $[0,N]$. Using (19) and (23), we obtain the state error bound

$$\lim_{k\to\infty} \| \delta x_k(i) \|_{\lambda_d} \le b_{x0} + b_B a^{-1}\left(\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1}\right) \cdot \frac{\varepsilon}{1-(\rho_1+\rho_2)} + b_w k_1 \quad (24)$$

and the output error bound

$$\lim_{k\to\infty} \| \delta y_k(i) \|_{\lambda_d} \le b_c[b_{x0} + b_B a^{-1}\left(\frac{1-a^{-(\lambda-1)N}}{a^{\lambda-1}-1}\right) \cdot \frac{\varepsilon}{1-(\rho_1+\rho_2)} + b_w k_1] + b_v . \quad (25)$$

Therefore, (24) and (25) implies that the state error and output error are bounded. This completes the proof.

4. Simulation Example

In the following, we shall consider linear discrete-time system

$$\begin{bmatrix} x_1(i+1) \\ x_2(i+1) \end{bmatrix} = \begin{bmatrix} 0.9953 & 0.0905 \\ -0.0905 & 0.8144 \end{bmatrix} \begin{bmatrix} x_1(i) \\ x_2(i) \end{bmatrix} + \begin{bmatrix} 0.0047 \\ 0.0905 \end{bmatrix} u(i) + \begin{bmatrix} 0.02 \\ 0 \end{bmatrix} w(i) \quad (26)$$

$$y(i) = [0 \ \ 1] \begin{bmatrix} x_1(i) \\ x_2(i) \end{bmatrix} + 0.05\, v(i), \quad (27)$$

where $w(i)$ and $v(i)$ are state disturbance and measurement noise that are random numbers whose elements are normally distributed with mean 0 and variance 1. We assume that the initial state error is normally distributed random numbers bounded by 0.1.

Also, suppose that the desired output trajectory is given by

$$y_d(i) = 2\sin(0.04\pi i) \qquad i=1,2,\cdots,50. \quad (28)$$

Γ is chosen as 5.0 based on the condition $\| I-\Gamma CB \| \le \rho < 1$. Let us assume that $\Phi=-0.3$. When $\Gamma=5.0$, $\Phi=-0.3$, $\Lambda=0$ and $\Theta=0$, the PD-type learning algorithm shows good performance as $\sum_{i=1}^{50} | e_{50}(i) | = 5.22$ and $\sum_{k=1}^{50}\sum_{i=1}^{50} | e_k(i) | = 361.67$. The result in Figure 1 shows the sum of error according to the parameters Λ and Θ at 50th iteration, that is, $\sum_{i=1}^{50} | e_{50}(i) |$. When $\Lambda=-0.2$ and $\Theta=0.1$, we can obtain the best result as 1.735. Figure 2 shows that the total sum of error, $\sum_{k=1}^{50}\sum_{i=1}^{50} | e_k(i) |$ is bounded and the best result is 263.65 when $\Lambda=-0.3$ and $\Theta=0.1$. From this result, we can see that the 2nd-order PD-type learning law is robust to state disturbance, measurement noise and initial state error. Also, we can find that the tracking performance can be improved by choosing the suitable parameters of learning law and depends on the bounds on initial state error, state disturbance and measurement noise.

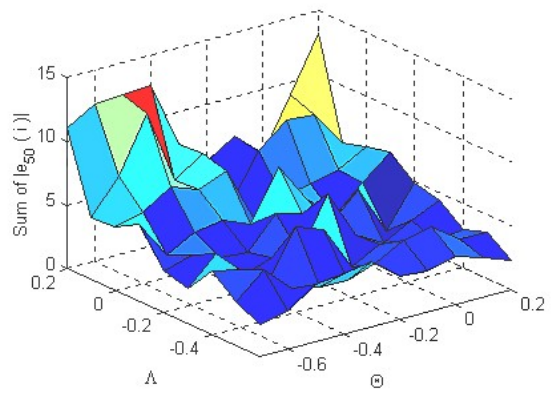

Fig. 1. $\sum_{i=1}^{50} | e_{50}(i) |$ under state disturbances, measurement noise and initial state error.

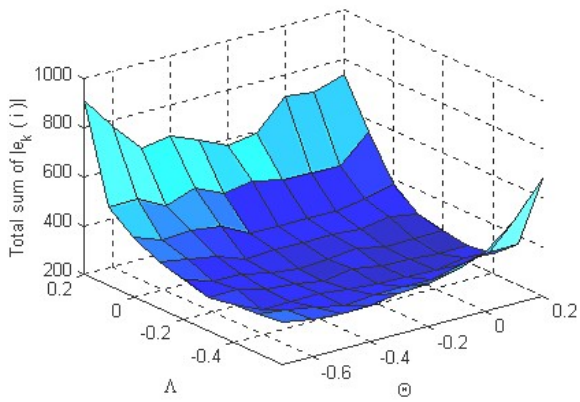

Fig. 2. $\sum_{k=1}^{50}\sum_{i=1}^{50} | e_k(i) |$ under state disturbances, measurement noise and initial state error.

5. Conclusion

In this paper, the robustness property of 2nd-order ILC method for a class of linear and nonlinear discrete-time dynamic systems is investigated. It is proved that the bounds of input error, state error and output error depend on the bounds of initial state error, state disturbance and measurement noise. Also, it is shown that, whenever initialization error, state disturbances and measurement noise tend to zero, the state, output, and input error converge uniformly to zero. A numerical example is given to show the robustness of the 2nd-order ILC method for the discrete-time dynamic system. Robustness of the ILC method for nonlinear systems of more general form will be studied against initial state error, state disturbance and measurement noise.

References

[1] S. Arimoto, S. Kawamura and F. Miyazaki, “Bettering Operation of Robots by Learning”, Journal Robotic System, Vol. 1, pp. 123-140, 1984.
[2] S. Arimoto, “Mathematical Theory of Learning with Applications to Robot Control”, in Adaptive and Learning Systems, edited by K.S.Narendra, New York: Plenum Press, 1986.
[3] J. E. Hauser, “Learning Control for a Class of Nonlinear Systems”, Proc. 26th IEEE Conf. Decision and Control, pp. 859-860, 1987.
[4] M. Kawato, Y. Uno, M. Isobe and R. Suzuki, “A Hierarchical Neural Network Model for Voluntary Movement with Application to Robotics”, IEEE Control Systems Magazine, Vol. 8, pp. 8-16, 1988.
[5] P. Bondi, G. Casalina and L. Gambardella, “On the Iterative Learning Control Theory for Robotic Manipulators”, IEEE J. of Robotics Automation, Vol. 4, pp. 14-21, 1988.
[6] S. R. Oh, Z. Bien and I. H. Suh, “An Iterative Learning Control Method With Application For The Robot Manipulator”, IEEE J. Robotics Automation, Vol. 4, pp. 508-514, 1988.
[7] Z. Bien, D. H. Hwang and S.R. Oh, “A Nonlinear Iterative Learning Method for Robot Path Control”, Robotica, Vol. 9, pp. 387-392, 1991.
[8] D. H. Hwang, Z. Bien and S.R. Oh, “Iterative Learning Control Method For Discrete-time Dynamic Systems”, IEE Proc. Part. D, Vol. 138, pp. 139-144, 1991.
[9] J. W. Lee, H. S. Lee and Z. Bien, “Iterative Learning Control with Feedback Using Fourier Series with Application to Robot Trajectory Tracking”, Robotica, Vol. 11, pp. 291-298, 1993.
[10] Z. Bien and K. M. Huh, “Higher-Order Iterative Learning Control Algorithm”, IEE Proc. Part. D, Vol. 136, pp. 105-112, 1989.
[11] K. H. Lee and Z. Bien, “Initial Condition Problem of Learning Control”, IEE Proc. Part. D, Vol. 138, pp. 525-528, 1991.
[12] H. S. Lee and Z. Bien, “Study on Robustness of Iterative Learning Control with Non-zero Initial Error”, Int. Journal of Control, Vol. 63, pp. 345-359, 1996.
[13] H. S. Lee, “Study on the Robustness and Convergence Properties of Iterative Learning Control,” Ph.D., Dissertation, KAIST, Korea, 1996.
[14] G. Heinzinger, D. Fenwick, B. Paden and F. Miyazaki, “Stability of Learning Control with Disturbances and Uncertain Initial Conditions”, IEEE Tr. on Automatic Control, Vol. 37, pp. 110-114, 1992.
[15] Y. Chen, M. Sun, B. Huang, and H. Dou, “Robust Higher Order Repetitive Learning Control Algorithm for Tracking Control of Delayed Repetitive Systems”, Proc. 31th IEEE Conf. Decision and Control, Arizona, USA, pp. 2504-2510, 1992.
[16] S. S. Saab, “On the P-type Learning Control”, IEEE Tr. on Automatic Control, Vol. 39, pp. 2298-2302, 1994.
[17] S. S. Saab, “A Discrete-Time Learning Control Algorithm for a Class of Linear Time-Invariant Systems”, IEEE Tr. on Automatic Control, Vol. 40, pp. 1138-1142, 1995.
[18] S. K. Tso and L. X. Y. Ma, “A Self-Contained Iterative Learning Controller for Feedback Control of Linear Systems”, Proc. of the Asian Control Conference, Vol. 1, pp. 545-548, 1994.
[19] K. H. Park, Z. Bien and D. H. Hwang, “A Study on the PID-type Iterative Learning Controller against Initial State Error,” International Journal of Systems Science, Vol. 30, pp. 49-59, 1999.
[20] Y. T. Kim and Z. Bien, “2nd-Order PD-type Learning Control Algorithm”, Journal of Fuzzy Logic and Intelligent Systems. Vol. 14, No. 2, pp. 247-252, 2004.
[21] A. W. Naylor and G. R. Sell, Linear Operator Theory in Engineering and Science, Holt, Rinehart and Winston, Inc., 1971.

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