

Analysis of Compression Ignition Combustion in a Schnurle-Type Gasoline Engine - Comparison of performance between direct injection and port injection systems -

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A two-stroke Schnurle-type gasoline engine was modified to enable compression-ignition in both the port fuel injection and the in-cylinder direct injection. Using the engine, examinations of compression-ignition operation and engine performance tests were carried out. The amount of the residual gas and the in-cylinder mixture conditions were controlled by varying the valve angle rate of the exhaust valve (VAR) and the injection timing for direct injection conditions. It was found that the direct injection system is superior to the port injection system in terms of exhaust gas emissions and thermal efficiency, and that almost the same operational region of compression-ignition at medium speeds and loads was attained. Some interesting combustion characteristics, such as a shorter combustion period in higher engine speed conditions, and factors for the onset of compression-ignition were also examined.

Key Words : Gasoline Engine, Compression-Ignition Engine, Two-Stroke Engine,
Fuel Injection, Homogeneous Charge Compression Ignition

1. Introduction

As reductions of both the exhaust gas emissions and the fuel consumption rate are required for internal combustion engines, compression-ignition combustion has been actively examined as one of the solutions. A combination with a two-stroke Schnurle-type gasoline engine, in particular, can reduce the cycle-to-cycle variation and the amount of HC (Hydrocarbon) emissions (Ishibashi, 2000; Iida, 1999; Onishi et al., 1979).

Thereby, this system has been commercialized for a motorcycle engine. Ishibashi et al. (1998) showed that stabilized combustion at partial loads, an improvement of the fuel consumption in LA4 mode by 55% and a reduction of HC emissions by 85% could be attained by varying the EOR (Exhaust Opening Rate). However, compression-ignition combustion at low engine speeds and low loads was not accomplished and the ignition-timing control was impossible.

If stable compression-ignition combustion is possible with the in-cylinder direct injection system, the irregular combustion at low loads and the short-circuiting, which are drawbacks of a two-stroke engine, can be resolved. The authors propose a new concept: the optimized performance can be achieved by homogeneous spark-ignition combustion at high loads, homogeneous

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compression-ignition combustion at medium loads and stratified compression-ignition combustion at low loads.

In this research, the authors first tested the in-cylinder direct injection method to realize the concept. The feasibility of the compression-ignition combustion at medium loads and speeds, the effects of the fuel supply method on the performance of compression-ignition combustion, and the possibility to control the ignition timing by varying the injection timing were examined.

2. Experimental Apparatus

2.1 Test engine

The test engine was a water-cooled single-cylinder Schnurle-type two-stroke gasoline engine whose cylinder head, intake port, and scavenging port have been modified to enable operation in in-cylinder direct injection or port injection. Major specifications are listed in Table 1 and the setup of the engine is shown in Fig. 1. The figure shows the locations of the pressure transducers and temperature sensors. The pressure in the cylinder was measured with a piezo-type sensor (KISTLER 601A), and the scavenging pressure and the exhaust pressure were measured with a semiconductor sensor (TOYODA PMS-5) and a strain-gauge sensor (KYOWAPE-5KRMT), respectively. Also the scavenging and exhaust temperatures were measured with K-type thermocouples.

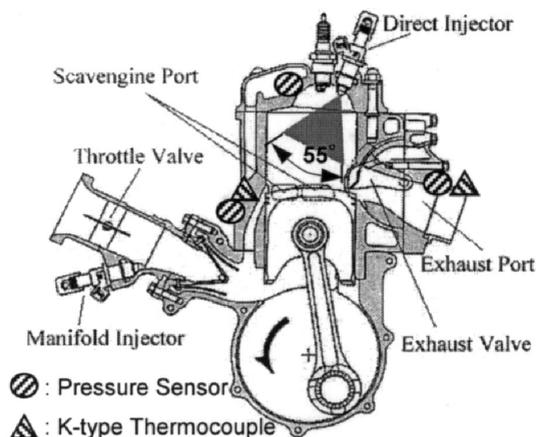


Fig. 1 Test engine configuration

Table 1 Engine specifications

Engine Type	2-Stroke Single Cylinder
Fuel Supply	Port Injection, Direct Injection
Bore×Stroke	$\phi 66.4 \times 72.0$ mm
Displacement	250 cc
Effective C. R.	6.7~9.6
Exh. Port Open	56~93 deg. BBDC
Scav. Port Open	55 deg. BBDC
Exh. Port Area	13~3 cm ²

A swirl-type injector for in-cylinder direct injection was employed with a spray angle of 55° and with fuel pressure of 5 Mpa, while a hole-type injector for the port injection was employed with fuel pressure of 0.3 MPa. In case of direct injection, as shown in Fig. 1, the injector attachment angle was set to 25° so that it faces the scavenging port at the opposite of the exhaust port (Moriyoshi et al., 1999), in order to reduce the short-circuiting and prevent the deterioration of the maximum output power.

Regular gasoline was used for the experimental fuel. The exhaust valve located at the exhaust port was used to adjust the amount of the internal EGR (Exhaust Gas Recirculation). The VAR (Valve Angle Rate of the Exhaust Valve) could be arbitrarily controlled. The exhaust port has the area of 3 and 13cm² for VAR=0 and 100%, respectively.

CO, CO₂, and HC concentrations of the exhaust gas were measured by a portable gas analyzer (HORIBA MEXA-554J), while O₂ concentration was measured by an oxygen meter (COSMO XPO-318-E). The air-fuel ratio of the exhaust gas was calculated using a carbon balance method with the measurements of CO, CO₂, and HC for port injection conditions. In case of direct injection, an accurate measurement is difficult because the fuel and air short-circuiting makes the air-fuel ratio inside the cylinder different from that at the exhaust. Therefore, the concentration of CO₂ while changing the air-fuel ratio from rich to lean range was measured and the stoichiometric air-fuel ratio inside the cylinder was determined, based on a comparison with the equilibrium calculation re-

Table 2 Test conditions

r.p.m.	Port Injection				Direct Injection							
	3000 r/min		4000 r/min		3000 r/min				4000 r/min			
θ_{th}	20%	40%	20%	40%	20%		40%		20%		40%	
IT VAR	-	-	-	-	273	180	273	180	273	180	273	180
100%	×	×	N.A.	N.A.	×	×	×	×	N.A.	N.A.	N.A.	N.A.
75%	×	×	△	△	×	×	×	×	△	×	△	△
50%	△	●	●	●	△	△	×	△	×	●	●	●
25%	●	●	●	●	×	●	×	●	●	●	●	●
0%	●	●	●	●	●	●	●	●	●	●	●	●

θ_{th} : Throttle Opening Rate, VAR: Valve Angle Rate of Exhaust Valve, IT: Injection Timing (deg. ATDC)

●: Compression Ignition (CI), ●: Spark-assisted Compression Ignition, ×: Spark Ignition (SI)

△: Irregular combustion in compression ignition, N. A.: No SI/CI operational

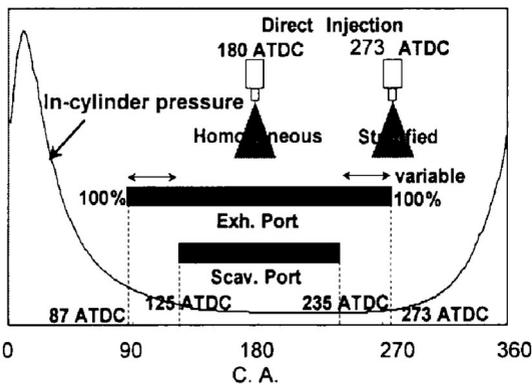


Fig. 2 Injection timing of in-cylinder direct injection

sult for the peak of CO_2 concentration (Agrawal et al., 1975).

2.2 Experimental procedure

Experiments were conducted in both port injection and direct injection at 3000 and 4000 r/min as shown in Table 2. The throttle opening rate (θ_{th}) was set to 20 and 40% with a fixed equivalence ratio of 1.0. The VAR and the injection timing for direct injection conditions were parametrically varied.

The injection timing for direct injection is shown in Fig. 2. The injection timing was set to 273 and 180 deg. ATDC. When the injection was set to 273 deg. ATDC, corresponding to the exhaust port close timing, the in-cylinder mixture is

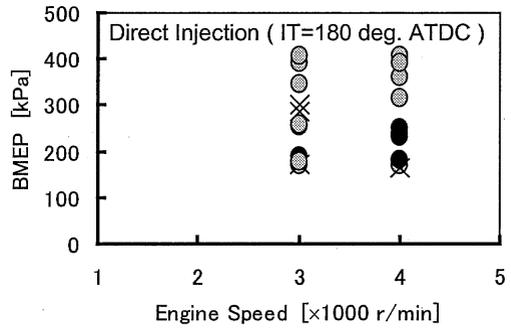
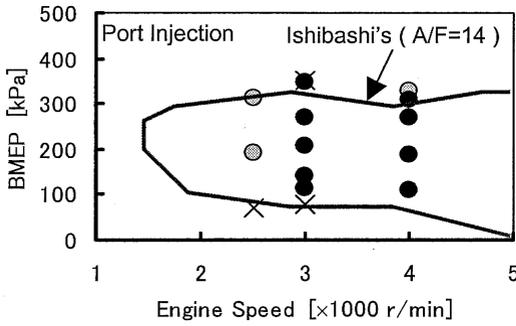
heterogeneous. At 180 deg. ATDC, as both exhaust and scavenging ports are open, the in-cylinder mixture would become nearly homogeneous. In case of port injection, the mixture is almost homogeneous. The test results on whether compression-ignition combustion is possible or not in each condition are shown in Table 2. The possibility was evaluated based on whether the engine remains in stable state even when the ignition power is disconnected. Compression-ignition combustion supported by spark ignition causes the in-cylinder pressure variation similar to that of compression-ignition combustion although the engine becomes unstable when the spark is stopped. The coolant temperature was maintained at 90°C during the experiment.

Indicated mean effective pressure (IMEP), indicated thermal efficiency, coefficient of variation (COV) of IMEP, heat release rate, mass burnt fraction, and the combustion period were obtained on the basis of the 100 cycle-averaged in-cylinder pressure for each experimental condition.

3. Results and Discussion

3.1 Range of compression-ignition combustion

The test results on whether compression-ignition combustion is possible or not in each condition are shown in Table 2. Figure 3 shows the operational region of compression-ignition for both port injection and direct injection (180 deg.



$\theta_{th}=20\%$, 40% , $A/F=14.6$

● : CI operational condition, ● : Spark-assisted CI operational condition, × : Only SI operational condition

Fig. 3 Operational region of compression ignition with port and direct injections

ATDC). The solid line indicates the range of compression-ignition (air to fuel ratio fixed to 14) obtained by Ishibashi (2000) for a comparison. The lower limit of BMEP was controlled by the minimum injection amount.

Both the port injection and direct injection results show almost the same operational region of compression-ignition at medium speeds and loads (3000~4000 r/min, BMEP = 180~350 kPa). The operational region of stable compression-ignition without spark-assist is broader in port injection than in direct injection. In the case of direct injection, the range was broader with the injection timing of 180 deg. ATDC toward lower engine speeds than with 273 deg. ATDC.

3.2 Effect of fuel injection method

Figures 4 and 5 show the BMEP, charging efficiency and BSHC as a function of VAR for both the direct injection and port injection at 3000 and 4000 r/min, respectively ($\theta_{th}=20\%$). The solid symbols indicate conditions that compression-ignition was possible and the gray ones indicate conditions that spark-assisted compression-ignition was possible. BMEP of direct injection at 273 deg. ATDC shows rather lower values compared with those of port injection, while almost the same BMEP was achieved between 180 deg. ATDC injection and port injection for both 3000 and 4000 r/min. This is probably due to that the in-cylinder mixture becomes more homogeneous as the injection timing is set earlier. Also, BMEP tends to decrease as VAR increases re-

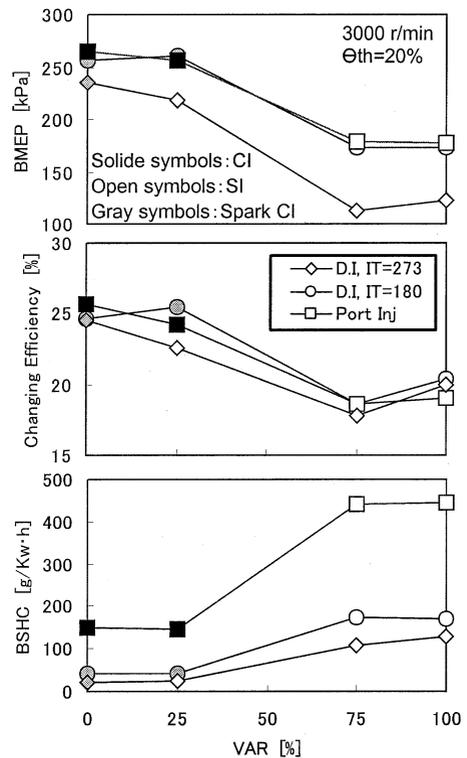


Fig. 4 Comparison of BMEP, charging efficiency, BSHC results between port and direct injections (3000 r/min)

gardless of the fuel injection system. Since the variation of BMEP for VAR shown is similar to that of charging efficiency, the charging efficiency was found to influence BMEP.

Charging efficiency at 3000 and 4000 r/min tends to decrease as VAR increases regardless of the fuel injection system. This could be inferred

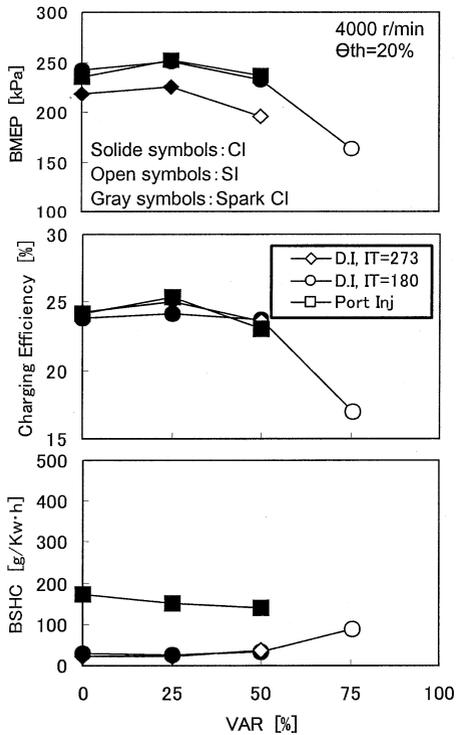


Fig. 5 Comparison of BMEP, charging efficiency, BSHC results between port and direct injections (4000 r/min)

since the short-circuiting increases as the opening period of the exhaust port increases. This process was predicted by numerical analyses at 2500 and 4000 r/min in the previous study (Moriyoshi et al., 1999). Charging efficiency is dependent on the injection timing of direct injection since the injection method affects the in-cylinder gas flow.

In the condition of the injection timing 180 deg. ATDC at 3000 and 4000 r/min, where BMEP is high, the direct injection was found to significantly reduce BSHC by 73% for VAR=25 and 0% compared to the port injection, and by 63% for VAR=100% (3000 r/min). Furthermore, at 3000 r/min, the condition of injection timing 273 deg. ATDC shows lower BSHC than that of 180 deg. ATDC. This could be inferred because short-circuiting was reduced as the fuel was injected after the exhaust port closed timing at the injection timing of 273 deg. ATDC.

Figure 6 shows the BSFC characteristic as a function of BMEP for each CI condition. The injec-

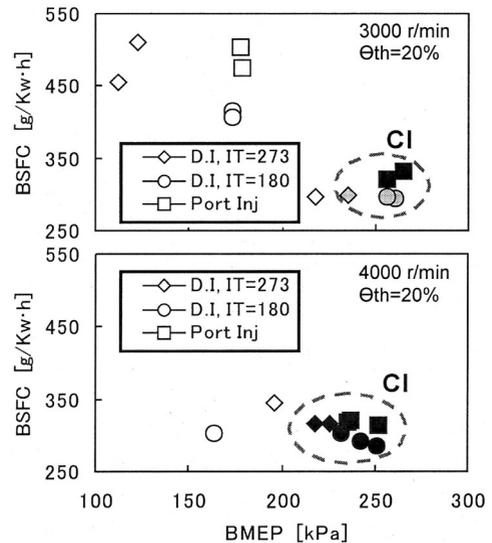


Fig. 6 BSFC vs. BMEP for port and direct injections

tion timing of 180 deg. ATDC, where compression-ignition operation was possible, shows better result than that of port injection for both 3000 and 4000 r/min. This could be inferred because short-circuiting was significantly reduced and the combustion pattern during compression-ignition operation was stabilized.

In conclusion, at 3000 and 4000 r/min in medium loads (BMEP = 180~350 kPa), compression-ignition combustion is possible by direct injection in the same range as the port injection, and consequently both low pollution and high thermal efficiency are achieved with high BMEP and low BSFC and BSHC.

3.3 Factors influencing compression-ignition combustion in direct injection

(1) Scavenging efficiency - The scavenging efficiency, significantly influential to compression-ignition combustion, was estimated using an experimental formula proposed by Ishibashi (2000). The trapping efficiency was obtained from the oxygen concentration of the exhaust gas, based on Watson law for port injection (Tomituka, 1985) by measuring values of fuel amount and the intake air in accordance with the definition of the trapping efficiency for direct injection. Then, the scavenging efficiency was estimated from the tra-

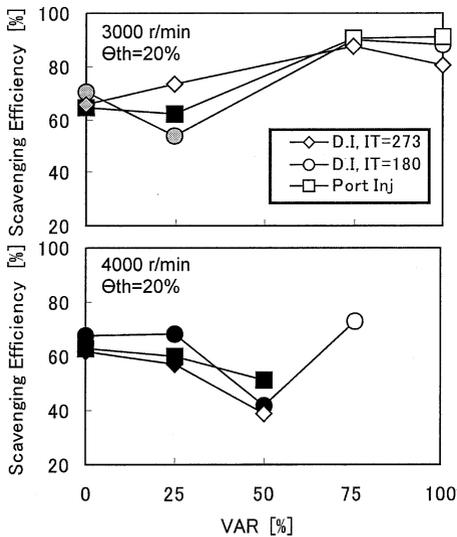


Fig. 7 Scavenging efficiency vs. VAR for port and direct injections

pping efficiency using Ishibashi's experimental formula.

Figure 7 shows the scavenging efficiency as a function of VAR for both direct injection and port injection at $\theta_{th} = 20\%$ and the engine speeds of 3000 and 4000 r/min. The scavenging efficiency was lower at 4000 r/min than at 3000 r/min for both port injection and direct injection of 273 deg. ATDC with $\theta_{th} = 20\%$, where compression-ignition operation was possible. This is because the duration of the scavenging port opening becomes relatively short at 4000 r/min. For the region where compression-ignition operation was possible by direct injection ($\theta_{th} = 20\%$, 4000 r/min, and IT = 273 deg. ATDC), it was found that the scavenging efficiency was lower than that of port injection by approximately 3~5%. This means more heat is transferred from the residual gas to the fresh mixture. Meanwhile, results with the injection timing of 180 deg. ATDC show no apparent characteristics, as the fuel injection affects the scavenging flow when the fuel is injected with the scavenging and exhaust ports open.

Figure 8 shows the scavenging efficiency and the delivery ratio for both direct injection and port injection at $\theta_{th} = 20$ and 40% and engine speeds of 3000 and 4000 r/min. It was also found

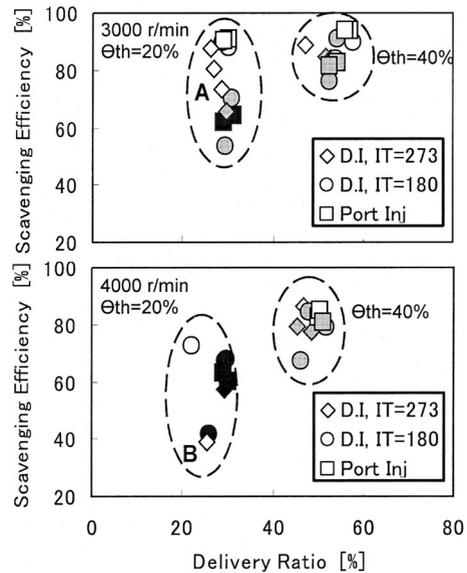


Fig. 8 Scavenging efficiency vs. delivery ratio for port and direct injections

that the scavenging efficiency increased with the increase in the throttle opening rate (and eventually the trapping ratio) regardless of the injection type and the injection timing. This might be disadvantageous to compression-ignition combustion considering that the heat from the residual gas is important for compression-ignition operation. In fact, Fig. 8 shows that compression-ignition operation is possible with the scavenging efficiency below 70% at $\theta_{th} = 20\%$, while it is difficult with a scavenging efficiency above 70% at $\theta_{th} = 40\%$. Even when $\theta_{th} = 20\%$ and the scavenging ratio is lower than 70%, there exist conditions where compression-ignition combustion does not occur, e. g. point B at 273 deg. ATDC injection. Therefore, factors influential to compression-ignition operation are not limited to only the scavenging efficiency.

(2) Gas temperature inside the cylinder - The gas temperature inside the cylinder is an influential factor. It should be noted, however, that the gas temperature is dependent on the scavenging efficiency and trapping ratio.

The gas temperature was estimated from the mean temperatures of the exhaust gas and scavenging gas, scavenging efficiency, and the com-

pression ratio, as suggested by Iida (1999). The details are described in the appendix. Figure 9 shows the estimated in-cylinder gas temperature at TDC for the variation of the IMEP at $\theta_{th} = 20^\circ$ and engine speeds of 3000 and 4000 r/min, for both the port injection and direct injection. The minimum temperature, where compression-ignition operation becomes possible, tends to increase as engine speed increases. In the condition above 1120 K at 3000 r/min and above 1180 K at 4000 r/min, compression-ignition occurs regardless of the fuel injection system and the variation of IMEP is stabilized below 10%.

In case of point A, however, at 3000 r/min and the injection timing of 273 deg. ATDC with direct injection, compression-ignition operation was impossible although the scavenging efficiency was equal to 70% and the gas temperature was higher than 1120 K. Also, in case of point B, at 4000 r/min and the injection timing of 273 deg. ATDC, compression-ignition operation was impossible although the scavenging efficiency was lower than 70% and the gas temperature was higher than 1180 K. This is probably due to that the rich region caused by the stratification of the in-cylinder mixture coincided with the low-temper-

ature region.

In conclusion, in case of port injection, the scavenging efficiency and temperature inside the cylinder is indispensable in predicting whether compression-ignition combustion is possible or not. In case of direct injection, another condition of the spatial distribution of fuel concentration and temperature inside the cylinder is indispensable in predicting the occurrence of compression-ignition.

3.4 Compression-ignition combustion characteristics

Combustion analyses for both the port injection and direct injection were conducted in order to clarify the characteristics of compression-ignition combustion. Figure 10 shows the temporal variations of the apparent heat release rate measured at $\theta_{th} = 20^\circ$, VAR = 0%, and engine speeds of 3000 and 4000 r/min, where compression-ignition occurs. The in-cylinder gas temperature at TDC is indicated in the figure.

The onset timing of combustion in the port injection at 3000 r/min is later compared to that at 4000 r/min. This is inferred because the in-cylinder gas temperature is higher at 4000 r/min. At 3000 r/min, the onset timing of combustion in direct injection at both 180 and 273 deg. ATDC was earlier than that of the port injection. Although almost the same temperature was estimated for both the direct injection and the port injection at 273 deg. ATDC, compression-ignition occurred from the point where the rich mixture region coincided with the high-temperature region due to heterogeneous distributions of temperature and concentration. Stratification is influential even at 4000 r/min because the onset timing of combustion in both the port injection and direct injection was almost the same although the temperature inside the cylinder was 1227 K, which is higher than the temperature in case of the port injection. It was found that when the spatial distribution of the in-cylinder mixture is homogeneous, the onset timing of compression-ignition combustion can be easily estimated and that when the spatial distribution of the in-cylinder mixture is heterogeneous, the onset timing of compression-

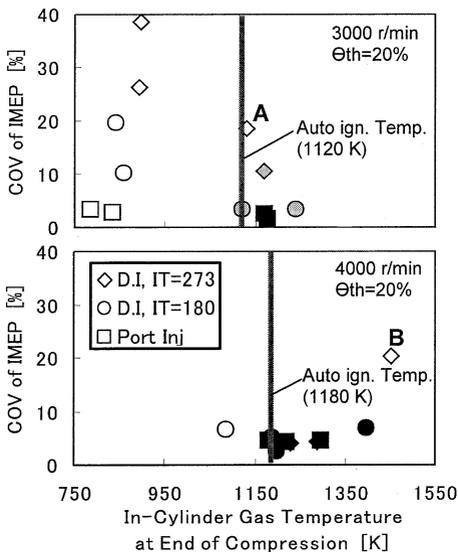


Fig. 9 COV of IMEP vs. estimated in-cylinder gas temperature at TDC for port and direct injections

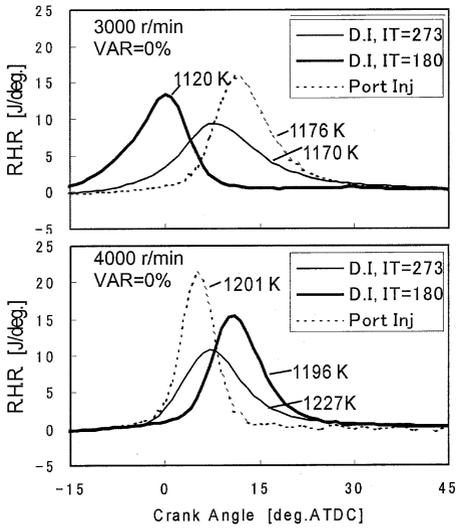


Fig. 10 Comparison of R.H.R for port and direct injections

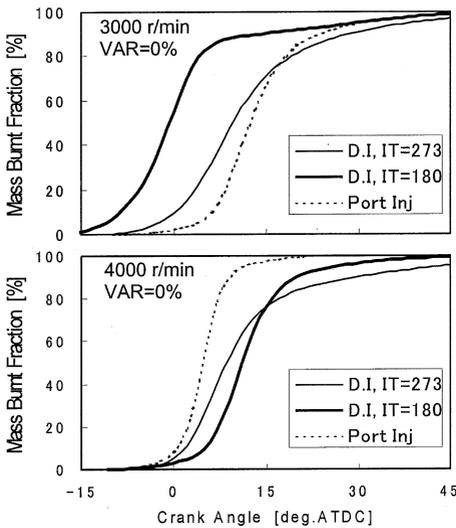


Fig. 11 Comparison of mass burnt fraction for port and direct injections

ignition combustion cannot be easily estimated. Then the heat release pattern can be controlled by the variation of the injection timing for direct injection.

For detailed identification during the combustion period, Fig. 11 shows the mass burnt fraction measured in the same conditions as Fig. 10. In the port injection conditions, the mass burnt duration from 10 to 90% is shorter at 4000 r/min than at 3000 r/min. This was caused by more homoge-

neous mixture, leading to a quick combustion. The previous simulation evaluated this phenomenon (Moriyoshi, et al. 1999). In case of direct injection, at the injection timing of 180 deg. ATDC and 4000 r/min, the mass burnt duration from 10 to 70% became longer than that of port injection by 3° and the duration from 70 to 90% became longer by 3°. In case of 4000 r/min with the injection timing of 273 deg. ATDC, meanwhile, the duration from 10 to 70% became longer by 6° and the duration from 70 to 90% became longer by 17°.

In conclusion, it was found that the combustion period becomes shorter as the mixture becomes homogeneous by means of earlier injection timing (Kaneko et al., 2001). However, in the later stage of combustion after 80% mass burnt, the combustion period in direct injection was longer than that of port injection at both engine speeds. For this reason, it is possible that the stratification of the in-cylinder mixture resulting from direct injection enables compression-ignition combustion enhanced at a portion with rich mixture and high temperature, while only flame propagation occurs in a portion with lean mixture and low temperature.

In addition, at VAR = 0%, the duration from 10 to 90% heat release was shorter at 4000 r/min in terms of the crank angle. It is inferred because the amount of heat reception at 4000 r/min is larger with the increase in the in-cylinder gas temperature resulting from the deterioration of the scavenging efficiency and the rise of the scavenging and exhaust gas temperatures. In fact, Fig. 9 shows that in-cylinder gas temperature of 4000 r/min is higher than that of 3000 r/min.

4. Conclusions

A modified Schnurle-type gasoline engine was built and tested, in order to determine the conditions where compression-ignition combustion can be conducted in both the port injection and in-cylinder direct injection. Experiments were conducted while varying the area of the exhaust port at the engine speeds of 3000 and 4000 r/min, the throttle opening rate at 20 and 40% and the eq-

ivalence ratio of 1.0. The following conclusions were obtained by investigation on the operational region of compression-ignition combustion for both the port injection and direct injection, the factors influential to performance characteristics, and the combustion characteristics.

(1) The operational region where compression-ignition combustion can be achieved was almost the same each other regardless of the injection type at medium speeds and loads (3000~4000 r/min, BMEP = 180~350 kPa). In case of direct injection, however, the injection timing needs to be set earlier. BMEP values were almost the same each other regardless of the type of injection, while BSFC and BSHC were much lower in direct injection due to the reduction of the short-circuiting.

(2) In case of port injection, whether compression-ignition occurs or not, can be determined by the scavenging efficiency and the presumed gas temperature at TDC. In case of direct injection, another condition of the spatial distribution of fuel concentration and temperature inside the cylinder is indispensable to predict the occurrence of compression-ignition.

(3) When the spatial distribution of the in-cylinder mixture is homogeneous, the onset timing of compression-ignition combustion can be easily estimated. On the other hand, when the spatial distribution of the in-cylinder mixture is heterogeneous, the onset timing of compression-ignition combustion cannot be easily estimated.

(4) In case of compression-ignition combustion with direct injection, it can be inferred that a flame propagation occurs in the ending part of combustion, corresponding to the low heat release rate (after 80% mass burnt). This causes a longer combustion period in comparison with port injection, because some region with lean mixture and low temperature is left at last.

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APPENDIX

Definitions concerning gas exchanging state

Mf: Mass of fresh air trapped in the cylinder after exhaust close

Mr: Mass of residual gas trapped in the cylinder after exhaust close

Mg: Mass of total gas trapped in the cylinder after exhaust close

Ms: Mass of intake air for one cycle

Mh: Reference mass required to fill the -isplacement under the reference conditions.

L ($=M_s/M_h$): Delivery ratio
 η_{tr} ($=M_f/M_s$): Trapping efficiency
 η_c ($=M_f/M_h=L \times \eta_{tr}$): Charging efficiency
 η_s ($=M_f/M_g$): Scavenging efficiency
 C_{rel} ($=M_g/M_h$): Relative charge
 K ($=M_s/M_g=L/C_{rel}$): Corrected delivery ratio

How to estimate the in-cylinder gas temperature at TDC

M_f : Mass of fresh air trapped in the cylinder
 M_r : Mass of residual gas trapped in the cylinder
 T_f : Temperature of scavenging gas
 T_e : Temperature of exhaust gas
 T_r : Temperature of residual gas
 T_i : Temperature of in-cylinder gas after the scavenging process
 C_p : Isobaric specific heat
 C_{ps} : C_p of scavenging gas

C_{pr} : C_p of residual gas
 C_{pi} : C_p of in-cylinder gas after the scavenging process
 C_{pe} : C_p of exhaust gas

From the enthalpic balance in exhaust process,
 $M_f C_{pr} T_r + M_r C_{ps} T_f = (M_f + M_r) C_{pe} T_e$

$C_{pr} = C_{ps} = C_{pe}$,

$T_r = (T_e - T_f) / \eta_s + T_f$

Where, $\eta_s = M_f / (M_f + M_r)$

From the entalpic balance in scavenging process,
 $M_f C_{ps} T_f + M_r C_{pr} T_r = (M_f + M_r) C_{pi} T_i$

$C_{ps} = C_{pr} = C_{pi}$,

$T_i = \eta_s T_f + (1 - \eta_s) T_r$

Assuming the adiabatic compression process,

$T = T_i \epsilon^{k-1}$

Thus,

$T = (\eta_s T_f + (1 - \eta_s) T_r) \epsilon^{k-1}$