Time-Discretization of Non-Affine Nonlinear System with Delayed Input Using Taylor-Series

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In this paper, we propose a new scheme for the discretization of nonlinear systems using Taylor series expansion and the zero-order hold assumption. This scheme is applied to the sampled-data representation of a non-affine nonlinear system with constant input time-delay. The mathematical expressions of the discretization scheme are presented and the ability of the algorithm is tested for some of the examples. The proposed scheme provides a finite-dimensional representation for nonlinear systems with time-delay enabling existing controller design techniques to be applied to them. For all the case studies, various sampling rates and time-delay values are considered.

Key Words : Non-Affine, Nonlinear, Taylor-Series, Time-Delay, Time-Discretization

1. Introduction

Control systems with time-delay are likely to gain in importance in the near future as Internet technology further develops and evolves. There are two reasons why time-delay receives special attention in the field of control systems. Firstly, time-delays are increasing due to the increased reliance on communication and the complex computations involved in control systems. The use of digital controllers in communication systems and their increased computational requirements induces this time-delay. In embedded control systems, the effect of time-delay due to communication and increased computation cannot be ignored. Secondly, control systems with time-delays exhibit complex behavior because of their infinite dimensionality, even in the case of linear time-invariant systems that have constant time-delays in the input or states have infinite dimensionality when expressed in the continuous-time domain. For these reasons, during the last few decades it was not possible to apply a controller design technique having any time-delays in the variables. Thus, it is necessary to develop a control system design scheme that resolves these time-delays.

The engineering literature dealing with time-delayed systems is very extensive. Most of this literature deals with linear time-delay control systems, and, in particular, with the stability and robustness related to time-delay. In a study by

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Choi et al. (1999), the authors proposed a new control scheme applicable to systems with time-delay, which is based on the conventional position-position feedback-type controller. The stability of this control system is proved using scattering theory and compared with that of conventional systems. Jeong and Lee (1995) proposed a method of designing a robust time-delayed teleoperator robot system based on optimization. The proposed teleoperator control system deals with the robustness of teleoperation, especially, during the contact phase.

Recently, research into the technique of Time Delay Control (TDC) utilizing the estimated uncertainties of general nonlinear systems based on the time-delay method has been actively pursued. Choi and Baek (2002) studied a magnetic levitation system required to have a large operating range in many applications. TDC was applied to a single-axis magnetic levitation system and a reduced-order observer was utilized to estimate the states in the control law, with the exception of the measurable states. Lee and Chang (1999) studied the input/output linearization (IOL) method using TDC and a time-delay observer. This method enables the IOL method to be applied to plants even when not all of the states of the plant are measurable or the measured plant output is very noisy. In Byeon and Song (1997), a position control system was developed for a throttle actuator system that uses one throttle actuation for small volumes and a DC servo motor to provide a fast response. In order to drive the DC motor, the PWM signal generator and PWM amplifier were built and interfaced to the motor and controller. Also, the time-delay control (TDC) law was used as a basic control algorithm in this study. A method of varying the reference model of the TDC with respect to the degree of change in target throttle angle is proposed by them. To apply TDC to a real system, Kwon et al. (2002) designed a Time Delay Controller to guarantee stability. Earlier research had established the sufficient stability condition of the TDC for general manufacturing plants. A new sufficient stability condition for the TDC of general manufacturing plants with finite time-delay is proposed.

Hong and Wu (1994) derived sufficient conditions for the zeros of the polynomial to be either inside the unit disk in the complex plane or at least for one zero not to be inside the unit disk by examining the coefficients of a given polynomial in the linear discrete system. Kang and Park (1999) experimentally confirmed the fundamental dynamic properties of an electrodynamic structure. They examined the discretization effects required for the conversion of continuous properties such as mass, stiffness and surface charge into discrete quantities. In the systems considered, the linearized characteristics are well-matched with the characteristics of the nonlinear systems in the sense that the linearized effects dominate over the high-order nonlinear terms.

In general, most if not all industrial controllers are currently implemented digitally. In the design of model-based digital controllers, both for process and non-process type of systems, two general approaches are available. In the first approach, a continuous-time controller is designed based on a continuous-time system model, followed by a digital redesign of the controller in the discrete-time domain in order to approximate the performance of the original continuous-time controller. In the second approach, a direct digital design strategy can be followed based on a discrete-time model (sampled-data representation) of the system, in which the controller is directly designed in the discrete-time domain. It is apparent that this alternative approach is attractive when dealing directly with the issue of sampling. Indeed, the effect of sampling on the system-theoretic properties of a continuous-time system is very important because these properties are associated with the design objectives. It should be emphasized that in both design approaches, the time-discretization of either the controller or the system model is necessary. Furthermore, it should be noted that in the controller design for time-delay systems, the first approach is troublesome, because of the infinite-dimensional nature of the underlying system dynamics. As a result, the second approach becomes more desirable and will be pursued in the present study.

This paper extends the well-known time-discretization technique for linear time-delay systems (Franklin et al., 1998; Vaccaro, 1995) and affine nonlinear systems (Kazantzis et al., 2003) to non-affine nonlinear systems. The proposed discretization method applies the Taylor series expansion according to the mathematical structure developed for a delay-free nonlinear system (Kazantzis and Kravaris, 1997; Kazantzis and Kravaris 1999). Conventional numerical techniques such as the Euler and Runge-Kutta methods have traditionally been used for obtaining the sampled-data representation for original continuous-time systems (Franklin et al., 1998), which do not have delay. All of these approaches require small time steps, in order to provide the required level of accuracy. Another interesting result for the discretization of delay-free nonlinear systems can be found in the Carleman linearization method (Svoronos et al., 1994). However, this method is useful only for low-dimensional systems. The dimension of a discretized system increases rapidly depending on the required accuracy of the continuous model and the dimension of the continuous system.

In particular, this paper makes the contribution: propose a new method for the discretization of non-affine nonlinear systems with time-delay in the control input. Since the resulting discrete system is finite-dimensional, existing nonlinear control system design techniques can be directly applied to it.

The discretization of an affine nonlinear system without time-delay will be considered in Sec. 2. The discretization of a linear system with time-delay is discussed in Sec. 3, and in Sec. 4 the affine nonlinear system with time-delay is derived based on the linear system discussed in Sec. 3. The sampled-data representation of the time-delayed non-affine nonlinear system, which will be the main idea of this paper, is derived in Sec. 5. An example is simulated in Sec. 6 and the performance of the proposed method is evaluated.

2. Time-Discretization of Delay-Free Affine Nonlinear Systems

Initially, delay-free ($D=0$) affine nonlinear systems are considered with a state-space representation of the form:

$$\frac{dx(t)}{dt}=f(x(t))+g(x(t))u(t), \tag{1}$$

where $x\in X\subset R^n$ is the vector of states and X is an open and connected set, $u\in R$ is the input variable and D is a constant time-delay. It is assumed that $f(x)$, $g(x)$ are real analytic vector fields on X.

An equidistant grid on the time axis with mesh $T=t_{k+1}-t_k>0$ is considered, where $[t_k, t_{k+1})=[kT, (k+1)T)$ is the sampling interval and T is the sampling period. It is assumed that the system described in Eq. (1) is driven by an input that is piecewise constant over the sampling interval, i. e. the zero-order hold (ZOH) assumption holds true:

$$u(t)=u(kT)\equiv u(k)=constant \tag{2}$$

for $kT\leq t<kT+T$.

Under the ZOH assumption and within the sampling interval, the solution described in Eq. (1) is expanded in a uniformly convergent Taylor series (Grobner, 1967) and the resulting coefficients can be easily computed by taking successive partial derivatives of the right-hand-side of Eq. (1):

$$\begin{aligned} x(k+1)&=x(k)+\sum_{l=1}^{\infty}\frac{T^l}{l\,!}\frac{d^l x}{dt^l}\Big|_{t_k}\\ &=x(k)+\sum_{l=1}^{\infty}A^{[l]}(x(k),\, u(k))\frac{T^l}{l\,!}, \end{aligned} \tag{3}$$

where $x(k)$ is the value of the state vector x at time $t=t_k=kT$ and $A^{[l]}(x, u)$ are determined recursively by:

$$\begin{aligned} &A^{[l]}(x,\, u)=f(x)+ug(x)\\ &A^{[l+1]}(x,\, u)=\frac{\partial A^{[l]}(x,\, u)}{\partial x}(f(x)+ug(x)) \end{aligned} \tag{4}$$

with $l=1, 2, 3, \cdots$.

The Taylor series expansion of Eq. (3) can offer either an **exact sampled-data representation (ESDR)** of Eq. (1) by retaining the full infinite series representation of the state vector:

$$x(k+1)=\Phi_T(x(k),\ u(k))$$
$$=x(k)+\sum_{l=1}^{\infty}A^{[l]}(x(k),\ u(k))\frac{T^l}{l\,!} \quad (5)$$

or an **approximate sampled-data representation (ASDR)** of Eq. (1) resulting from a truncation of the Taylor series of order N:

$$x(k+1)=\Phi_T^N(x(k),\ u(k))$$
$$=x(k)+\sum_{l=1}^{N}A^{[l]}(x(k),\ u(k))\frac{T^l}{l\,!} \quad (6)$$

where the subscript of the map Φ_T^N denotes the dependence on the sampling period T of the sampled-data representation obtained under the above discretization scheme, and the superscript N denotes the finite series truncation order associated with the ASDR of Eq. (6).

3. Time-Discretization of Linear Systems with Time-Delay

It is possible to extend the application of the Taylor discretization method into the nonlinear continuous-time systems, which have constant time-delay in the input. Here, we recall the procedures used for the discretization of the linear system since the discretization of the nonlinear system can follow the same principle as that used for the linear system.

$$\frac{dx(t)}{dt}=Ax(t)+bu(t-D) \quad (7)$$

where A, b are constant matrices of appropriate dimensions and D is the system's constant time-delay (dead-time) that directly affects the input. It is generally recognized that for any time interval $I=[t_i,\ t_f)$, such that $u=u_c=$constant, the following formula holds true:

$$x(t_f)=\exp(A(t_f-t_i))x(t_i)$$
$$+u_c\int_{t_i}^{t_f}\exp(A(t_f-\tau))\,b\cdot d\tau. \quad (8)$$

Furthermore, let:

$$D=qT+\gamma \quad (9)$$

where $q\in\{0,\ 1,\ 2,\ \ldots\}$ and $0<\gamma\leq T$. Equivalently, the time-delay D is customarily represented as an integer multiple of the sampling period plus a fractional part of T (Chen, 1984; Franklin et al., 1998). Under the ZOH assumption and the above notation, it is rather straightforward to verify that the "delayed" input variable attains the following two distinct values within the sampling interval (Franklin et al., 1998):

$$u(t-D)=\begin{cases}u(kT-qT-T)\equiv u(k-q-1) & \text{if } kT\leq t<kT+\gamma\\ u(kT-qT)\equiv u(k-q) & \text{if } kT+\gamma\leq t<kT+T.\end{cases} \quad (10)$$

As can be readily inferred from Eq. (10), the input variable $u(t)$ remains constant within the subintervals: $[kT,\ kT+\gamma)$, $[kT+\gamma,\ kT+T)$, to which the above formula Eq. (8) is successively applied. In this way, one readily obtains:

If $kT\leq t<kT+\gamma$, then

$$x(kT+\gamma)=\exp(A\gamma)x(kT)$$
$$+u(k-q-1)\int_{kT}^{kT+\gamma}\exp(A(kT+\gamma-\tau))\,b\cdot d\tau \quad (11)$$

If $kT+\gamma\leq t<kT+T$, then

$$\begin{aligned}x(kT+T)&=\exp(A(T-\gamma))x(kT+\gamma)\\&\quad+u(k-q)\int_{kT+\gamma}^{kT+T}\exp(A(kT+T-\tau))\,b\cdot d\tau\\&=\exp(A(T-\gamma))\exp(A\gamma)x(kT)+\exp(A(T-\gamma))\\&\quad u(k-q-1)\int_{kT}^{kT+\gamma}\exp(A(kT+\gamma-\tau))\,b\cdot d\tau\\&\quad+u(k-q)\int_{kT+\gamma}^{kT+T}\exp(A(kT+T-\tau))\,b\cdot d\tau\\&=\exp(AT)x(kT)+\Gamma_1\cdot u(k-q-1)+\Gamma_0\cdot u(k-q)\end{aligned} \quad (12)$$

where it can easily be verified that the integrals

$$\Gamma_1=\int_{T-\gamma}^{T}\exp(A\tau)\,bd\tau \text{ and } \Gamma_0=\int_{0}^{T-\gamma}\exp(A\tau)\,bd\tau$$

are independent of the discrete-time index k (Franklin et al., 1998; Vaccaro, 1995). The above expression Eq. (12) represents the sampled-data representation of the original continuous-time system Eq. (7) with time-delay D. Notice, that the value of the state vector at $(k+1)T$ is a linear combination of the states evaluated at kT and the past values of the input variable u at $(k-q-1)T$ and $(k-q)T$.

4. Time-Discretization of Affine Nonlinear Systems with Time-Delay

Single-input nonlinear continuous-time systems can be expressed with the following state-space representation of the form:

$$\frac{dx(t)}{dt}=f(x(t))+g(x(t))u(t-D). \quad (13)$$

Motivated by the linear approach described above, a similar line of thinking is adopted for the nonlinear case as well. Indeed, the following sampled-data representation can be obtained for every subinterval from Eq. (3) by applying the Taylor series discretization method to affine nonlinear systems with delayed input. First, the state vector at $kT+\gamma$ for subinterval $[kT, kT+\gamma)$ (Kazantzis et al., 2003):

$$x(kT+\gamma)=x(kT)+\sum_{l=1}^{\infty}A^{l}(x(kT), u(k-q-1))\frac{\gamma^{l}}{l!}=\Phi_{\gamma}(x(kT), u(k-q-1)) \quad (14)$$

where the map Φ_{r} can be derived through a direct application of formula Eq. (3), and the subsequent calculation of the corresponding Taylor coefficients can be realized through the recursive formulas described in Eq. (4). The state vector $(k+1)T$ can also be obtained by using the state vector at $kT+\gamma$ and the input $u(k-q)$ can be obtained by using the Taylor discretization method for the subinterval $[kT+\gamma, kT+T)$.

$$\begin{aligned}x(kT+T)&=x(kT+\gamma)+\sum_{l=1}^{\infty}A^{l}(x(kT+\gamma), u(k-q))\frac{(T-\gamma)^{l}}{l!}\\&=\Phi_{T-\gamma}(x(kT+\gamma), u(k-q))\end{aligned} \quad (15)$$

The functional representation of the coefficient $A^{[l]}$ of $\Phi_{T-\gamma}$ is same as that described in the previous procedures, the thus making the computation unnecessary. The sampled-data representation of the original system Eq. (13) is obtained by combining Eqs. (14) and (15).

$$\begin{aligned}x(k+1)&=\Phi_{\gamma}(x(k), u(k-q-1))+\sum_{l=1}^{\infty}A^{l}(\Phi_{\gamma}(x(k), u(k-q-1)), u(k-q))\frac{(T-\gamma)^{l}}{l!}\\&=\Phi_{T}^{D}(x(k), u(k-q-1), u(k-q))\\&=\Phi_{T-\gamma}(\Phi_{\gamma}(x(k), u(k-q-1)), u(k-q))\end{aligned} \quad (16)$$

5. Time-Discretization of Non-Affine Nonlinear Systems with Time-Delay

The equations which describe a single input non-affine nonlinear system are as follows (Wei Lin, 1995);

$$\dot{x}=f_0(x)+g_1(x)u+g_2(x)u^2+\cdots+g_l(x)u^l \quad (17)$$

where $x\in R^n$ is the state, $u\in R$ is the control input,

$$f_0: R^n\rightarrow R^n,\ g_i: R^n\rightarrow R^n,\ i=1, 2, \ldots, l$$

and $f: R^n\times R\rightarrow R^n$ are smooth mappings.

A non-affine system has nonlinear control inputs, whereas an affine system has linear control inputs. Furthermore, a non-affine nonlinear system also can be discretized using Taylor series expansion. In this study, the mathematical structure of the discretized non-affine nonlinear system can be considered to be the same as that in the affine case, since the input u is assumed to be constant in the sampling interval. All of the partial differentials used in the affine case can also be used in the non-affine case. Thus, the related equations are as follows;

$$\begin{aligned}x(k+1)&=x(k)+\sum_{l=1}^{\infty}\frac{T^l}{l!}\left.\frac{d^lx}{dt^l}\right|_{t_k}\\&=x(k)+\sum_{l=1}^{\infty}A^{[l]}(x(k), u(k))\frac{T^l}{l!}\\\dot{x}&=f_0(x)+g_1(x)u+g_2(x)u^2+\cdots+g_1(x)u^l\\&=A^{[1]}(x, u)=f(x, u)\\\ddot{x}&=\dot{f}_0(x)\dot{x}+\dot{g}_1(x)u\dot{x}+\dot{g}_2(x)u^2\dot{x}+\cdots+\dot{g}_l(x)u^l\dot{x}\\&=(\dot{f}_0(x)+\dot{g}_1(x)u+\dot{g}_2(x)u^2+\cdots+\dot{g}_l(x)u^l)\dot{x}\\&=A^{[2]}(x, u)=\frac{\partial A^{[1]}(x, u)}{\partial x}f(x, u)\\&\vdots\end{aligned} \quad (18)$$

The generalized coefficients can be represented as follows;

$$\begin{aligned}A^{[1]}(x, u)&=f(x, u)\\A^{[l+1]}(x, u)&=\frac{\partial A^{[l]}(x, u)}{\partial x}f(x, u).\end{aligned} \quad (19)$$

Now we will consider the time-discretization of a non-affine nonlinear system with delayed input. Single-input non-affine nonlinear systems with input delay can be expressed in the following state-space representation of the form:

$$\frac{dx(t)}{dt}=f(x(t), u(t-D)). \quad (20)$$

From the ZOH assumption and Eqs. (18) and (19), we obtain the followings;

If $kT \leq t < kT+\gamma$, then

$$x(kT+\gamma)=x(kT)+\sum_{l=1}^{\infty}A^l(x(kT),\, u(k-q-1))\frac{\gamma^l}{l!} \\ =\Phi_\gamma(x(kT),\, u(k-q-1)). \tag{21}$$

If $kT+\gamma \leq t < kT+T$, then

$$x(kT+T)=x(kT+\gamma)+\sum_{l=1}^{\infty}A^l(x(kT+\gamma),\, u(k-q))\frac{(T-\gamma)^l}{l!} \\ =\Phi_{T-\gamma}(x(kT+\gamma),\, u(k-q)). \tag{22}$$

Also, when the Taylor series expansion is applied to each subinterval, the sampled-data representations for the non-affine nonlinear system are identical to those described in Eqs. (14) and (15). Finally, the discretization method using Taylor series expansion can be used for the non-affine nonlinear system.

6. Simulations

The performance of the proposed time-discretization of non-affine nonlinear systems with input time-delay using the Taylor series expansion method is evaluated by applying it to a non-affine system. The systems considered exhibit nonlinear behavior and it is studied for a broad range of values of the sampling period and input delays. Reference solutions for the system is required to validate the proposed time-discretization method. In this paper the Matlab ODE solver is used to obtain reference solutions. The ODE solver was written based on 4th or 5th order Runge-Kutta method. The discrete values obtained at every time step using the proposed time-discretization method are compared to the values obtained using the Matlab ODE solver at the corresponding time steps. The propriety using Matlab ODE solver as the reference model is shown in (Kazantzis et al., 2003). The partial derivative terms involved in the Taylor series expansion are determined recursively. For the case study considered these partial derivative terms are calculated using Maple and the corresponding Maple code listings are included in the Appendix.

The system considered in this paper is assumed to be a single-input non-affine nonlinear system (Wei Lin, 1995).

$$\dot{x}_1=-\dot{x}_1^3+x_1 exp(x_2)\,u(t-D)^2 \\ \dot{x}_2=x_2^2 u(t-D) \tag{23}$$

The above equation is globally asymptotically stabilized by the smooth state feedback control law

$$u=-\frac{x_2^3}{1+x_1^2 exp(x_2)} \tag{24}$$

The system runs for 100second with the initial conditions $x_1(0)=1.0$ and $x_2(0)=-1.0$. Two parameters, the sampling periods and the time delays, are considered while investigating the performance of the proposed algorithm. The numerical experiments are performed for a fixed truncation order, various input time-delays and various sampling periods. Throughout this example, the truncation order is chosen as $N=3$ for all simulations since the simulation results show that truncation orders greater than 3 do not significantly improve the accuracy.

First, how the sampling period effects the proposed algorithm will be investigated. Three different sampling periods, 0.1, 0.05 and 0.01sec, are investigated and also it is assumed that the corresponding input delays are 0.05, 0.025 and 0.005sec, respectively. Figure 1 shows the errors of the state x_1 and x_2 between the response of the proposed algorithm and the Matlab solution. The dotted line indicates the errors of states when $T=0.1$, $D=0.05$, the thin line indicates the errors when $T=0.05$, $D=0.025$, and finally the thick line indicates the error when $T=0.01$, $D=0.005$.

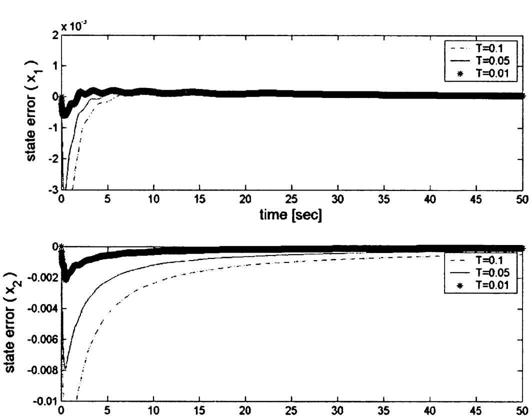

Fig. 1 Response of the proposed algorithm.

Table 1 Average error between the Taylor series and Matlab for x_1

	T=0.1, D=0.05	T=0.05, D=0.025	T=0.01, D=0.005
absolute error	5.66×10^{-4}	2.76×10^{-4}	9.28×10^{-5}
relative error	1×10^{-3}	6.2×10^{-4}	4.8×10^{-4}

Table 2 Average errors between the Taylor series and Matlab for x_2

	T=0.1, D=0.05	T=0.05, D=0.025	T=0.01, D=0.005
absolute error	2.3×10^{-3}	1.1×10^{-3}	2.9×10^{-4}
relative error	4.1×10^{-3}	2.1×10^{-3}	5.55×10^{-4}

The average errors of states x_1 and x_2 between the response of the Taylor method and Matlab solution are summarized in Table 1 and Table 2. The performance of the proposed algorithm is better when the sampling periods are smaller.

In the following simulations, the effects of time-delay are investigated for three cases; delay is smaller than one sampling period, delay is greater than one sampling period and less than two sampling periods and the delay is greater than two sampling periods and less than three sampling periods. The error responses are depicted in Fig. 2 which shows that the proposed algorithm is able to discretize the time-delay nonlinear systems. The simulation is accomplished for sampling period T=0.01 and the time-delays are 0.005, 0.015 and 0.025sec. The thick line indicates the relative error between the response of the proposed algorithm and the Matlab solution when T=0.01, D=0.005, the thin line indicates the same error when T=0.01, D=0.015, and finally the dotted line indicates the error when T=0.01, D=0.025. The results show that as the time-delays are increases the errors are getting larger.

The performance of the proposed scheme is investigated by considering the numerical values of the responses in the above simulation. Table 3 shows the responses computed using the Matlab solution and the proposed scheme when the sampling period is T=0.01 and the input time-delay is D=0.005(q=0); input time-delay is smaller than the sampling period. The numerical differences between the response of the Matlab solution and the proposed scheme for state x_1 lie in the range 0.0006 to 0.0003, and 0.0022 to 0.0001 for state x_2. Table 4 shows that results obtained for a sampling period T=0.01 and delay D=0.015 (q=1). The numerical differences for state x_1 are from 0.0012 to 0.0003 and the differences for state x_2 are from 0.0035 to 0.0001. Similarly the system is simulated for T=0.01 and delay D=0.025 (q=2). These discrete-values are shown in Table 5. The numerical differences for state x_1 range form 0.002 to 0.0001 and those for state x_2 range from 0.005 and 0.0001. The above numerical experiments for various combinations of the time-

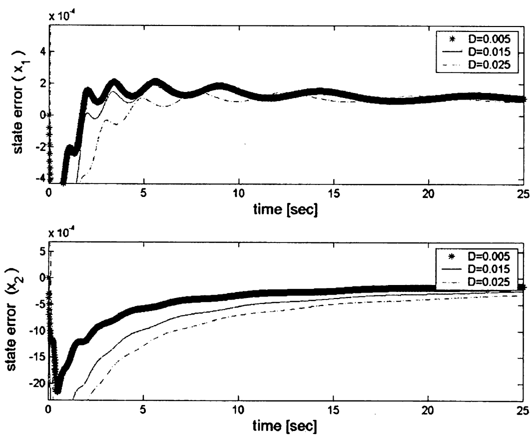

Fig. 2 The relative errors for various time-delays

Table 3 The computation results for T=0.01, D=0.005

Time step	state x_1		state x_2	
	Matlab	Maple	Matlab	Maple
1000	0.2268	0.2266	−0.4032	−0.4029
2000	0.1611	0.1610	−0.3377	−0.3375
3000	0.1315	0.1314	−0.3045	−0.3044
4000	0.1138	0.1137	−0.2830	−0.2829
5000	0.1017	0.1017	−0.2674	−0.2673
6000	0.0928	0.0927	−0.2553	−0.2553
7000	0.0858	0.0858	−0.2455	−0.2455
8000	0.0802	0.0802	−0.2374	−0.2373
9000	0.0756	0.0756	−0.2304	−0.2304
10000	0.0717	0.0717	−0.2244	−0.2243

Table 4 The computation results for $T=0.01$, $D=0.015$

Time step	state x_1		state x_2	
	Matlab	Maple	Matlab	Maple
1000	0.2268	0.2267	−0.4032	−0.4027
2000	0.1611	0.1610	−0.3377	−0.3374
3000	0.1315	0.1314	−0.3045	−0.3043
4000	0.1138	0.1137	−0.2830	−0.2829
5000	0.1017	0.1017	−0.2674	−0.2673
6000	0.0928	0.0927	−0.2553	−0.2552
7000	0.0858	0.0858	−0.2455	−0.2455
8000	0.0802	0.0802	−0.2374	−0.2373
9000	0.0756	0.0756	−0.2304	−0.2304
10000	0.0717	0.0717	−0.2244	−0.2243

Table 5 The computation results for $T=0.01$, $D=0.025$

Time step	state x_1		state x_2	
	Matlab	Maple	Matlab	Maple
1000	0.2268	0.2267	−0.4032	−0.4026
2000	0.1611	0.1610	−0.3378	−0.3374
3000	0.1315	0.1314	−0.3045	−0.3043
4000	0.1138	0.1138	−0.2830	−0.2828
5000	0.1017	0.1017	−0.2674	−0.2673
6000	0.0928	0.0927	−0.2553	−0.2555
7000	0.0858	0.0858	−0.2455	−0.2454
8000	0.0802	0.0802	−0.2374	−0.2373
9000	0.0756	0.0756	−0.2304	−0.2303
10000	0.0717	0.0717	−0.2244	−0.2243

delay and the sampling-period demonstrate that the Taylor expansion scheme discretized non-affine nonlinear systems with input time-delays quite accurately.

7. Conclusions

In this paper, we propose a new scheme for the discretizaion of non-affine nonlinear systems that have time-delays in their inputs, in order to obtain their sampled-data values. This scheme is derived using the Taylor series expansion to yield a solution to the continuous-time system and the input is assumed to be constant in the sampling-interval. The mathematical structure of the derived system can be expressed in the finite dimension with input time-delay. The performance of the proposed time-discretization approach is evaluated using an example of a non-affine nonlinear system. The input is nonlinear, the sampling time is assumed to be fixed and various time-delays are considered in the computer simulation. The computer simulation results show that the proposed scheme performs well in the discretization of non-affine nonlinear systems.

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Appendix

▶ A Maple Code for the first case (T=0.1, D=0.05)

```
> A[1]:=-x1^3+x1*exp(x2)*u^2;
> B[1]:=x2^2*u;
> for i from 1 to 3 do
> A[i+1]:=diff(A[i], x1)*A[1]+diff(A[i], x2)*B[1];
> B[i+1]:=diff(B[i], x1)*A[1]+diff(B[i], x2)*B[1];
> end do:
> T:=0.1:d:=0.05:
> H1:=sum(A[j]*d^j/j!, j=1..3):
> H2:=sum(A[j]*(T-d)^j/j!, j=1..3):
> H3:=sum(B[j]*d^j/j!, j=1..3): H4:=sum(B[j]*(T-d)^j/j!, j=1..3):
> x1[0]:=1 : x2[0]:=-1 :u[-1]:=0:
> for k from 0 to 1000 do
> u[k]:=evalf(-(x2[k]^3)/(1+x1[k]^2*exp(x2[k])));
> m1[k]:=evalf(x1[k]+subs((x1=x1[k], x2=x2[k], u=u[k-1]), H1));
> m2[k]:=evalf(x2[k]+subs((x1=x1[k], x2=x2[k], u=u[k-1]), H3));
> x1[k+1]:=evalf(m1[k]+subs((x1=m1[k], x2=m2[k], u=u[k]), H2));
> x2[k+1]:=evalf(m2[k]+subs((x1=m1[k], x2=m2[k], u=u[k]), H4));
> end do:
```