A Simple Analytical Model for the Study of Optical Bistability Using Multiple Quantum Well p-i-n Diode Structure

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***Abstract*—A simple analytical model has been presented for the study of the optical bistability using a GaAs-$Al_{0.32}Ga_{0.68}As$ multiple quantum well (MQW) p-i-n diode structure. The calculation of the optical absorption is based on a semi-emperical model which is accurately valid for a range of wells between 5 and 20 *nm* and the electric field $F < 200 kV/cm$. The electric field dependent analytical expression for the responsivity is presented. An attempt has been made to derive the analytical relationship between the incident optical power (P_{in}) and the voltage V across the device when the diode is reverse biased by a power supply in series with a load resistor. The relationship between P_{in} and P_{out} (i.e. transmitted optical power) is also presented. Numerical results are presented for a typical case of well size $L_Z = 10.5 nm$, barrier size $L_B = 9.5 nm$, optical wave length $\lambda = 851.7 nm$ and electric field $F \leq 100 kV/cm$. It has been shown that for the values of P_{in} within certain range, the device changes its state in such a way that corresponding to every value of P_{in}, two stable states and one unstable state of V as well as of P_{out} are obtained which shows the optically controlled bistable nature of the device.**

I. INTRODUCTION

Modern optoelectronic devices based on electro-absorption have been of interest for more than a decade because of their nonlinear optical properties. Such devices most often employ p-i-n epitaxial structures in which the main optically active component is the intrinsic (i) region. It consists of a multiple quantum well structure (MQWS) composed of alternating layers of very thin (on the order of 10 *nm*) nominally undoped GaAs well layers and $Al_xGa_{1-x}As$ barrier -layers. The most important property of the MQWS is the existence of clearly- resolved exciton absorption peaks near the optical absorption edge at room temperature [1-2] whereas most normal semiconductors show well-resolved peaks only at low temperature. But, when an electric field is applied perpendicular to the quantum well layers, peaks of the exciton resonances move to the lower photon energy (i.e. to longer wavelengths) which results in the change in transition energy near the absorption edge. This phenomenon in quantum wells is known as *Quantum Confined Stark Effect* (QCSE)[3-5].

However, the general principle of the p-i(MQWS)-n diode structure is that when a reverse bias is applied across the diode in series with a load as shown in fig-1, the photocurrent passing through the electronic circuit influences the voltage across the device; which, in turn, influences the optical absorption in the device; and the optical absorption again influences the photocurrent in the circuit. Thus, a feedback process is established which is optoelectronic in nature. That is to say that this feedback mechanism exists only at the presence of some optical power incident on the device. The behavior of the device depends greatly on the nature of the electronic circuit and the sign of the feedback. If the device is operated under a positive feedback region, it shows the optical bistability or oscillation.

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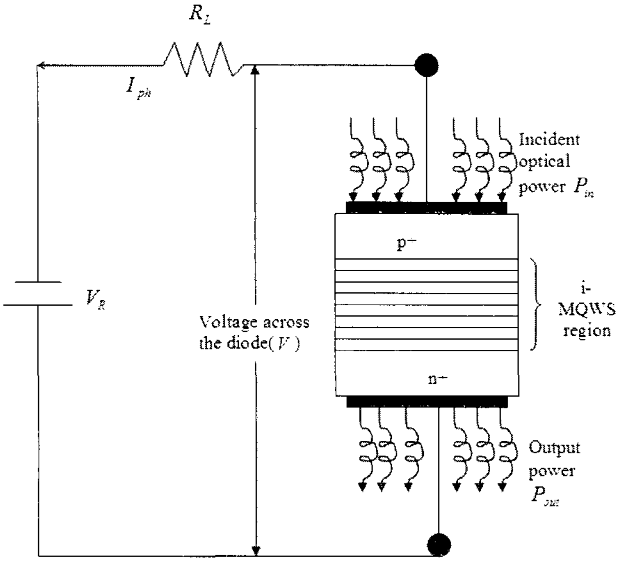


Fig. 1. Schematic diagram for optical bistability in p-i(MQWS)-n diode. R_L is a load resistor connected in series with a power supply V_R to make the diode reverse biased, P_{in} is the incident optical power and P_{out} is the transmitted power output from the device.

The optical bistability using a p-i(MQWS)-n diode reported earlier [6-7] are mainly based on the experimental results. The results for analyzing optical bistable nature of quantum well self-electrooptic effect devices (QW-SEED) reported by Miller *et. al.* [7] was dependent on the measured *responsivity* obtained by suitable experimental set-up. In this paper, an attempt has been made to explain the behavior of such devices using simple analytical expressions. The model presented in this paper is very simple in the sense that the MQWS considered for the present study is very simple as compared to the earlier reported works [6,7]. Simple analysis is also presented to obtain the optical bistability in such a structure. However, the purpose of the present paper is not to provide any new theoretical analysis or to improve the accuracy of other analytical models. The paper presents a simple design tool for the modeling of optical bistability utilizing the QCSE effect in the MQWS region of a simple p-i(MQWS)-n diode.

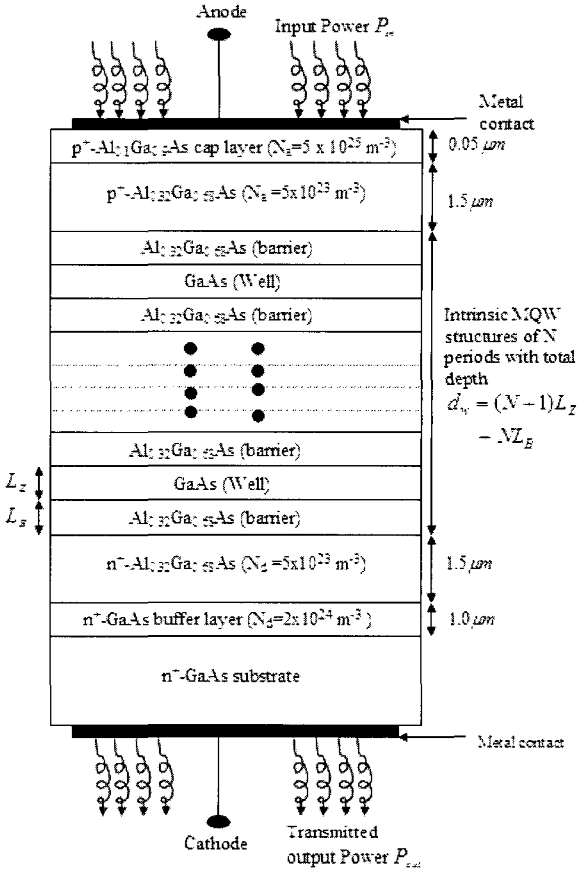


Fig. 2. Schematic structure of the MQW p-i-n diode. L_Z and L_B are the well size and barrier size respectively.

The structure for the p-i(MQWS)-n diode considered in this paper is shown in fig-2. The sample under consideration may be grown by molecular beam epitaxy on a Si –doped n$^+$ GaAs as the substrate. A buffer layer of n$^+$-GaAs with 1.0 μm thickness and doping concentration $N_d = 2\times10^{24}$ m^{-3} is developed on the substrate. Another layer of n$^+$- Al$_{0.32}$Ga$_{0.68}$As with $N_d = 5\times10^{23}$ m^{-3} and 1.5 μm thickness is grown on the buffer layer. Then the active part of the device, the MQWS region consisting of alternate thin layers GaAs and Al$_{0.32}$Ga$_{0.68}$As of well thickness $5nm \le L_Z \le 20nm$ and barrier thickness $9.5\mu m \le L_B \le 10.5\mu m$ is developed. If the MQWS is composed of N periods, the total thickness of the MQWS region is $d_w = NL_Z$ + $(N+1)L_B$. On the intrinsic MQWS region, a $1.5\mu m$ thick layer of p$^+$-Al$_{0.32}$Ga$_{0.68}$ with doping concentration of $N_a = 5\times10^{23}$ m^{-3} is also grown. For providing contact, a cap layer of p$^+$-Al$_{0.1}$Ga$_{0.9}$As of 0.05 μm thickness with $N_a = 5\times10^{25}$ m^{-3} is grown on the previous layer. Standard metallizations may be made on the cap layer and the substrate to provide metallic contacts for anode and cathode of the diode; respectively. Optical power P_{in} with light perpendicular to the layers is incident on the metal contact layer. A suitable antireflection coating is employed on the metal layer to reduce the reflection of light from the surface. It is to be noted that this structure is a simpler one as compared to the SEED [7]. However, the details of the structure is discussed elsewhere [8-10].

The main difficulty for studying the optical behavior of the p-i(MQWS)-n diode lies in obtaining an analytic expression for the optical absorption coefficient (α) which is a function of electric field (F), operating wavelength (λ) and the size of the well (L_Z). In most of the modern optoelectronic devices based on electroabsorption (i.e. electric field dependent optical absorption), the well size (L_Z) is normally considered to be in the range of $5nm \le L_Z \le 20nm$. In this paper, we have considered the expression for α using the semi-empirical model proposed by Lengyel et. al. [8], which is most accurately valid for the above range of L_Z and $F \le 200kV/cm$. The effect of discontinuity in effective masses at the heterostructure interfaces (i.e. different effective masses of an electron (a hole) in the well and barrier regions) are also considered as in [11].

The layout of this paper is as follows. Section II contains the analytical model for characterizing the behavior of the device. The semi-empirical model used for the calculation of the electric field dependent optical absorption co-efficient (α) in the MQWS region of the diode is presented. An analytical expression for the *responsivity* (S) of the device is derived. The relationship between *input optical power* (P_{in}) and *transmitted power* (P_{out}) has also been presented in this section. Section III presents some numerical results for a typical case of $L_Z = 10.5nm$, $\lambda = 851.7\ nm$ and $F \le 100kV/cm$. Section IV contains the summary and conclusion of the present work.

II. ANALYTICAL MODEL FOR OPTICAL BISTABILITY

In this section, we have presented the expressions for the optical absorption coefficient, responsivity and the relation between the input power and voltage across the diode. The relationship between the input power and transmitted power and the condition for bistability are also presented when the diode is reverse biased by a power supply in series with a load resistor, as shown in fig-1.

The Optical Absorption Coefficient ($\alpha(F,\lambda,L_Z)$): To study the characteristic behavior of the p-i(MQWS)-n diode analytically, one needs an analytical expression for the optical absorption coefficient (α) which is a function of operating wavelength (λ), electric field (F) and the well size (L_Z). The absorption spectrum α at the fundamental absorption edge can be expressed as the product of the peak value of the exciton absorption α_{hh} which is a function of F and L_Z ; and a suitable line shape function to include the effects of the broadening mechanisms [12]. It is, however, observed that a good agreement between the theoretical and experimental results for the absorption spectrum may be obtained by considering a Lorentzian line shape function. Thus, optical absorption coefficient in the MQWS region of the diode may be expressed as [8]

$$\alpha(F,\lambda,L_Z) = \alpha_{hh}(F,L_Z)\left\{1+\left[\frac{\left(E_0(F,L_Z)-\frac{hc}{\lambda}\right)}{\Gamma_{hh}(F,L_Z)}\right]^2\right\}^{-1} \quad (1)$$

$$for \quad \lambda > \frac{hc}{E_0}$$

where F is the electric field, λ is the optical wavelength, $c = 2.998\times10^8 m/s$ is the velocity of light, $h = 6.626\times10^{-34} J-s$ is the Planck's constant, $E_0(F,L_Z)$ is the energy of the *highest heavy hole* (1hh) exciton and $\Gamma_{hh}(F,L_Z)$ is its *half width at half maximum* (HWHM). The $\alpha_{hh}(F,L_Z)$ represents the peak value of the absorption exciton and may be described by the semi-empirical relation [8]

$$\alpha_{hh}(F,L_Z) = \frac{C_{fit}}{L_Z}\left|\int_{-\infty}^{\infty} \psi_e(z)\psi_h^*(z)\ dz\right|^2 + \alpha_{bulk} \quad (2)$$

where C_{fit} and α_{bulk} are constants and $\psi_{e(h)}$ is the perturbed wave function for electron (hole) which may be expressed as a linear combination of unperturbed wave functions and can be given by [13]

$$\psi_{e(h)}(z)=\sum_{i=1}^{n} a_{ie(h)}\psi^{0}_{ie(h)}(z) \qquad (3)$$

where $a_{ie(h)}$ and $\psi^{0}_{ie(h)}(z)$ are the expansion coefficients and unperturbed wave functions corresponding to the i^{th} energy level of the electron (hole) in the conduction (valence) band. In the above equation 'n' is the maximum number of bound energy states under the unperturbed condition in the conduction (valence) band for electron (heavy hole) and may be given by [14]

$$n = Maximum \;\; value \;\; of \left\{ 1+Int\left[\left(\frac{2m^{*}_{we(h)} \; \Delta E_{c(v)} \; L_Z^2}{\pi^2 \; \hbar^2} \right)^{\frac{1}{2}} \right] \right\} \qquad (4)$$

where $m^{*}_{we(h)}$ is the effective mass of an electron (heavy hole) in the well region, $\Delta E_{c(v)}$ is the band discontinuity in the conduction (valence) band and $[x]$ denotes the integral part of 'x'. The expansion coefficients $a_{ie(h)}$ may be determined numerically by using the method as described in ref [13].

It has been observed that if we consider C_{fit} = 16000 nm and $\alpha_{bulk} = 5500/cm$, a good agreement of the calculated result of $\alpha_{hh}(F,L_Z)$ with the experimental result may be obtained [8]. Further, only the highest heavy hole and the lowest electronic energy levels and their wave functions $\psi_{e(h)}$ are considered. Because, only these determine the optical absorption edge in the MQWS region with the above mentioned well sizes and electric fields.

Although the heavy hole exciton energy (i.e. $E_0(F,L_Z)$) for the fundamental edge and the expansion coefficients (i.e. $a_{ie(h)}$'s) may be calculated by using numerical method as described in [13], but for the sake of analytical purposes, we may express $E_0(F,L_Z)$ as [14]

$$E_0(F,L_Z)=E_g+E_{1c}(L_Z)+E_{1hh}(L_Z)-\Delta E_0(F,L_Z) \qquad (5)$$

where E_g is the band-gap energy of GaAs, $E_{1c}(L_Z)$ is the lowest unperturbed electronic energy level measured from the bottom of the conduction band , $E_{1hh}(L_Z)$ is the highest heavy hole energy level measured from the top of the valence band under unperturbed condition, as shown in fig-3(a). $\Delta E_0(F,L_Z)$ is the change in $E_0(F,L_Z)$ under the perturbed condition which is described in fig-3(b). It may be noticed from fig-3(b) that $E^{/}_{1c}$ and $E^{/}_{1hh}$ are dependent on the applied electric field. Instead of considering the field dependent relations of $E^{/}_{1c}$ and $E^{/}_{1hh}$ separately, the total change between the two energy levels due to the perturbation is described by $\Delta E_0(F,L_Z)$ in eqn.(5). The energy levels of electron and heavy hole under unperturbed condition (i.e. $E_{1c}(L_Z)$ and $E_{1hh}(L_Z)$ as shown in fig-3(a)) may be determined by solving the Schrodinger's equation for the unperturbed potential. For the simplification purposes, the binding energy of the exciton is neglected in the above relation. If the applied electric field is small enough such that $F \ll \frac{\hbar^2 \; \pi^2}{2 \; q \; m^{*}_{we(h)} \; L_Z^3}$, using the second order perturbation theory, $\Delta E_0(F,L_Z)$ can be given as [14]

$$\Delta E_0(F,L_Z)=\frac{1}{24\pi^2}\left(\frac{15}{\pi^2}-1\right)\frac{q^2F^2L_Z{}^4\left(m^{*}_{we}+m^{*}_{wh}\right)}{\hbar^2} \qquad (6)$$

where q is the charge of electron, m^{*}_{we} and m^{*}_{wh} are the effective masses of electron and heavy hole in the well region, respectively. The electric field may be made to be small enough by considering the MQWS region with a large value of N.

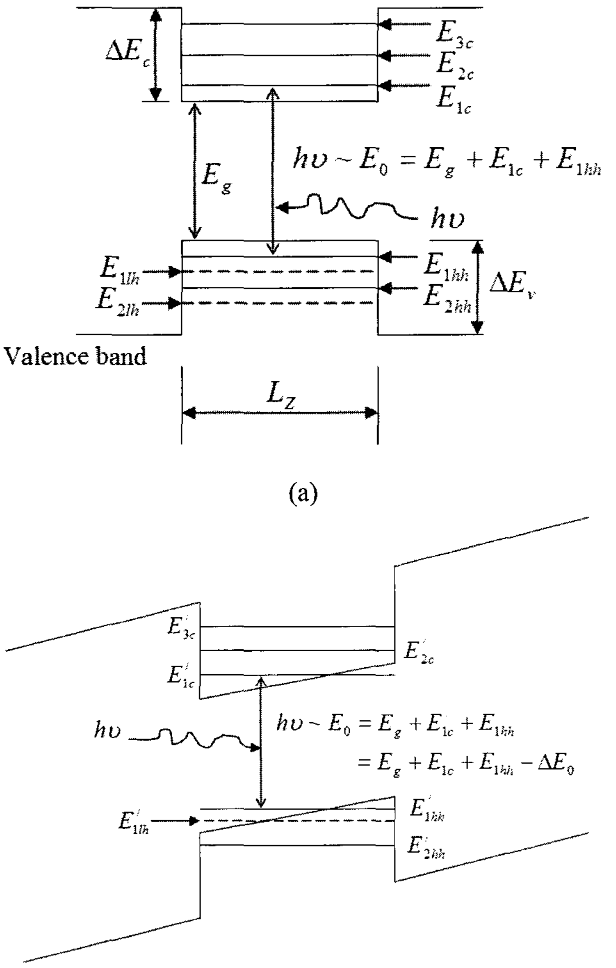
The semi-analytical relation for the HWHM for $5 \;\; nm \le L_Z \le 20 \;\; nm$ may be given as [8]

$$\Gamma_{hh}(F,L_Z)=7.374-0.511 \; L_Z+0.0182 \; L_Z{}^2-0.054 \; F+0.0161 \; F^2 \qquad (7)$$

where Γ_{hh} is in meV , L_Z is in nm , and F is in mV/nm . Thus the optical absorption coefficient in the MQWS region of the diode may be determined by using eqns. (2)-(7) in eqn.(1).

Expression for Responsivity $S(F,\lambda,L_Z)$: Since the intrinsic MQW region is sandwiched by p^+-$Al_{0.32}Ga_{0.68}As$ and n^+- $Al_{0.32}Ga_{0.68}As$ layers (see fig-2), it may be assumed that the MQWS region is completely depleted of carriers. Hence, if a reverse bias voltage V

is applied across the device, the whole voltage will appear across the intrinsic MQWS region because a negligible amount of voltage drop takes place across the heavily doped layers of the structure. Thus the electric field in the MQWS region may de given as [9]



(a)

(b)

Fig. 3. Discrete energy state diagram of a single quantum well: (a) in the absence of electric field and (b) when an electric field is applied perpendicular to the layers of the device.

$$F = F_0 + \frac{V}{d_w} \tag{8}$$

where F_0 is the built-in field and may be given by

$$F_0 = \frac{V_{pn}}{d_w} \tag{9}$$

in which V_{pn} is the difference in Fermi levels of the p^+ and n^+ $Al_{0.32}Ga_{0.68}As$ materials. The V_{pn} may be determined by using the Joyce-Dixon approximation [15] and may be given by

$$V_{pn} = E_{BG} + 0.026\left[\ln\left(\frac{n\ p}{N_c N_v}\right) + \frac{1}{\sqrt{8}}\left(\frac{n}{N_c} + \frac{p}{N_v}\right)\right]$$
$$= 1.7386 \tag{10}$$

where V_{pn} is in volts, $E_{BG} = 1.823\ eV$ is the band-gap energy of $Al_{0.32}Ga_{0.68}As$, $n = N_d = 5\times10^{23}\ m^{-3}$ (i.e. the donor concentration in the n^+ $Al_{0.32}Ga_{0.68}As$ region), $p = N_a = 5\times10^{23}\ m^{-3}$ (i.e. the acceptor concentration in the p^+ $Al_{0.32}Ga_{0.68}As$ region) and ; $N_c\ \left(=7.34\times10^{23}\ m^{-3}\right)$ and $N_v\ \left(=1.131\times10^{25}\ m^{-3}\right)$ are the effective densities of states for electron and hole, respectively.

The photocurrent flowing through the circuit in fig-1 may be given as [9]

$$I_{ph}(F,\lambda,L_Z) = \frac{\lambda(1-R)\ P_{in}(\lambda)\ \eta\ q}{hc}\left[1-\exp\left(-\alpha\left(F,\lambda,L_Z\right)\ d_w\right)\right] \tag{11}$$

where R is the reflection coefficient of the metal surface, η is the quantum efficiency and d_w is the width of the intrinsic MQW region. Since, F can be expressed in terms of the voltage V across the diode, we may write the analytical expression for the responsivity as

$$S(V,\lambda,L_Z) = \frac{I_{ph}\left(\frac{V+V_{pn}}{d_w},\lambda,L_Z\right)}{P_{in}(\lambda)}$$
$$= \frac{\lambda(1-R)\ \eta\ q}{hc}\left[1-\exp\left(-d_w\ \alpha\left(\left(\frac{V+V_{pn}}{d_w}\right),\lambda,L_Z\right)\right)\right] \tag{12}$$

Relation Between V and $P_{in}(\lambda)$: To study the optical bistability of MQWS under consideration, electronic circuit may be set-up as shown in fig-1. In this circuit, R_L is the load resistance and V_R is a power supply to make the p-i(MQWS)-n diode reverse biased.

Since the same photocurrent generated by the incident power $P_{in}(\lambda)$ flows through the load R_L, we may also write

$$I_{ph}(F,\lambda,L_Z)=\frac{V_R-V}{R_L} \tag{13}$$

where V is the voltage across the diode. Thus from eqns. (11) and (13) we may write

$$V=V_R-\frac{\lambda(1-R)\ P_{in}(\lambda)\ \eta\ q\ R_L}{hc}\left[1-\exp\left(-d_w\ \alpha\left(\left(\frac{V+V_{pn}}{d_w}\right),\lambda,L_Z\right)\right)\right] \tag{14}$$

Note that from eqn.(14), it is very difficult to derive the expression for V explicitly in terms of $P_{in}(\lambda)$. A reverse method may be employed to establish the relation between V and $P_{in}(\lambda)$. For a particular value of V, α may be calculated first and using the values of V and α, $P_{in}(\lambda)$ may be computed from eqn. (14). However, for a particular value of $P_{in}(\lambda)$, using numerical technique, V may be calculated from eqn(14).

Using eqns. (12) and (13), we may also express the relation between V and $P_{in}(\lambda)$ in terms of the responsivity as follows.

$$S(V,\lambda,L_Z)=\frac{I_{ph}\left(\left(\frac{V+V_{pn}}{d_w}\right),\lambda,L_Z\right)}{P_{in}(\lambda)} \\ =\frac{V_R-V}{R_L P_{in}(\lambda)} \tag{15}$$

Thus, we write

$$V=V_R-S(V,\lambda,L_Z)\ P_{in}\ R_L \tag{16}$$

Hence, if the responsivity $S(V,\lambda,L_Z)$ at a particular value of V is known, P_{in} may be computed by substituting the values of $S(V,\lambda,L_Z)$ and V in eqn. (16).

Relation between P_{out} and P_{in}: When the power P_{in} is incident on the diode as shown in fig-1, a certain fraction of P_{in} is absorbed in the MQW region which generates the photocurrent and the remaining power is transmitted out from the device as shown in the figure. If P_{out} be the transmitted power, then we may write

$$P_{out}(P_{in},\lambda,L_Z)=P_{in}(\lambda)\ \exp\left(-d_w\ \alpha\left(\left(\frac{V+V_{pn}}{d_w}\right),\lambda,L_Z\right)\right) \tag{17}$$

Note that V itself is a function of P_{in}. Hence, to determine P_{out} from the above equation the following steps may be followed.

i) For a particular value of V, compute α

ii) Using the values of V and α, compute $S(V,\lambda,L_Z)$

iii) Compute P_{in} either from eqn.(14) or (15)

iv) Subtitute the values of α and P_{in} in eqn. (17) and thus P_{out} may be obtained

However, using numerical method, V may be obtained from eqn.(14) for a particular value of P_{in} and substituting the values of P_{in} and V in eqn.(17), we may also determine P_{out}.

Condition for optical bistability: When the optical power P_{in} is incident on the device as shown in fig-1, a feedback mechanism is established which is trully optoelectronic in nature as mentioned earlier. If a condition is created so that the responsivity decreases with the increase in voltage across the device, then a positive feedback is established due to the *negative differential conductance* characteristic of the device and optical bistability in the device may be obtained. It can be shown that for bistability to exist for a particuar MQW p-i-n diode structure and at a given V_R and λ, the inequality

$$\frac{dS}{dV}<-\frac{S}{V_R-V}$$

$$\Rightarrow -\frac{dS}{dV}>\frac{S}{V_R-V}$$

$$\Rightarrow f(V)=\left[(V-V_R)\frac{dS}{dV^{l}}-S\right]>0 \tag{18}$$

must be satisfied for some range of V [7]. In other words, as long as the responsivity S increases with the increase in V, positive feedback can not take place. But, if S decreases with the increase in V, a condition for the positive feedback in the circuit may be achieved depending on the rest of the circuit. Note that, because of the complexity involved in S, it is very difficult to get a closed analytical expression for $f(V)$ to determine the range of V over which the bistability may occur. However, the above equation may be solved numerically to determine the range of V for which optical bistability may be obtained.

III. RESULTS AND DISCUSSIONS

In this section we present some numerical results for a MQW p-i-n diode structure using the values of different parameters as mentioned in table-I. All the calculations for determining the optical absorption coefficient have been carried out in a similar way as discussed in [8]. $E_{1c} = 34.21\ meV$ and $E_{1h} = 5.75\ meV$ are obtained by solving the Schrodinger's equation for unperturbed potential. The peak value of the absorption exciton $\alpha_{hh}(F) = \alpha_{hh}(F, 10.5nm)$ is calculated from eqn.(2) for different electric fields $F \le 100\ kV/cm$. Using the curve fitting technique of MATLAB, it may be approximated as a polynomial given by

$$\alpha_{hh}(F) = -1.742\times10^{-6}F^5 + 2.883\times10^{-4}F^4 + 6.516\times10^{-3}F^3 - 2.694F^2 - 41.65F + 20909 \qquad (19)$$

where $\alpha_{hh}(F)$ is in cm^{-1} and F is expressed in kV/cm. The actual and approximated values of $\alpha_{hh}(F)$ are shown in fig-4. From the figure, it is observed that as the electric field is increased, the corresponding value of $\alpha_{hh}(F)$ is decreased. Substituting eqn.(19) in eqn.(1) we get an simplified analytical expression for α. Using this expression for α in eqn. (12), the *responsivity* is determined and is shown in fig-5. It is seen from the figure that initially *responsivity* increases with the increase in the voltage V across the diode but starts to of V we observe the *negative differential conductance* characteristic of the p-i-n diode which establishes a positive feedback mechanism in the electronic circuit shown in fig-1. The optical bistability may be obtained over this range of V.

The range of V over which at least one unstable point must exist for the bistability, may be obtained from eqn. (18). But, due to the complexity involved in the analytical expression for responsivity $S(V) = S(V, 851.7nm, 10.5nm)$, using the curve fitting technique of MATLAB the computed responsivity may be expressed as a polynomial given by Using eqn.(20) in eqn.(18), the function $f(V)$ is computed and is shown in fig-6. From the figure we observe that condition for bistability (i.e. $f(V) > 0$) is satisfied for $V \ge 4.75$ *volts*. In fig-7, variation of the voltage V across the device due to the incident power P_{in} is shown. From the figure, it is observed that, initially when $P_{in} = 0$, the whole reverse

Table 1. Parameter values used for numerical calculations.

Symbols	Descriptions	Values considered
N	No. of periods	1 50
L_Z	Width of a quantum well	$10.5nm$
L_B	Width of the barrier	$9.5nm$
m_0	The rest mass of an electron	$9.109\times10^{-31}kg$
q	Charge of an electron	$1.602\times10^{-19}C$
$m^*_{w(b)e}$	Effective mass of electrons in the well (barrier) region	$0.067m_0$ (in GaAs well) and $0.0936m_0$ (in $Al_{0.32}Ga_{0.68}As$ barrier)
$m^*_{w(b)h}$	Effective mass of heavy holes in the well (barrier) region	$0.48m_0$ (in GaAs well) and $0.5792m_0$ (in $Al_{0.32}Ga_{0.68}As$ barrier)
ΔE_c	Barrier height in the conduction band	$240\ meV$
ΔE_v	Barrier height in the valence band	$160\ meV$
E_g	Band gap energy of GaAs	$1.424\ eV$
F	Electric field	$F \le 100\ kV/cm$
λ	Operating optical wave length	$851.7\ nm$
η	Quantum efficiency	1.0
R	Reflection coefficient of the metal surface	0
V_R	Supply voltage to make the diode reverse biased	$8.0\ volts$
R_L	Load resistor connected in series with V_R	$1.0\ M\Omega$

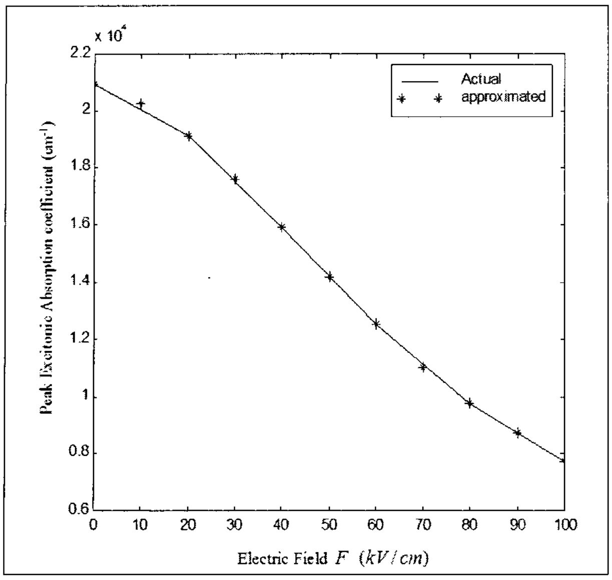


Fig. 4. Comparison of the calculated and approximated values of the peak excitonic absorption coefficient (α_{hh}) for $L_Z = 10.5\ nm,\ L_B = 9.5\ nm,\ R_L = 1.0\ M\Omega,\ V_R = 8.0\ volts$ and $\lambda = 851.7\ nm$.

voltage V_R appears across the diode (see fig-1) because there is no voltage drop across R_L since no photo current is generated by the device under this condition. But, when P_{in} is increased, photo current is also increased which results in the voltage drop V to decrease. But, for any value P' (say) of P_{in} in the range of $P_1 \le P_{in} \le P_2$ as shown in the figure, we observe three voltage levels A, B and C across the device where the middle point B corresponds to the unstable state and the other two voltage levels (i.e. A and C) represent two stable states and thus the optical bistability is obtained. If $P_{in} > P_2$, a stable value of V is obtained corresponding to every P_{in}. Also note that the region of unstability is $V_1 \le V \le V_2$ which satisfies the condition for bistability described by eqn.(18) (see fig-6). The change in the transmitted optical power(P_{out}) due to the change in the incident optical power (P_{in}) is shown in fig-8. From the figure, it is observed that corresponding to every value of P_{in} within the range $P_1 \le P_{in} \le P_2$, there are two values of P_{out} and again bistability is obtained. Outside the above range of P_{in} , a stable value of P_{out} is obtained and P_{out} increases with the increase in P_{in} .

It is to be noted that for $V_1 \le V \le V_2$, the corresponding range of input power P_{in} (i.e. the value of P_1 and P_2) over which optical bistability takes place may be

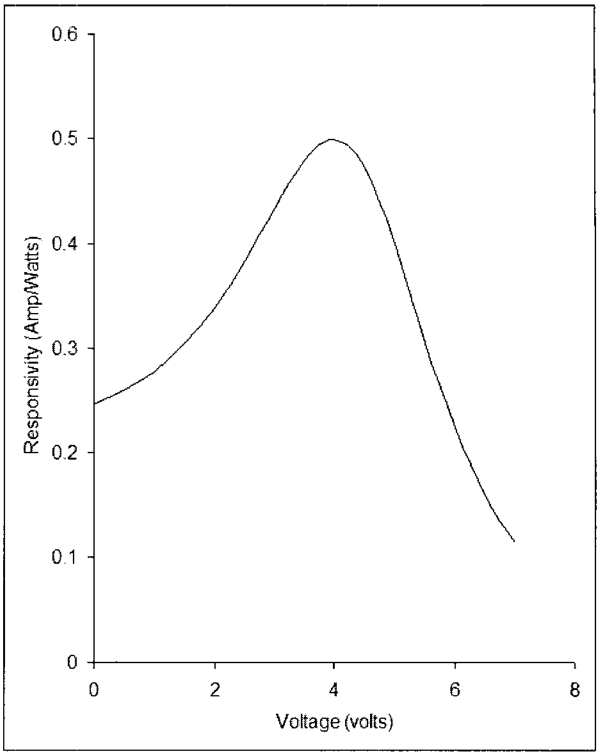


Fig. 5. Variation of responsivity $S(V)$ with the voltage V across the diode structure under reverse biased condition.

$$S(V) = 0.0002\ V^8 - 0.002\ V^7 + 0.0094\ V^6 - 0.0266\ V^5 + 0.0452\ V^4 - 0.0434\ V^3 + 0.0335\ V^2 + 0.0159\ V + 0.2461 \tag{20}$$

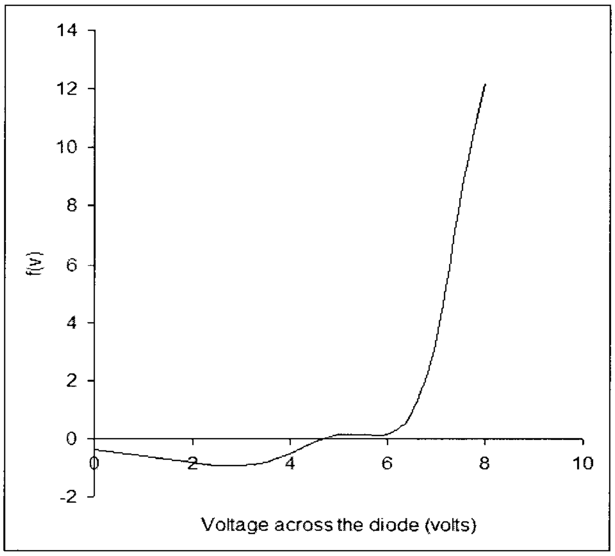


Fig. 6. The function $f(V)$ is plotted as a function of the voltage V across the device to verify the condition of the optical bistability in the MQW diode structure.

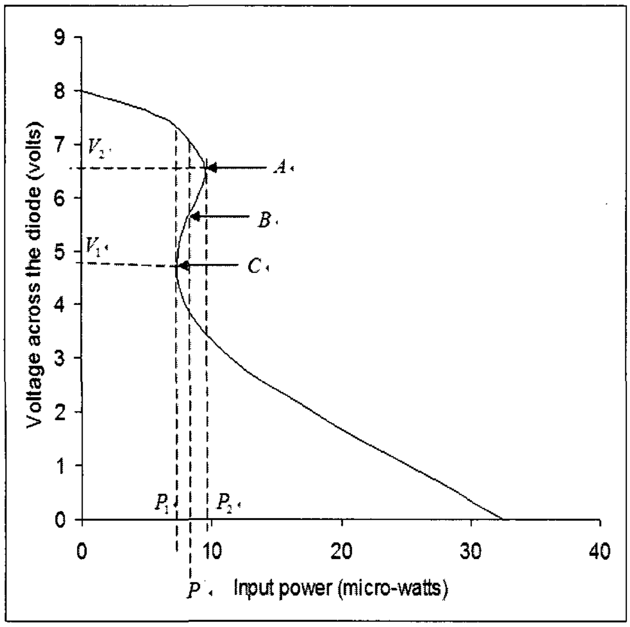


Fig. 7. Variation of the voltage V across the diode due to the change in the optical input power P_{in}. It is observed that for any value $P^{/}$ of P_{in} within certain range $P_1 \le P_{in} \le P_2$, three voltage levels are obtained. The middle point B is unstable and other two points A and C are stable states and hence the optical bistability is obtained.

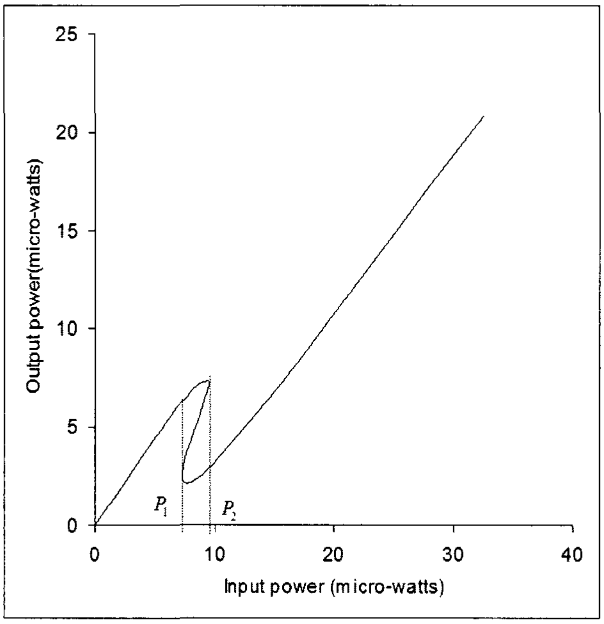


Fig. 8. Variation of output power P_{out} due to the change in the incident power P_{in} and again bistability is observed.

determined from eqn.(16) for fixed values of R_L and V_R. Further, since bistability is basically a cusp catastrophe, hence if the system is bistable with respect to the variation of one parameter, we can expect that it will be bistable with respect to others also [7]. Thus, form eqns.(12) and (16), we can expect to see the bistability with respect to any parameter among V_R, R_L, V, P_{in}, P_{out} etc. provided eqn.(18) is satisfied over some range of V and other conditions are held fixed.

IV. SUMMARY AND CONCLUSIONS

Using the concept of internal optoelectronic feedback mechanism combined with the quantum confined stark effect in MQWS, a simple analytical model is presented for the study of optical bistability in optoelectronic absorption based modulators. The device under consideration is a simple p-i-n diode with the MQWS of alternate thin layers of GaAs/$Al_{0.32}Ga_{0.68}As$ in the intrinsic region (i) of the diode. The diode is operated under reverse biased condition by connecting a power supply in series with a load resistor. When optical power is incident on the device a feedback mechanism is established within the electronic circuit which is truly optoelectronic in nature. If the feedback is made to be a positive one, the optical bistability in such devices may be observed.

In this paper, an attempt has been made to present a simple analytical model for studying the behavior of such devices quantitatively. Assuming that the absorption edge is mainly determined by the first heavy hole energy level in the valence band and the first electron energy level in the conduction band, the analytical expression for optical absorption coefficient is presented using a semi-empirical model which is valid for well sizes in the range of $5nm \le L_Z \le 20nm$ and electric field $F < 200kV/cm$. Analytical expressions for the responsivity, the input power and voltage across the diode and; the input and transmitted power are presented in this paper. Analytical results for showing the optically controlled bistable nature of the device are discussed for $L_Z = 10.5nm$ and $F \le 100kV/cm$.

It is observed that as the input power is increased from dark condition (i.e. $P_{in} = 0$), the voltage across the device is decreased but the out put power is increased. When P_{in} exceeds certain value, for every value of P_{in} within certain range, two values of voltage across the device as well as two values of the transmitted power from the device are obtained which shows the optical bistability in such devices.

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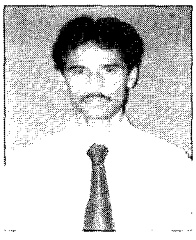

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