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Noninformative Priors for the Common Scale Parameter in the Inverse Gaussian Distributions[1]

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Abstract

In this paper, we develop the noninformative priors for the common scale parameter in the inverse gaussian distributions. We developed the first and second order matching priors. Next we revealed that the second order matching prior satisfies a HPD matching criterion. Also we showed that the second order matching prior matches alternative coverage probabilities up to the second order. It turns out that the one-at-a-time reference prior satisfies a second order matching criterion. Some simulation study is performed.

Keywords : Common Scale, Inverse Gaussian, Matching Prior, Reference Prior

1. Introduction

Consider k independent inverse gaussian populations with parameters μ_i and λ. Let $X_{ij}, j = 1, \cdots, n_i$ denote observations from the ith inverse gaussian population, $i = 1, \cdots, k$. Then the inverse gaussian distribution is given by,

$$f(x_{ij}) = \sqrt{\frac{\lambda}{2\pi}} x_{ij}^{-\frac{3}{2}} \exp\left\{-\frac{\lambda(x_{ij}-\mu_i)^2}{2\mu_i^2 x_{ij}}\right\}, x_{ij} > 0, i = 1, \cdots, k, j = 1, \cdots, n_i, \qquad (1)$$

where $\mu_i > 0$ and $\lambda > 0$. Because of the versatility and flexibility in modelling right-skewed data, the inverse gaussian distribution has potential useful

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applications in a wide variety of fields such as biology, economics, reliability theory, life testing and social sciences as discussed in Chhikara and Folks (1978, 1989) and Seshadri (1999). Tweedie (1957a, 1957b) established many important statistical properties of the inverse gaussian distribution and discussed the similarity between statistical methods based on the inverse gaussian distribution and those based on the normal theory.

The common scale parameter λ is of interest. This parameter λ is shown in the analysis of reciprocals (Tweedie, 1957a; Fries and Bhattacharyya, 1983) and regression models (Whitmore, 1979).

The present paper focuses on noninformative priors for λ. We consider Bayesian priors such that the resulting credible intervals for λ have coverage probabilities equivalent to their frequentist counterparts. Although this matching can be justified only asymptotically, our simulation results indicate that this is indeed achieved for small or moderate sample sizes as well.

This matching idea goes back to Welch and Peers (1963). Interest in such priors revived with the work of Stein (1985) and Tibshirani (1989). Among others, we may cite the work of DiCiccio and Stern (1994), Datta and Ghosh (1995a,b, 1996), Mukerjee and Ghosh (1997) and Mukerjee and Reid (1999).

On the other hand, Ghosh and Mukerjee (1992), and Berger and Bernardo (1989,1992) extended Bernardo's (1979) reference prior approach, giving a general algorithm to derive a reference prior by splitting the parameters into several groups according to their order of inferential importance. This approach is very successful in various practical problems. Quite often reference priors satisfy the matching criterion described earlier.

The outline of the remaining sections is as follows. In Section 2, we develop first order and second order probability matching priors for λ. We revealed that the second order matching prior matches the alternative coverage probabilities up to the same order, and is a HPD matching prior up to the same order. Also we derive the reference priors for the parameters. It turns out that the one-at-a-time reference prior satisfies a second order matching criterion. We provide that the propriety of the posterior distribution for the reference priors as well as second order matching prior. In Section 4, simulated frequentist coverage probabilities under the proposed priors are given.

2. The Noninformative Priors

2.1 The Matching Priors

For a prior π, let $\theta_1^{1-\alpha}(\pi;\boldsymbol{X})$ denote the $(1-\alpha)$th percentile of the posterior distribution of θ_1, that is,

$$P^{\pi}[\theta_1 \le \theta_1^{1-\alpha}(\pi;\boldsymbol{X}) \mid \boldsymbol{X}] = 1-\alpha, \tag{2}$$

where $\theta = (\theta_1, \cdots, \theta_t)^T$ and θ_1 is the parameter of interest. We want to find priors π for which

$$P[\theta_1 \le \theta_1^{1-\alpha}(\pi;\boldsymbol{X}) \mid \theta] = 1-\alpha + o(n^{-u}). \tag{3}$$

for some u, as n goes to infinity. Priors π satisfying (3) are called matching priors. If $u = 1/2$, then π is referred to as a first order matching prior, while if $u = 1$, π is referred to as a second order matching prior.

We now begin to find such matching priors π. The likelihood function of parameters $(\lambda, \mu_1, \cdots, \mu_k)$ for the model (1) is given by

$$L(\lambda, \mu_1, \cdots, \mu_k) \propto \lambda^{\frac{N}{2}} \exp\left\{-\frac{\lambda}{2}\sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{(x_{ij}-\mu_i)^2}{\mu_i^2 x_{ij}}\right\}, \tag{4}$$

where $N = n_1 + \cdots + n_k$. Based on (4), the Fisher information matrix is given by

$$I = Diag\left\{\frac{N}{2}\lambda^{-2}, n_1\lambda\mu_1^{-3}, \cdots, n_k\lambda\mu_k^{-3}\right\} \tag{5}$$

From the above Fisher information matrix I, λ is orthogonal to $(\mu_1, \cdots, \mu_k)$ in the sense of Cox and Reid(1987). Following Tibshirani(1989), the class of first order probability matching prior is characterized by

$$\pi_m^{(1)}(\lambda, \mu_1, \cdots, \mu_k) \propto \lambda^{-1} d(\mu_1, \cdots, \mu_k), \tag{6}$$

where $d(\mu_1, \cdots, \mu_k) > 0$ is an arbitrary function differentiable in its arguments.

The class of prior given in (6) can be narrowed down to the second order probability matching priors as given in Mukerjee and Ghosh (1997). A second order probability matching prior is of the form (6), and also d must satisfy an additional differential equation (cf (2.10) of Mukerjee and Ghosh (1997)), namely

$$\frac{1}{6} d(\mu_1, \cdots, \mu_k)\frac{\partial}{\partial\lambda}\left\{I_{11}^{-\frac{3}{2}} L_{1,1,1}\right\} + \sum_{v=1}^{k}\frac{\partial}{\partial\mu_v}\left\{I_{11}^{-\frac{1}{2}} L_{11v} I^{vv} d(\mu_1, \cdots, \mu_k)\right\} = 0, \tag{7}$$

where

$$L_{1,1,1}=E[(\frac{\partial \log L}{\partial\lambda})^3]=-N\lambda^{-3}, L_{11v}=E[\frac{\partial^3 \log L}{\partial\lambda^2\partial\mu_v}]=0,$$

where, $v=1,\cdots,k$ and $I_{11}=\frac{N}{2}\lambda^{-2}$

Then (7) simplifies to

$$-\frac{N^{-1/2}2^{3/2}}{6}d(\mu_1,\cdots,\mu_k)\frac{\partial}{\partial\lambda}\{\lambda^3\cdot\lambda^{-3}\}=0.$$

Thus the resulting second order probability matching prior is

$$\pi_m^{(2)}(\lambda,\mu_1,\cdots,\mu_k)\propto\lambda^{-1}d(\mu_1,\cdots,\mu_k). \tag{8}$$

If $\pi(\cdot)$ is second order matching for θ then Bayesian credible sets of the form$(-\infty,\theta^{1-\alpha}(\pi;\boldsymbol{X})]$ for θ have correct frequentist coverage as well, with margin of error
$o(n^{-1})$. In this case, such Bayesian credible sets can also be interpreted as frequentist confidence sets. From the frequentist point of view, however, the probability for a confidence set to include an alternative value of the parameter of interest is as important as that of the true coverage. Such an alternative coverage probability indicates how selective a confidence set is. So Mukerjee and Reid (1999) studied that a prior satisfying (3) matches $P[\theta_1+\beta(I^{11}/n)^{1/2}\le\theta_1^{1-\alpha}(\pi;\boldsymbol{X})\mid\theta]$ with the corresponding posterior probability, up to the same order and for each β and α, where the scalar β is free from n,θ and $\boldsymbol{X}$. If a matching prior matches the alternative coverage probabilities then there is a stronger justification for calling it noninformative in so far as agreement with a frequentist is concerned. In general a second order matching prior may or may not match the alternative coverage probabilities up to the same order of approximation.

Under orthogonal parametrization, Mukerjee and Reid (1999) gives the simple differential equations that a second order probability matching prior matches alternative coverage probabilities up to the second order.

Since

$$L_{111}=E[\frac{\partial^3 \log L}{\partial\lambda^3}]=N\lambda^{-3}, L_{11j}=E[\frac{\partial^3 \log L}{\partial\lambda^2\partial\mu_j}]=0, j=1,\cdots,k,$$

$$L_{1,11}=E[\frac{\partial \log L}{\partial\lambda}\frac{\partial^2 \log L}{\partial\lambda^2}]=0, L_{j,11}=E[\frac{\partial \log L}{\partial\mu_j}\frac{\partial^2 \log L}{\partial\lambda^2}]=0, j=1,\cdots,k,$$

and $I_{11} = \frac{N}{2}\lambda^{-2}$, thus the differential equations ((3.18)-(3.20) of Mukerjee and Reid (1999)) are simplified to

$$\frac{\partial}{\partial\lambda}\left\{I_{11}^{-\frac{3}{2}} L_{111}\right\} = \frac{\partial}{\partial\lambda}\left\{\left(\frac{2}{N}\right)^{3/2}\lambda^{3} \cdot N\lambda^{-3}\right\} = 0, \frac{\partial}{\partial\lambda}\left\{I_{11}^{-\frac{3}{2}} L_{1,11}\right\} = \frac{\partial}{\partial\lambda}\left\{I_{11}^{-\frac{3}{2}} \cdot 0\right\} = 0,$$

$$\sum_{j=1}^{k} \frac{\partial}{\partial\mu_j}\left\{L_{11j} I^{jj} I_{11}^{-\frac{1}{2}} d(\mu_1, \cdots, \mu_k)\right\} = \sum_{j=1}^{k} \frac{\partial}{\partial\mu_j}\left\{0 \cdot I^{jj} I_{11}^{-\frac{1}{2}} d(\mu_1, \cdots, \mu_k)\right\} = 0,$$

and

$$\sum_{j=1}^{k} \frac{\partial}{\partial\mu_j}\left\{L_{j,11} I^{jj} I_{11}^{-\frac{1}{2}} d(\mu_1, \cdots, \mu_k)\right\} = \sum_{j=1}^{k} \frac{\partial}{\partial\mu_j}\left\{0 \cdot I^{jj} I_{11}^{-\frac{1}{2}} d(\mu_1, \cdots, \mu_k)\right\} = 0.$$

Therefore the second order matching prior (8) matches the alternative coverage probabilities up to the second order.

There are alternative ways through which matching can be accomplished. One such approach (DiCiccio and Stern, 1994; Ghosh and Mukerjee, 1995) is matching through the HPD region. Specifically, if $\tilde{\pi}$ denotes the posterior distribution of θ_1 under a prior π, and $k_\alpha \equiv k_\alpha(\pi; \boldsymbol{X})$ is such that

$$P^{\pi}[\tilde{\pi}(\theta_1 \mid \boldsymbol{X}) \geq k_\alpha \mid \boldsymbol{X}] = 1 - \alpha + o(n^{-u}), \tag{9}$$

then the HPD region for θ_1 with posterior coverage probability $1 - \alpha + o(n^{-u})$ is given by

$$H_\alpha(\pi; \boldsymbol{X}) = \{\theta_1 : \tilde{\pi}(\theta_1 \mid \boldsymbol{X}) \geq k_\alpha\}. \tag{10}$$

DiCicco and Stern (1994) and Ghosh and Murkerjee (1995) characterized priors π for which

$$P[\theta_1 \in H_\alpha(\pi; \boldsymbol{X}) \mid \theta] = 1 - \alpha + o(n^{-u}), \tag{11}$$

for all θ and all $\alpha \in (0, 1)$. They found necessary and sufficient conditions under which π satisfies (11).

Recently, Datta, Ghosh and Mukerjee (2000) provided a theorem which establishes the equivalence of second order matching priors and HPD matching priors within the class of first order matching priors. The equivalence condition is that $I_{11}^{-3/2}L_{111}$ dose not depend on θ_1. Since $L_{111}=E[\partial^3 \log L/\partial\lambda^3]=N\lambda^{-3}$, thus $I_{11}^{-3/2}L_{111}=N^{-1/2}2^{3/2}$. So $I_{11}^{-3/2}L_{111}$ does not depend on λ. Therefore the second order probability matching prior (8) is a HPD matching prior up to the same order.

2.2 The Reference Priors

Reference priors introduced by Bernardo (1979), and extended further by Berger and Bernardo (1992) have become very popular over the years for the development of noninformative priors. In this Section, we derive the reference priors for different groups of ordering of $(\lambda,\mu_1,\cdots,\mu_k)$. Then due to the orthogonality of the parameters, following Datta and Ghosh (1995), choosing rectangular compacts for each $\lambda,\mu_1,\cdots,\mu_k$ when λ is the parameter of interest, the reference priors are given as follows.

If λ is the parameter of interest, then the reference prior distributions for different groups of ordering of $(\lambda,\mu_1,\cdots,\mu_k)$ are:

Group ordering	Reference prior
$\{(\lambda,\mu_1,\cdots,\mu_k)\}$,	$\pi_1 \propto \lambda^{-\frac{2-k}{2}}\mu_1^{-\frac{3}{2}}\cdots\mu_k^{-\frac{3}{2}}$
$\{\lambda,\mu_1,\cdots,\mu_k\},\{\lambda,(\mu_1,\cdots,\mu_k)\},\{(\mu_1,\cdots,\mu_k),\lambda\}$,	$\pi_2 \propto \lambda^{-1}\mu_1^{-\frac{3}{2}}\cdots\mu_k^{-\frac{3}{2}}$.

Remark 1. In the above reference priors, the one-at-a-time reference prior satisfies a second order matching criterion. But Jeffreys' prior is not a second order matching prior.

In the above results (8), the second order probability matching priors are given by

$$\pi_m^{(2)}(\lambda,\mu_1,\cdots,\mu_k)\propto\lambda^{-1}d(\mu_1,\cdots,\mu_k),$$

where d is any smooth function of $\mu_1,\cdots,\mu_k$. However every function is not permissible in the construction of priors. For instance, we consider any function of the form$(\mu_1\cdots\mu_k)^{-a}$. If a is negative integer, then the posterior distribution of λ is proper. But the condition of propriety in this form strongly depend on the a. Moreover there does not seem to be any improvement in the coverage probabilities with this posterior distribution. So we consider a particular second order matching prior where $d=\mu_1^{-3/2}\cdots\mu_k^{-3/2}$. Because this matching prior is the one-at-a-time reference prior. The matching prior is given by

$$\pi_m^{(2)}(\lambda,\mu_1,\cdots,\mu_k)=\lambda^{-1}\mu_1^{-\frac{3}{2}}\cdots\mu_k^{-\frac{3}{2}}. \tag{12}$$

Remark 2. We show that the prior (12) is joint probability matching when λ, $\mu_1,\cdots,\mu_{k-1}$ and μ_k are of interest. Write $\theta=(\lambda,\mu_1,\cdots,\mu_k)$. Let $t_1(\theta)=\lambda$, $t_2(\theta)=\mu_1,\cdots,t_k(\theta)=\mu_{k-1}$ and $t_{k+1}(\theta)=\mu_k$. Following the notation of Datta (1996), $P(\theta)=Diag\{1,1,\cdots,1\}$. Thus condition (7) of Datta (1996) is satisfied. Moreover the prior (12) is the unique solution to the equations of (2) of Datta (1996). Thus the prior (12) is joint probability matching prior for $(\lambda,\mu_1,\cdots,\mu_k)$. So this matching prior can be used for the Bayesian inference in the analysis of reciprocals and regression models.

3. Implementation of the Bayesian Procedure

We investigate the propriety of posteriors for a general class of priors which include the Jeffreys' prior and the second order matching prior (12). We consider the class of priors

$$\pi_m^{(2)}(\lambda,\mu_1,\cdots,\mu_k)=\lambda^{-a}\mu_1^{-b}\cdots\mu_k^{-b}, \tag{13}$$

where $|a|\geq 0$ and $b>0$. The following general theorem can be proved.

Theorem 1. The posterior distribution of $(\lambda,\mu_1,\cdots,\mu_k)$ under the general prior (13) is proper if $N-2a-kb+k+2>0$ and $b>1$.

Proof. Under the general prior (13), the joint posterior for $\lambda,\mu_1,\cdots,\mu_k$ given $\boldsymbol{x}$ is

$$\pi(\lambda,\mu_1,\cdots,\mu_k\mid\boldsymbol{x})\propto\lambda^{\frac{N-2a}{2}}\mu_1^{-b}\cdots\mu_k^{-b}\exp\left\{-\frac{\lambda}{2}\sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{(x_{ij}-\mu_i)^2}{\mu_i^2x_{ij}}\right\}. \tag{14}$$

Integrating with respect to λ, $(\mu_1,\cdots,\mu_k)$ has the posterior

$$\pi(\mu_1,\cdots,\mu_k \mid \boldsymbol{x}) \propto \mu_1^{-b}\cdots\mu_k^{-b}\left[\sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{(x_{ij}-\mu_i)^2}{\mu_i^2 x_{ij}}\right]^{-\frac{N-2a+2}{2}}$$
$$\leq x_{max}^{\frac{N-2a+2}{2}}\mu_1^{-b}\cdots\mu_k^{-b}\left[\sum_{i=1}^{k}\sum_{j=1}^{n_i}\frac{(x_{ij}-\mu_i)^2}{\mu_i^2}\right]^{-\frac{N-2a+2}{2}}, \qquad (15)$$

provided $N-2a+2>0$ and $x_{max}=max_{1\leq i\leq k, 1\leq j\leq n_i}\{x_{ij}\}$. For (15), substituting $t_i=\mu_i^{-1},\ i=1,\cdots,k,$ then

$$\pi(t_1,\cdots,t_k \mid \boldsymbol{x}) \propto t_1^{b-2}\cdots t_k^{b-2}[\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij}t_i-1)^2]^{-\frac{N-2a+2}{2}}$$
$$\leq c_1 t_1^{b-2}\cdots t_k^{b-2}\prod_{i=1}^{k}[\sum_{j=1}^{n_i}(x_{ij}t_i-1)^2]^{-\frac{N-2a+2}{2k}},$$

where c_1 is a constant. For $t_i\in(0,t_{i0}], i=1,\cdots,k,$ the integral $\int_0^\infty t_i^{b-2}[\sum_{j=1}^{n_i}(x_{ij}t_i-1)^2]^{-\frac{N-2a+2}{2k}}dt_i$ is proper if $b>1$. Also, for $t_i\in(t_{i0},\infty)$, $i=1,\cdots,k,$ $\int_0^\infty t_i^{b-2}[\sum_{j=1}^{n_i}(x_{ij}t_i-1)^2]^{-\frac{N-2a+2}{2k}}dt_i$ is proper if $(N-2a+2)/k-b+1>0$, so that $N-2a-kb+k+2>0$. This completes the proof. □

Theorem 2. Under the general prior (13), the marginal posterior density of λ is given by

$$\pi(\lambda \mid \boldsymbol{x}) \propto \prod_{i=1}^{k}\left\{\Gamma(\frac{b-1}{2})\,{}_1F_1[\frac{b-1}{2},\frac{1}{2},\frac{n_i\lambda}{2\bar{x}_i}]+\sqrt{\frac{2n_i\lambda}{\bar{x}_i}}\Gamma(\frac{b}{2})\,{}_1F_1[\frac{b}{2},\frac{3}{2},\frac{n_i\lambda}{2\bar{x}_i}]\right\}$$
$$\times\lambda^{-\frac{N-2a-bk+k}{2}}\exp\left\{-\frac{\lambda}{2}[\sum_{i=1}^{k}\sum_{j=1}^{n_i}x_{ij}^{-1}]\right\},$$

where $\bar{x}_i=\sum_{j=1}^{n_i}x_{ij}/n_i,\ i=1,\cdots,k$ and ${}_1F_1[\cdot,\cdot,\cdot]$ is Kummer confluent hypergeometric function.

The normalizing constant for the marginal density of λ requires a one dimensional integration. Therefore we can have the marginal posterior density of λ, and so we can compute the marginal moment of λ. In Section 4, we investigate the frequentist coverage probabilities for the Jeffreys' prior and the one-at-a-time reference prior, respectively.

4. Numerical Studies and Discussion

We evaluate the frequentist coverage probability by investigating the credible interval of the marginal posteriors density of λ under the noninformative prior π given in Section 3 for several configurations $(\lambda, \mu_1, \cdots, \mu_k)$ and $(n_1, \cdots, n_k)$. That is to say, the frequentist coverage of a $(1-\alpha)th$ posterior quantile should be close to $1-\alpha$. This is done numerically. Table 1 gives numerical values of the frequentist coverage probabilities of 0.05(0.95) posterior quantiles for the our prior. The computation of these numerical values is based on the following algorithm for any fixed true $(\lambda, \mu_1, \cdots, \mu_k)$ and any prespecified probability value α. Here α is 0.05(0.95). Let $\theta_1^{\pi}(\alpha \mid \boldsymbol{X})$ be the

Table 1: Frequentist Coverage Probabilities of 0.05 (0.95) Posterior Quantiles for λ

λ	n_1, n_2, n_3	π_1	π_2
0.5	3, 3, 3	0.277(0.995)	0.074(0.973)
	3, 5, 5	0.199(0.992)	0.063(0.967)
	5, 5, 5	0.179(0.989)	0.065(0.962)
	5, 10, 10	0.130(0.979)	0.057(0.950)
	10, 10, 10	0.104(0.980)	0.044(0.955)
1	3, 3, 3	0.259(0.993)	0.059(0.968)
	3, 5, 5	0.185(0.990)	0.055(0.958)
	5, 5, 5	0.159(0.988)	0.048(0.958)
	5, 10, 10	0.113(0.977)	0.044(0.946)
	10, 10, 10	0.109(0.977)	0.046(0.946)
5	3, 3, 3	0.208(0.991)	0.040(0.947)
	3, 5, 5	0.170(0.984)	0.050(0.942)
	5, 5, 5	0.151(0.979)	0.047(0.943)
	5, 10, 10	0.124(0.979)	0.053(0.946)
	10, 10, 10	0.107(0.975)	0.048(0.947)
10	3, 3, 3	0.221(0.990)	0.041(0.943)
	3, 5, 5	0.169(0.987)	0.048(0.946)
	5, 5, 5	0.159(0.984)	0.051(0.948)
	5, 10, 10	0.126(0.979)	0.057(0.950)
	10, 10, 10	0.106(0.979)	0.051(0.954)

posterior α-quantile of λ given x. That is to say, $F(\lambda^{\pi}(\alpha \mid \boldsymbol{X}) \mid \boldsymbol{X}) = \alpha$, where $F(\cdot \mid \boldsymbol{X})$ is the marginal posterior distribution of λ. Then the frequentist coverage probability of this one sided credible interval of λ is

$$P_{(\lambda,\mu_1,\cdots,\mu_k)}(\alpha;\lambda) = P_{(\lambda,\mu_1,\cdots,\mu_k)}(0 < \lambda \leq \lambda^{\pi}(\alpha \mid \boldsymbol{X})). \tag{16}$$

The estimated $P_{(\lambda,\mu_1,\cdots,\mu_k)}(\alpha;\lambda)$ when $\alpha = 0.05\ (0.95)$ is shown in Table 1 for the $k=3$ case.

In particular, for fixed $(\lambda, \mu_1, \mu_2, \mu_3)$, we take 5,000 independent random samples of $\boldsymbol{X}$ from the model (1). In our simulation, we take $(\mu_1, \mu_2, \mu_3) = (1, 2, 3)$.

For the cases presented in Table 1, we see that the one-at-a-time reference prior π_2 matches the target coverage probability much more accurately than the Jeffreys' prior π_1 for small values of n_1, n_2 and n_3, and values of λ . Note that the one-at-a-time reference prior satisfies a second order matching criterion but the Jeffreys' prior is not matching prior. Thus we recommend to use the one-at-a-time reference prior π_2 in the sense of asymptotic frequentist coverage property.

In the inverse gaussian populations, we have found a prior which is a second order matching prior and reference prior for the common scale parameter. It turns out that the one-at-a-time reference prior satisfies the second order matching criterion. Also we revealed that the one-at-a-time reference prior is a joint probability matching prior for $(\lambda, \mu_1, \mu_2, \mu_3)$. Thus we recommend the use of the on-at-a-time reference prior for the Bayesian inference in the analysis of reciprocals and regression models.

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