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Testing Two Exponential Means Based on the Bayesian Reference Criterion

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Abstract

We consider the comparison of two one-parameter exponential distributions with the complete data as well as the type II censored data. We adapt Bayesian test procedure for nested hypothesis based on the Bayesian reference criterion. Specifically we derive the expression for the Bayesian reference criterion to solve our problem. Also we provide numerical examples using simulated data sets to illustrate our results.

Keywords : Bayesian hypotheses testing, exponential means, noninformative priors, type II censoring

1. Introduction

The comparison of two exponential distributions is often important in statistical analyses of lifetime data. In one-parameter exponential distributions, this amounts to a comparison of their failure rates or means. Usual F test by Cox(1953) can be utilized for this purpose in the frequentist sense. For type II censored data, the frequentist approach for this problem has been well-summarized in the literatures (see for example, Lawless, 1982).

In Bayesian testing problem, the Bayes factor under proper or informative priors have been very successful. But the results may well be very sensitive to the particular choice of such priors. To overcome this kind of problem, noninformative priors are often used in Bayesian inference. For example, noninformative priors such as Jeffrey's(1961) priors or reference priors (Berger and Bernardo 1989, 1992) are typically used, but they are improper. Based on intrinsic Bayes factor introduced by Berger and Pericchi(1996), the comparison of exponential models has

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been studied by Kim et al.(2000).

Bernardo(1999) introduced a new model selection criterion, called the Bayesian reference criterion using the reference prior. By Bernardo(1999), H_0 is rejected, if, and only if, the posterior mean exceeds some specified numbers. By Bernardo(1999), nested hypothesis testing problem in the one exponential distribution with one parameter is better described as specific decision problem about the choice of a useful model. When formulated within the framework of decision theory, they have a natural, fully Bayesian coherent solution. Reference analysis (Bernardo 1979b; Berger and Bernardo 1989, 1992) is successfully used to provide a non-subjective Bayesian solution by Bernardo(1999).

The objective of this paper is to derive Bayesian test for nested hypothesis in two exponential distributions with one parameter based on the Bayesian reference criterion. We consider two types of data such as the uncensored data and the type II censored data.

Specifically we derive expression for the Bayesian reference criterion to solve our problem. Also we provide numerical examples using simulated data sets to illustrate our results.

2. Bayesian Hypotheses Testing for Two Exponential Distributions

If $x = \{x_1, \cdots, x_n\}$ is a random sample from a model $p_x(x \mid \theta)$, where $\theta \in \Theta$ is one-dimensional and there are no nuisance parameters, then $\delta(\theta_0, \theta)$ will typically be a piecewise invertible function of θ and hence (see Bernardo 1999, Proposition 1 in the Appendix) the relevant reference prior will simply be Jeffreys' prior, $\pi_\delta(\theta) \propto i(\theta)^{1/2}$, where $i(\theta)$ is Fisher's information function. Thus, in terms of the natural parameterization, defined as $\phi = \phi(\theta) = \int i(\theta)^{1/2} d\theta$, the reference prior $\pi_\delta(\phi)$ will be uniform.

For the comparison of one sample, the Bayesian reference criterion(BRC) is obtained from the following procedure. To decide whether or not some data x are compatible with the (null) hypothesis $\theta = \theta_0$, assuming that the data have been generated from the model $p_x(\cdot \mid \theta, \omega)$, $\theta \in \Theta$, $\omega \in \Omega$;

(i) compute the logarithmic discrepancy between the assumed model and its closest approximation under the null

$$\delta(\theta_0, \theta, \omega) = inf_{\omega_0 \in \Omega} \int p_x(y \mid \theta, \omega) \log \frac{p_x(y \mid \theta, \omega)}{p_x(y \mid \theta_0, \omega_0)} dy;$$

(ii) derive the reference posterior expectation

$$d_r(x, \theta_0) = \int \int \delta(\theta_0, \theta, \omega) \pi_\delta(\theta, \omega \mid x) d\theta d\omega;$$

(iii) for some d^*, reject the hypothesis $\theta = \theta_0$ if, and only if, $d_r(x, \theta_0) > d^*$, where values such as $d^* = 2.5$ (mild evidence against θ_0) or $d^* = 5$ (significant evidence against θ_0) may conveniently be chosen for scientific communication.

The choice of d^* is determined by the utility gain which may be expected by using the null model when it is true; i.e., the larger that gain, the larger d^*. $d_r(x, \theta_0)$ close to 1 may be expected if M_0 is true. d_r-values over 2.5 should raise some doubts on the use of M_0, and that d_r-values over 5 should typically be regarded as significant evidence against the suitability of using M_0 as a proxy to M_1.

2.1 The uncensored case

We now consider two exponential distributions with one parameter each other. Let $y_i = \{y_{i1}, y_{i2}, \cdots, y_{in_i}\}$ be a random sample of exponential with parameter θ_i, so that $p_{\theta_i}(y_i \mid \theta_i) = \theta_i^{n_i} \exp[-n_i \bar{y}_i \theta_i]$, and the sample mean $\bar{y}_i$ is sufficient, for $i = 1, 2$. We suppose that y_1 and y_2 are independent. To test whether or not the value $\theta_0 = \theta_1 = \theta_2$ is compatible with those observations, where θ_0 is common parameter when $\theta_1 = \theta_2$, we first derive the corresponding logarithmic discrepancy,

$$\begin{aligned}\delta(\theta_0, \theta_1, \theta_2) &= \int p(y \mid \theta_1, \theta_2) \log \frac{p(y \mid \theta_1, \theta_2)}{p_{\theta_0}(y \mid \theta_0)} dy \\ &= n_1\left[\frac{\theta_0}{\theta_1} - 1 - \log\left(\frac{\theta_0}{\theta_1}\right)\right] + n_2\left[\frac{\theta_0}{\theta_2} - 1 - \log\left(\frac{\theta_0}{\theta_2}\right)\right],\end{aligned}$$

where y is $\{y_1, y_2\} = \{y_{11}, \cdots, y_{1n_1}, y_{21}, \cdots, y_{2n_2}\}$.

Since there are no nuisance parameters, the reference prior will simply be Jeffrey's prior, that is $\pi_{\theta_1, \theta_2}(\theta_1, \theta_2) \propto i(\theta_1, \theta_2)^{1/2}$ where $i(\theta_1, \theta_2)$ is Fisher information. The reference prior is $\pi_{\theta_1, \theta_2}(\theta_1, \theta_2) = \frac{1}{\theta_1 \theta_2}$ and $\delta = \delta(\theta_0, \theta_1, \theta_2)$

is a piecewise invertible function of θ_1 and θ_2. The reference prior when δ is the quantity of interest is $\pi_\delta(\theta_1, \theta_2) = \frac{1}{\theta_1\theta_2}$.

The posterior distribution of (θ_1, θ_2) is

$$\pi(\theta_1, \theta_2 \mid y) \propto \theta_1^{n_1-1}\exp(-n_1\bar{y}_1\theta_1)\theta_2^{n_2-1}\exp(-n_2\bar{y}_2\theta_2),$$

which is a joint distribution of Gamma distribution $Ga(\theta_1 \mid n_1, n_1\bar{y}_1)$ and Gamma distribution $Ga(\theta_2 \mid n_2, n_2\bar{y}_2)$, with unique mode at $\tilde{\theta}_1 = (n_1-1)/n_1\bar{y}_1$ and $\tilde{\theta}_2 = (n_2-1)/n_2\bar{y}_2$, respectively whenever $n_1 > 1$ and $n_2 > 1$. Using the fact that if θ has a $Ga(\theta \mid \alpha, \beta)$ distribution, then $E[\log\theta] = \psi(\alpha) - \log\beta$, where $\psi(x)$ is the digamma function, the reference posterior expectation of the logarithmic discrepancy is given by

$$\begin{aligned} d_r(y, \theta_0) &= n_1\left[\psi(n_1) - \log(n_1-1) + \frac{\theta_0}{\tilde{\theta}_1} - 1 - \log\left(\frac{\theta_0}{\tilde{\theta}_1}\right)\right] \\ &\quad + n_2\left[\psi(n_2) - \log(n_2-1) + \frac{\theta_0}{\tilde{\theta}_2} - 1 - \log\left(\frac{\theta_0}{\tilde{\theta}_2}\right)\right], \quad n_1 \geq 2, \ n_2 \geq 2. \end{aligned}$$

The above equation is composed to two parts. The first part is the reference posterior expectation of the logarithmic discrepancy when sample size is n_1, and the second part is the reference posterior expectation of the logarithmic discrepancy when sample size is n_2.

Note that $d_r(y, \theta_0)$ only depends on the data through the ratio $\theta_0/\tilde{\theta}_{i,}$ $i = 1, 2$, and that the procedure suggests that no testing of the parameter value is possible in one or two exponential models with only one observation.

2.2 The type Ⅱ censored case

A type Ⅱ censored sample is one for which the r smallest observations in a random sample of n items are observed. We are interested in comparing two exponential lifetime data under type Ⅱ censoring. We consider two samples of sizes n_1 and n_2 from exponential populations with parameters θ_1 and θ_2, respectively. Under type Ⅱ censoring the observed data consist of the ordered failure times $y_{i1} \leq y_{i2} \leq \cdots \leq y_{ir_i}$ and $(n_i - r_i)$ survivors, where $i = 1, 2$. Let θ_0 be the common parameter. We want to test the hypotheses of $H_0: \theta_0 = \theta_1 = \theta_2$ vs. $H_1: \theta_1 \neq \theta_2$, based on BRC. Let y be the total sample of $\{y_1, y_2\}$, where

$\mathbf{y}_1 = \{y_{11}, y_{12}, \cdots, y_{1r_1}\}$ and $\mathbf{y}_2 = \{y_{21}, y_{22}, \cdots, y_{2r_2}\}$. We see the following equations;

$$p_{\theta_1}(\mathbf{y}_1 \mid \theta_1) = \frac{n_1!}{(n_1 - r_1)!}\theta_1^{r_1}\exp\left[-\theta_1\left(\sum_{i=1}^{r_1} y_{1i} + (n_1 - r_1)y_{1r_1}\right)\right],$$

$$p_{\theta_2}(\mathbf{y}_2 \mid \theta_2) = \frac{n_2!}{(n_2 - r_2)!}\theta_2^{r_2}\exp\left[-\theta_2\left(\sum_{i=1}^{r_2} y_{2i} + (n_2 - r_2)y_{2r_2}\right)\right],$$

$$p_{\theta_0}(\mathbf{y} \mid \theta_0) = \frac{n_1!\,n_2!}{(n_1 - r_1)!(n_2 - r_2)!}\theta_0^{r_1 + r_2} \times \exp\left[-\theta_0\left(\sum_{i=1}^{r_1} y_{1i} + (n_1 - r_1)y_{1r_1} + \sum_{i=1}^{r_2} y_{2i} + (n_2 - r_2)y_{2r_2}\right)\right],$$

where $p(\mathbf{y} \mid \theta_1, \theta_2) = p_{\theta_1}(\mathbf{y}_1 \mid \theta_1)p_{\theta_2}(\mathbf{y}_2 \mid \theta_2)$. When $\theta_1 = \theta_2$, we first derive the corresponding logarithmic discrepancy,

$$\delta(\theta_0, \theta_1, \theta_2) = r_1\left[\frac{\theta_0}{\theta_1} - 1 - \log\left(\frac{\theta_0}{\theta_1}\right)\right] + r_2\left[\frac{\theta_0}{\theta_2} - 1 - \log\left(\frac{\theta_0}{\theta_2}\right)\right].$$

The reference prior is simply Jeffrey's prior, that is $\pi_{\theta_1, \theta_2}(\theta_1, \theta_2) \propto i(\theta_1, \theta_2)^{1/2}$ where $i(\theta_1, \theta_2)$ is Fisher information. The reference prior is $\pi_{\theta_1, \theta_2}(\theta_1, \theta_2) = \frac{1}{\theta_1\theta_2}$, and $\delta(\theta_0, \theta_1, \theta_2)$ is a piecewise invertible function of θ_1 and θ_2. The reference prior when $\delta(\theta_0, \theta_1, \theta_2)$ is the quantity of interest is $\pi_\delta(\theta_1, \theta_2) = \frac{1}{\theta_1\theta_2}$ where δ is $\delta(\theta_0, \theta_1, \theta_2)$. The reference posterior distribution of (θ_1, θ_2) is

$$\pi(\theta_1, \theta_2 \mid \mathbf{y}) \propto \theta_1^{r_1 - 1}\exp\left[-\theta_1\left(\sum_{i=1}^{r_1} y_{1i} + (n_1 - r_1)y_{1r_1}\right]\right. \times \theta_2^{r_2 - 1}\exp\left[-\theta_2\left(\sum_{i=1}^{r_2} y_{2i} + (n_2 - r_2)y_{2r_2}\right],\right.$$

which is a joint distribution of Gamma distribution $Ga\left(\theta_1 \middle| r_1, \sum_{i=1}^{r_1} y_{1i} + (n_1 - r_1)y_{1r_1}\right)$ and Gamma distribution $Ga\left(\theta_2 \middle| r_2, \sum_{i=1}^{r_2} y_{2i} + (n_2 - r_2)y_{2r_2}\right)$, with unique mode at $\tilde{\theta}_i = \frac{r_i - 1}{\sum_{j=1}^{r_i} y_{ij} + (n_i - r_i)y_{ir_i}}$, $i = 1, 2$, whenever $r_1 > 1$ and $r_2 > 1$. Then the reference posterior expectation of the logarithmic discrepancy is

$$d_r(y,\theta_0) = r_1\left[\psi(r_1) - \log(r_1-1) + \frac{\theta_0}{\tilde{\theta}_1} - 1 - \log\left(\frac{\theta_0}{\tilde{\theta}_1}\right)\right] + r_2\left[\psi(r_2) - \log(r_2-1) + \frac{\theta_0}{\tilde{\theta}_2} - 1 - \log\left(\frac{\theta_0}{\tilde{\theta}_2}\right)\right], \quad r_1 \geq 2, \ r_2 \geq 2.$$

This equation is also composed to two parts. The first part is the reference posterior expectation of the logarithmic discrepancy based on the r_1 smallest observations, and the second part is the reference posterior expectation of the logarithmic discrepancy based on the r_2 smallest observations. Notice that uncensored case is the special case of type Ⅱ censored case.
Note that $d_r(y, \ \theta_0)$ only depends on the data through the ratio $\theta_0/\tilde{\theta}_i, \ i=1, \ 2$ and that the procedure suggests that no testing of the parameter value is possible in one or two exponential models with only one observation.

3. Simulation Studies

Using BRC approach, we simulate the hypothesis testing problem for comparison of two exponential models, when two exponential distributions have equally or differently several configurations sample sizes under the condition of uncensored or type Ⅱ censored population.

The exact frequentist behavior of the proposed test under the null may be obtained from the fact that if y has an exponential sampling distribution with parameter θ, then $\bar{y}$ has a Gamma sampling distribution, $Ga(\bar{y} \mid n, \ n\theta)$ and, therefore, $z=\theta/\tilde{\theta}$ has a Gamma sampling distribution $Ga(z \mid n, \ n-1)$. Tables 1-5 reproduce the results obtained for several sample sizes under uncensored or type Ⅱ censored.

First, to see the result of tests by BRC for the hypotheses

$$H_0 : \ \theta_0 = \theta_1 = \theta_2 \ \text{vs.} \ H_1 : \ \theta_1 \neq \theta_2,$$

under uncensored samples, we take 1,000,000 independent random samples of y_1 and y_2. For y_1, sample size $n_1 = 2, 10$ is taken, and for y_2, sample size $n_2 = 2, 10, 100, 1000$ is taken.

Table 1 and Table 2 give correspondence between the threshold value d^* of the test statistic $d_r(y, \ \theta_0)$, and "type I" error probabilities, $P[d_r > d^* \mid H_0]$. Here, threshold value takes over the range of values from 1 to 9. For the cases presented in Table 1 and Table 2, we observe that "type I" error probabilities, $P[d_r > d^* \mid H_0]$ become smaller as sample size increases.

We observe that the significant d^*-value is larger than that proposed Bernardo (1999). This is due to the fact that the d_r value of the comparison of two exponential sample is the summation of two parts, that is, the d_r value of the comparison of one exponential sample when the sample size is n_1 and n_2.

In Table 2, we observe the similar result such as Table 1. When $d_r \geq 8.55$, we observe that type I error probabilities are 0.001447, 0.001011 and 0.000901 when the sample size of n_2 is 10, 100, and 1000.

Second, to see the performance of tests by BRC for the hypotheses

$$H_0 : \theta_0 = \theta_1 = \theta_2 \text{ vs. } H_1 : \theta_1 \neq \theta_2,$$

under type II censored samples, we take 1,000,000 independent random samples of y_1 and y_2. In particular, we simulate six cases, i.e.,

$$(n_1, n_2, r_1, r_2) = \begin{cases} (10,10,8,8), \\ (10,10,9,9), \\ (10,20,8,16), \\ (10,20,9,18), \\ (20,20,16,16), \\ (20,20,18,18). \end{cases}$$

Table 3, Table 4, and Table 5 give correspondence between the threshold value d^* of the test statistic $d_r(y, \theta_0)$, and 'type I' error probabilities, $P[d_r > d^* \mid H_0]$. Here, threshold value takes over the range of values from 1 to 9. For the cases presented in Table 3, Table 4, and Table 5, we see that 'type I' error probabilities, $P[d_r > d^* \mid H_0]$ is smaller as total or censored sample size increases.

The uncensored data are the special type of type II censored data. i.e., our simulation results recommend rejecting the null whenever $d_r \geq 8.55$. When $n_1 = 3$, $n_2 = 3$, $r_1 = 2$ and $r_2 = 2$, type I error probabilities, $P[d_r > d^* \mid H_0]$ are equal to the below table.

d^*	1.5	2	3	4	5
$P[d_r > d^* \mid H_0]$	1.000000	0.876567	0.571552	0.370887	0.240164
d^*	6	7	8	8.55	9
$P[d_r > d^* \mid H_0]$	0.154260	0.099787	0.063748	0.049971	0.040778

Therefore, with type II censored data, our procedure shows that results are similar to Table 1 and Table 2.

In Table 3, when $d_r \geq 8.55$, we observe that type I error probabilities are 0.001848 and 0.001548. In Table 4, the sample size of n_2 increases from 10 to 20. Then when $d_r \geq 8.55$, we observe that type I error probabilities are decreasing from 0.001408 to 0.001162.

We observe the similar results in Table 5, with Table 3 and Table4. This case also shows to be in agreement with the popular belief on decreasing the significance level as the sample size increases.

In summary, for comparison of two exponential modes with the complete data as well as the type II censored data, it rejects H_0 if, and only if, this posterior mean exceeds some specified number d^*. Bernardo suggested 2.5 or 5 for d^* value, but we observe that significant d^* value is 8.55 based on our simulation studies. Also, this is in agreement with the popular belief on decreasing the significance level as the sample size increases.

Table 1. With uncensored data, correspondence between the threshold value d^* of the test statistic $d_r(y, \theta_0)$, and "type I" error probabilities, $P[d_r > d^* \mid H_0]$, for sample sizes of $n_1 = 2$ and $n_2 = 2, 10, 100, 1000$.

	(n_1, n_2)			
d^*	(2 , 2)	(2 , 10)	(2 , 100)	(2 , 1000)
1.5	1.000000	0.935376	0.907857	0.905217
2.0	0.876567	0.691423	0.661308	0.657970
3.0	0.571552	0.382829	0.358154	0.356591
4.0	0.370887	0.216809	0.199426	0.198413
5.0	0.240164	0.123659	0.113705	0.112520
6.0	0.154260	0.071834	0.065520	0.065216
7.0	0.099787	0.041888	0.038022	0.038176
8.0	0.063748	0.024928	0.022523	0.022411
8.55	0.049971	0.018595	0.016966	0.016754
9.0	0.040778	0.014733	0.013365	0.013364

Table 2. With uncensored data, correspondence between the threshold value d^* of the test statistic $d_r(y, \theta_0)$, and "type I" error probabilities, $P[d_r > d^* \mid H_0]$, for sample sizes of $n_1 = 10$ and $n_2 = 10, 100, 1000$

	(n_1, n_2)		
d^*	(10 , 10)	(10 , 100)	(10 , 1000)
1.5	0.696599	0.656409	0.652994
2.0	0.447773	0.411056	0.408539
3.0	0.185051	0.161762	0.160400
4.0	0.076662	0.063635	0.062455
5.0	0.031530	0.024884	0.024703
6.0	0.013038	0.009838	0.009660
7.0	0.005404	0.003935	0.003838
8.0	0.002155	0.001559	0.001542
8.55	0.001447	0.001011	0.000901
9.0	0.000926	0.000645	0.000601

Table 3. With type II censored data, correspondence between the threshold value d^* of the test statistic $d_r(y, \theta_0)$, and "type I" error probabilities, $P[d_r > d^* \mid H_0]$, for sample size of $n_1 = 10$ and $n_2 = 10$.

d^*	$(r_1 = 8, r_2 = 8)$	$(r_1 = 9, r_2 = 9)$
1.5	0.719613	0.707226
2.0	0.469009	0.457227
3.0	0.199548	0.191510
4.0	0.084495	0.080220
5.0	0.036374	0.033696
6.0	0.015203	0.014212
7.0	0.006575	0.005810
8.0	0.002761	0.002473
8.55	0.001848	0.001548
9.0	0.001157	0.001012

Table 4. Under type Ⅱ censored data, correspondence between the threshold value d^* of the test statistic $d_r(y, \theta_0)$, and 'type Ⅰ' error probabilities, $P[d_r > d^* \mid H_0]$, for sample size of $n_1 = 10$ and $n_2 = 20$.

d^*	$(r_1 = 8,\ r_2 = 16)$	$(r_1 = 9,\ r_2 = 18)$
1.5	0.691229	0.681736
2.0	0.443407	0.434263
3.0	0.181994	0.175715
4.0	0.074639	0.071735
5.0	0.030784	0.029128
6.0	0.012616	0.011762
7.0	0.005201	0.004768
8.0	0.002192	0.001960
8.55	0.001408	0.001162
9.0	0.000887	0.000775

Table 5. Under type Ⅱ censored data, correspondence between the threshold value d^* of the test statistic $d_r(y, \theta_0)$, and 'type Ⅰ' error probabilities, $P[d_r > d^* \mid H_0]$, for sample sizes of $n_1 = 20$ and $n_2 = 20$.

d^*	$(r_1 = 16,\ r_2 = 16)$	$(r_1 = 18,\ r_2 = 18)$
1.5	0.661570	0.655674
2.0	0.416411	0.410328
3.0	0.164177	0.160770
4.0	0.065256	0.062810
5.0	0.025644	0.024771
6.0	0.010161	0.009863
7.0	0.004034	0.003788
8.0	0.001569	0.001572
8.55	0.000946	0.000896
9.0	0.000664	0.000594

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