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Exponentially Weighted Moving Average Control Charts for Dispersion Matrix[1)]

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Abstract

Exponentially Weighted Moving Average(EWMA) control chart for variance-covariance matrix of several quality characteristics based on accumulate-combine approach has proposed. Numerical computations show that multivariate EWMA chart based on accumulate-combine approach is more efficient than corresponding multivariate EWMA chart based on combine-accumulate approach.

Keywords : Accumulate-combine approach, ARL(average run length)

1. Introduction

Many situations in manufacturing process, there exist two or more related quality characteristics to define quality of output rather than a single quality characteristics. And shifts in the parameters of dispersion matrix for the related quality characteristics are often important.

There are two basic ways to use the past sample information in multivariate quality control chart. The first, which is called combine-accumulate approach, combines the multivariate data into a univariate statistic and then accumulate over past samples. The second, which is called accumulate-combine approach, accumulates past sample information for each process parameter and then combine the multivariate accumulations into a univariate statistic.

Pignatiello and Runger(1990) considered new multivariate CUSUM procedures that accumulate past sample information for each parameter and then form a univariate CUSUM statistic from the multivariate data for monitoring the mean

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vector. A multivariate EWMA(MEWMA) chart for mean vector $\underline{\mu}$ with accumulate-combine technique was proposed by Lowry et. al.(1992). By simulation, they showed that the performances of MEWMA procedure performs better than the multivariate CUSUM procedures of Crosier(1988) and Pignatiello and Runger(1990).

In this paper, we propose EWMA procedures to simultaneously monitor both the variances and correlation coefficients of dispersion matrix Σ with accumulate-combine approach where the target process mean vector $\underline{\mu}$ remained known constant.

2. Control Statistic based on Combine-Accumulate Approach

Assume that the process of interest has $p\,(p \geq 2)$ related quality characteristics represented by the random vector $\underline{X} = (X_1, X_2, \cdots, X_p)'$ and we obtain a sequence of random vectors $\underline{X}_1, \underline{X}_2, \cdots,$ where $\underline{X}_i = (\underline{X}'_{i1}, \cdots, \underline{X}'_{in})'$ is a sample of observations at the sampling time i and $\underline{X}_{ij} = (\underline{X}_{ij1}, \cdots, \underline{X}_{ijp})'$. It will be also assumed that the sequential observation vectors between samples are independent and identically distributed, and the process quality variables have a multivariate normal distribution. Therefore, the distribution of $\underline{X}$ is indexed by a set of parameters $\underline{\theta} = (\underline{\mu}, \Sigma)$ where $\underline{\mu}$ is the mean vector and Σ is the dispersion matrix. Let $\underline{\theta}_0 = (\underline{\mu}_0, \Sigma_0)$ be the known target process parameters for $\underline{\theta} = (\underline{\mu}, \Sigma)$ of p related quality characteristics.

The general multivariate statistical quality control chart can be considered as a repetitive tests of significance where each quality characteristic is defined by p quality variables $X_1, X_2, \cdots, X_p$. Therefore, we can obtain a sample statistic for monitoring Σ by using the likelihood ratio test(LRT) statistic for testing $H_0 : \Sigma = \Sigma_0$ vs $H_1 : \Sigma \neq \Sigma_0$ where the target mean vector of the quality variables $\underline{\mu}_0$ is known. The region above the upper control limit(UCL) corresponds to the LRT rejection region. For the ith sample, likelihood ratio λ_i can be expressed as

$$\lambda_i = n^{-\frac{np}{2}} \cdot |A_i \Sigma_0^{-1}|^{\frac{n}{2}} \cdot \exp\left[-\frac{1}{2}\, tr(\Sigma_0^{-1} A_i) + \frac{1}{2}\, np\right]. \tag{2.1}$$

where $A_i = \sum_{j=1}^{n} (\underline{X}_{ij} - \underline{\mu}_0)(\underline{X}_{ij} - \underline{\mu}_0)'$. Let TV_i be $-2\ln\lambda_i$, then

$$TV_i = tr(A_i \Sigma_0^{-1}) - n \ln|A_i| + n \ln|\Sigma_0| + np \ln n - np, \tag{2.2}$$

and, we can use the LRT statistic TV_i as the sample statistic for Σ.

3. Control Statistic based on Accumulate-Combine Approach

Lowry et al.(1992) asserted that MEWMA chart for $\underline{\mu}$ is a more straightforward generalization of the corresponding univariate procedure than the multivariate CUSUM statistics by Crosier(1988) and Pignatiello and Runger(1990). In this chapter, we propose two separate control statistics, based on accumulate-combine approach, for variances and for correlation coefficients.

3.1 Control Statistic for Variance Components

In univariate case, EWMA chart for σ^2 can be constructed by using the chart statistic

$$Y_{V,i} = (1-\lambda)Y_{V,i-1} + \lambda \sum_{j=1}^{n} \left(\frac{X_{ij}-\mu_0}{\sigma_0}\right)^2 \tag{3.1}$$

$i = 1, 2, 3, \cdots$ where μ_0, σ_0^2 are known parameters and $0 < \lambda \le 1$. By repeated substitution in (3.1), it can be shown that

$$Y_{V,i} = (1-\lambda)^i Y_{V,0} + \sum_{k=1}^{i} \lambda(1-\lambda)^{i-k} \sum_{j=1}^{n} \left(\frac{X_{kj}-\mu_0}{\sigma_0}\right)^2. \tag{3.2}$$

Multivariate EWMA chart for variances with accumulate-combine approach can be constructed by forming multivariate extension of the univariate EWMA chart. Therefore, we can define the vectors of EWMA's for variances

$$\underline{Y}_{V,i} = \begin{bmatrix} Y_{V,i1} \\ Y_{V,i2} \\ \vdots \\ Y_{V,ip} \end{bmatrix} = \begin{bmatrix} (1-\lambda_1)^i Y_{V,10} + \sum_{k=1}^{i} \lambda_1(1-\lambda_1)^{i-k} \left[\sum_{j=1}^{n} \left(\frac{X_{kj1}-\mu_{01}}{\sigma_{01}}\right)^2 - n \right] \\ (1-\lambda_2)^i Y_{V,20} + \sum_{k=1}^{i} \lambda_2(1-\lambda_2)^{i-k} \left[\sum_{j=1}^{n} \left(\frac{X_{kj2}-\mu_{02}}{\sigma_{02}}\right)^2 - n \right] \\ \vdots \\ (1-\lambda_p)^i Y_{V,p0} + \sum_{k=1}^{i} \lambda_p(1-\lambda_p)^{i-k} \left[\sum_{j=1}^{n} \left(\frac{X_{kjp}-\mu_{0p}}{\sigma_{0p}}\right)^2 - n \right] \end{bmatrix},$$

where $0 < \lambda_l \le 1$ $(l = 1, 2, \cdots, p)$ and $i = 1, 2, \cdots$.

Then the multivariate EWMA vectors can be expressed as

$$\underline{Y}_{V,i} = (I - \Lambda)\underline{Y}_{V,i-1} + \Lambda\, Z_i, \tag{3.3}$$

where $\underline{Y}_{V,0}=\underline{0}$, $\underline{Z}_i=(Z_{i1},\cdots,Z_{ip})'$ and $Z_{il}=\sum_{j=1}^{n}(\frac{X_{ijl}-\mu_{0l}}{\sigma_{0l}})^2-n\ (l=1,\cdots,p)$. By repeated substitution in (3.3), $\underline{Y}_{V,i}$ can be rewritten as

$$\underline{Y}_{V,i}=\sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\underline{Z}_i+(I-\Lambda)^i\underline{Y}_{V,0} \tag{3.4}$$

$i=1,2,\cdots$ where $\Lambda=diag(\lambda_1,\lambda_2,\cdots,\lambda_p)$, $0<\lambda_j\le 1\ (j=1,2,\cdots,p)$.

Then we can obtain the dispersion matrix of $\underline{Y}_{V,i}$ as

$$\Sigma_{\underline{Y}_i}=\sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\Sigma_{\underline{Z}}(I-\Lambda)^{i-k}\Lambda, \tag{3.5}$$

where $\Sigma_{\underline{Z}}$, dispersion matrix of $\underline{Z}$, is given in Theorem 3.1.

Theorem 3.1. When the process is in-control, the dispersion matrix of $\underline{Y}_{V,i}$ is given by

$$\Sigma_{\underline{Y}_i}=\sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\Sigma_{\underline{Z}}(I-\Lambda)^{i-k}\Lambda$$

and

$$\Sigma_{\underline{Z}}=2nR^{(2)},$$

where $R^{(2)}$ is used to denote the matrix whose (i,j)th component is the $2nd$ power of the (i,j)th component of R which is the correlation matrix of $\underline{X}=(X_1,X_2,\cdots,X_p)$.

Proof. It is easy to show that

$$\begin{aligned}\Sigma_{\underline{Y}_i}&=V[(I-\Lambda)Y_{V,i-1}+\Lambda\,\underline{Z}_i]\\&=\sum_{k=1}^{i}\Lambda(I-\Lambda)^{i-k}\Sigma_{\underline{Z}_i}(I-\Lambda)^{i-k}\Lambda.\end{aligned}$$

To show the form of $\Sigma_{\underline{Z}_i}$, we obtain the mean vector and variance-covariance matrix of $\underline{Z}_i$ when the process is in-control and $\underline{Y}_{V,0}=\underline{0}$. Since a random sample $(X_{i1l},X_{i2l},\cdots,X_{inl})'$ at the sampling time i follows a multivariate normal distribution, the statistic $\sum_{j=1}^{n}(\frac{X_{ijl}-\mu_{ol}}{\sigma_{ol}})^2=Z_{il}+n$ has a chi-squared distribution with n degrees of freedom. Thus $E(Z_{il})=0$ and $V(Z_{il})=2n$ for $l=1,2,\cdots,p$ and $i=1,2,\cdots$. Now, for $l \neq s$, we can derive as

$$\begin{aligned} Cov[Z_{il}, Z_{is}] &= Cov[Z_{il}+n, Z_{is}+n] \\ &= Cov\left[\sum_{j=1}^{n}\left(\frac{X_{ijl}-\mu_{0l}}{\sigma_{ol}}\right)^2, \sum_{j=1}^{n}\left(\frac{X_{ijs}-\mu_{0s}}{\sigma_{os}}\right)^2\right] \\ &= nCov\left[\left(\frac{X_{ijl}-\mu_{0l}}{\sigma_{0l}}\right)^2, \left(\frac{X_{ijs}-\mu_{0s}}{\sigma_{0s}}\right)^2\right]. \end{aligned}$$

If we let $U=\frac{X_{ijl}-\mu_{0l}}{\sigma_{0l}}$ and $V=\frac{X_{ijs}-\mu_{0s}}{\sigma_{0s}}$, then the random variables U and V have a bivariate normal distribution with $N_2(0,0,1,1,\rho_{u,v})$. Using the moment generating function of bivariate normal distribution, $E(U^2V^2)=1+2\rho^2_{u,v}$ and then we can easily obtain that

$$\begin{aligned} Cov(Z_{il}, Z_{is}) &= nCov(U^2, V^2) \\ &= n[E(U^2V^2)-E(U^2)E(V^2)] \\ &= 2n\rho^2_{u,v}. \end{aligned}$$

Thus, we found that the diagonal elements and corresponding off-diagonal elements of $\Sigma_{\underline{Z}_i}$ is $2n$ and $2n\rho^2_{i,j}$ from the above results. Therefore,

$$E(\underline{Z}_i)=\underline{0} \quad \text{and} \quad \Sigma_{\underline{Z}_i}=2nR^{(2)}. \qquad \square$$

Unless there is any reason to differently weight the elements of smoothing matrix Λ, all diagonal elements of Λ can be set to an equal value. Under the assumption $\lambda_1=\lambda_2=\cdots=\lambda_p=\lambda$, then we can simplify the variance-covariance matrix of $\underline{Y}_i$ as

$$\Sigma_{\underline{Y}_i}=\frac{\lambda[1-(1-\lambda)^{2i}]}{2-\lambda}\cdot\Sigma_{\underline{Z}}. \tag{3.6}$$

3.2 Control Statistic for Correlation Coefficients

Univariate EWMA chart for ρ_{12} which is based on $r_{12}=\sum_{j=1}^{n}(x_j-\mu_1)(y_j-\mu_2)/n\sigma_1\sigma_2$, an estimator of ρ_{12}, can be constructed as

$$Y_{C,i} = (1-\lambda)Y_{C,i-1} + \lambda \frac{\sum_{j=1}^{n}(X_{ij1}-\mu_{10})(X_{ij2}-\mu_{20})}{n\sigma_{10}\sigma_{20}}, \tag{3.7}$$

$i=1,2,\cdots$ and $0<\lambda\leq 1$. By repeated substitution, it can be shown that

$$Y_{C,i} = (1-\lambda)^i Y_{C,0} + \sum_{k=1}^{i}\lambda(1-\lambda)^{i-k}\frac{\sum_{j=1}^{n}(X_{kj1}-\mu_{10})(X_{kj2}-\mu_{20})}{n\sigma_{10}\sigma_{20}}. \tag{3.8}$$

To simultaneously monitor the correlation coefficients of p related quality variables, if we let the control statistic for ρ_{lm} be r_{lm} by suitable modification of the simple expression in (3.8), then the vector $\underline{\rho} = (\rho_{12}, \rho_{13}, \cdots, \rho_{1p}, \rho_{23}, \cdots, \rho_{2p}, \cdots, \rho_{p-1,p})'$ of EWMA's can be defined as

$$\begin{aligned} \underline{Y}_{C,i}' &= (r_{12}, r_{13}, \cdots, r_{1p}, r_{23}, \cdots, r_{2p}, \cdots, r_{p-2,p-1}, r_{p-1,p}) \\ &= (Y_{V,i1}, Y_{V,i2}, \cdots, Y_{V,i,p-1}, Y_{V,ip}, \cdots, Y_{V,i,2p-3}, \cdots, Y_{V,i,s-1}, Y_{V,i,s}). \end{aligned}$$

Then the vector $\underline{Y}_{C,i}$ can be rewritten as

$$\underline{Y}_{C,i} = \begin{bmatrix} (1-\lambda_1)^i Y_{i10} + \sum_{k=1}^{i}\lambda_1(1-\lambda_1)^{i-k}Z_{k12} \\ \vdots \\ (1-\lambda_{p-1})^i Y_{i,p-1,0} + \sum_{k=1}^{i}\lambda_{p-1}(1-\lambda_{p-1})^{i-k}Z_{k1p} \\ (1-\lambda_p)^i Y_{i,p,0} + \sum_{k=1}^{i}\lambda_p(1-\lambda_p)^{i-k}Z_{k23} \\ \vdots \\ (1-\lambda_{2p-3})^i Y_{i,2p-3,0} + \sum_{k=1}^{i}\lambda_{2p-3}(1-\lambda_{2p-3})^{i-k}Z_{k2p} \\ \vdots \\ (1-\lambda_s)^i Y_{i,s,0} + \sum_{k=1}^{i}\lambda_s(1-\lambda_s)^{i-k}Z_{k,p-1,p} \end{bmatrix}, \tag{3.9}$$

where $s=p(p-1)/2$, $0<\lambda_a\leq 1\ (a=1,2,\cdots,s)$ and

$$Z_{kmu} = \frac{\sum_{j=1}^{n}(X_{kjm}-\mu_{m0})(X_{kju}-\mu_{u0})}{n\sigma_{m0}\sigma_{u0}} - \rho_{mu0}. \ (m\neq u)$$

Multivariate EWMA chart based on the vector (3.9) can be expressed as

$$\underset{\sim}{Y}_{C,i} = \sum_{k=1}^{i} \Lambda(I-\Lambda)^{i-k} \underset{\sim}{Z}_k + (I-\Lambda)^i \underset{\sim}{Y}_0 , \qquad (3.10)$$

where $\underset{\sim}{Z}_k' = (Z_{k12}, Z_{k13}, \cdots, Z_{k1p}, Z_{k23}, \cdots, Z_{k2p}, \cdots, Z_{k,p-1,p})$, $\Lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_s)$ and $0 < \lambda_j \leq 1 \ (j=1,2,\cdots,s)$.

Prabhu and Runger(1997) stated that good choices for λ depend on the number of variables in the control scheme and the size of the shift in MEWMA chart for $\underset{\sim}{\mu}$ and they stated that values for λ from 0.1 to 0.5 are good choices. Under the assumption that $\lambda_1 = \lambda_2 = \cdots = \lambda_s = \lambda$, the multivariate EWMA vector in (3.10) can be written as

$$\begin{aligned} \underset{\sim}{Y}_{C,i} &= (1-\lambda) \underset{\sim}{Y}_{C,i-1} + \lambda \underset{\sim}{Z}_t \\ &= \sum_{k=1}^{i} \lambda(1-\lambda)^{i-k} \underset{\sim}{Z}_k + (1-\lambda)^i \underset{\sim}{Y}_{C,0} . \end{aligned} \qquad (3.11)$$

Theorem 3.2 If the process is in-control and $\underset{\sim}{Y}_{C,0} = \underset{\sim}{0}$, then the dispersion matrix of $\underset{\sim}{Y}_{C,i}$ is given by

$$\Sigma_{\underset{\sim}{Y}_i} = \left\{ \frac{\lambda[1-(1-\lambda)^{2i}]}{2-\lambda} \right\} \cdot \Sigma_{\underset{\sim}{Z}} \qquad (3.12)$$

and

$$\Sigma_{\underset{\sim}{Z}} = \begin{pmatrix} Var(Z_{i12}) & Cov(Z_{i12}, Z_{i13}) & \cdots & Cov(Z_{i12}, Z_{i,p-1,p}) \\ & Var(Z_{i13}) & \cdots & Cov(Z_{i13}, Z_{i,p-1,p}) \\ & & \ddots & \vdots \\ & Sym & & Var(Z_{i,p-1,p}) \end{pmatrix}, \qquad (3.13)$$

where

$$\begin{aligned} Var(Z_{ipq}) &= \frac{1+\rho_{pq0}^2}{n} \\ Cov(Z_{ipq}, Z_{ipr}) &= \frac{\rho_{qr0} + \rho_{pq0}\rho_{pr0}}{n} \\ Cov(Z_{ipq}, Z_{irw}) &= \frac{\rho_{pr0}\rho_{qw0} + \rho_{pw0}\rho_{qr0}}{n} \end{aligned}$$

and the subscripts p, q, r and w are different each other.

Proof

$$\begin{aligned} \Sigma_{\underset{\sim}{Y}_i} &= \sum_{k=1}^{i} Cov[\lambda(1-\lambda)^{i-k} \underset{\sim}{Z}_k] \\ &= \left\{ \frac{\lambda[1-(1-\lambda)^{2i}]}{2-\lambda} \right\} \cdot \Sigma_{\underset{\sim}{Z}} . \end{aligned}$$

To show the form of Σ_Z, it is necessary to derive that the following results by using the moment generating function(MGF) of quality vectors under multivariate normal distribution when the process is in-control such that

$$\begin{aligned} Var(Z_{ipq}) &= \frac{1}{n^2\sigma^2_{p0}\sigma^2_{q0}} Var[\sum_{j=1}^{n}(X_{ijp}-\mu_{p0})(X_{ijq}-\mu_{q0})] \\ &= \frac{1}{n(\sigma_{p0}\sigma_{q0})^2}(\sigma^2_{p0}\sigma^2_{q0}+\rho^2_{pq0}\sigma^2_{p0}\sigma^2_{q0}) \\ &= \frac{1+\rho^2_{pq0}}{n} \end{aligned}$$

$$\begin{aligned} Cov(Z_{pq}, Z_{pr}) &= Cov[\frac{\sum_{j=1}^{n}(X_{ijp}-\mu_{p0})(X_{ijq}-\mu_{q0})}{n\sigma_{p0}\sigma_{q0}}, \frac{\sum_{j=1}^{n}(X_{ijp}-\mu_{p0})(X_{ijr}-\mu_{r0})}{n\sigma_{p0}\sigma_{r0}}] \\ &= \frac{1}{n\sigma^2_{p0}\sigma_{q0}\sigma_{r0}} Cov[\ (X_{ijp}-\mu_{p0})(X_{ijq}-\mu_{q0})\ ,\ (X_{ijp}-\mu_{p0})(X_{ijr}-\mu_{r0})\ ] \\ &= \frac{\rho_{qr0}+\rho_{pq0}\rho_{pr0}}{n} \end{aligned}$$

and

$$\begin{aligned} Cov(Z_{pq}, Z_{rw}) &= \frac{1}{n\sigma_{p0}\sigma_{q0}\sigma_{r0}\sigma_{w0}} Cov[\ (X_{tjp}-\mu_{p0})(X_{tjq}-\mu_{q0})\ ,\ (X_{tjr}-\mu_{r0})(X_{tjw}-\mu_{w0})\ ] \\ &= \frac{\rho_{pr0}\rho_{qw0}+\rho_{pw0}\rho_{qr0}}{n}\ . \qquad \square \end{aligned}$$

4. EWMA Control Charts

In this section, we propose a procedure which uses two separate EWMA charts, based on accumulate-combine approach, for means and for variances. This proposed scheme signals if either of these two charts signals.

4.1 EWMA Chart based on Combine-Accumulate Approach

A multivariate EWMA chart based on LRT statistic in (2.2) is given by

$$Y_{TV,i} = (1-\lambda)Y_{TV,i-1} + \lambda TV_i, \tag{4.1}$$

where $Y_{TV,0} = \omega_{TV} \cdot I_{(\omega_{TV}\geq 0)}$ and $0 < \lambda \leq 1$. If the chart statistic $Y_{TV,i}$ exceeds UCL h_{TV}, the process is deemed out-of-control state and assignable causes are sought. Since it is difficult to obtain the exact distribution of TV_i when the process is in-control or out-of-control states, the performances of the proposed charts based on the sample statistic TV_i are obtained by simulation, and the parameter h_{TV} can be obtained to satisfy a specified in-control ARL.

4.2 EWMA Chart based on Accumulate-Combine Approach

Multivariate EWMA chart for variances based on accumulate-combine approach signals whenever

$$T^2_{1,i} = \underline{Y}_{V,i}{}' \, \Sigma_{\underline{Y}_i}{}^{-1} \, \underline{Y}_{V,i} \; > \; h_1, \tag{4.2}$$

where the parameter h_1 can be obtained to achieve a specified in-control ARL and $\Sigma_{\underline{Y}_i}$ is in (3.6). And multivariate EWMA chart for correlation coefficients based on accumulate-combine approach signals whenever

$$T^2_{2,i} = \underline{Y}_{C,i}{}' \, \Sigma_{\underline{Y}_i}{}^{-1} \, \underline{Y}_{C,i} > h_2. \tag{4.3}$$

where the parameter h_2 can be obtained to achieve a specified in-control ARL and $\Sigma_{\underline{Y}_i}$ is in (3.12).

The multivariate EWMA scheme based on accumulate-combine approach for simultaneously monitoring both variances and correlation coefficients signals whenever $T^2_{1,i} > h_1$ or $T^2_{2,i} > h_2$. Since it is difficult to obtain the joint distribution of $T^2_{1,i}$ and $T^2_{2,i}$, we obtain the parameters h_1, h_2 and performances of this scheme by simulation.

5. Concluding Remarks

In order to evaluate the performances and compare the properties of the proposed charts, the charts should have the same ARL when the process is in-control and some kinds of standards for comparison are necessary.

For simplicity in our numerical computation, we assume that target mean vector $\underline{\mu}_0 = \underline{0}'$, all diagonal elements of Σ_0 are 1 and off-diagonal elements of Σ_0 are 0.3. The numerical results were obtained when the ARL of in-control state was approximately equal to 200, and the sample size for each variable was five for $p = 3 \sim 4$.

Since the performance of the charts depends on the components of Σ, it is not possible to investigate all of the different ways in which Σ could change. Hence, we consider the following typical types of shifts for comparison in the process parameters.

(1) V_i : σ_{10} of Σ_0 is increased to $[1 + (4i-3)/10]$.

(2) C_i : ρ_{120} and ρ_{210} of Σ_0 are changed to $[0.3 + (2i-1)/10]$

(3) (V_i, C_i) for $i=1,2,3$.
(4) S_i : Σ_0 is changed to $c_i\Sigma_0$ where $c_i=[1+(3i-2)/10]^2$

After the design parameters h values of the proposed EWMA charts have been determined, the numerical performances of the proposed types of shifts were obtained by simulation with 10,000 runs.

The properties and comparison of the proposed procedures are given in [Table 1] and [Table 2]. From the numerical results, we found the following properties. Small value of smoothing constant λ is more effective in detecting small or moderate changes in EWMA chart for both accumulate-combine approach and combine-accumulate approach.

We also found that the EWMA procedures based on accumulate-combine approach exhibit far smaller ARL values when compared to the corresponding EWMA procedures based on combine-accumulate approach.

[Table 1] ARL performances of EWMA chart for Σ ($p=3$)

types of shift	C-A Approach			A-C Approach		
	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0.3$	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0.3$
in-control	200.0	200.1	200.0	200.0	200.0	200.0
V_1	170.7	173.0	176.1	51.6	65.2	74.6
V_2	29.4	24.9	24.8	3.9	4.5	5.1
V_3	11.8	8.1	6.9	1.9	2.1	2.2
C_1	178.8	181.3	183.9	110.2	136.0	149.3
C_2	77.8	84.2	93.7	20.4	30.4	41.7
C_3	24.4	21.3	23.3	8.6	11.0	14.4
(V_1, C_1)	157.9	162.8	167.4	40.7	53.0	62.1
(V_2, C_2)	25.9	21.2	20.9	3.7	4.2	4.7
(V_3, C_3)	9.7	6.5	5.5	1.8	2.0	2.1
S_1	132.2	138.0	143.5	20.9	26.1	30.7
S_2	18.0	13.5	12.7	2.2	2.6	2.8
S_3	7.6	4.9	4.1	1.2	1.2	1.3
S_4	4.4	2.9	2.3	1.1	1.1	1.1

[Table 2] ARL performances of EWMA chart for Σ ($p=4$)

types of shift	C-A Approach			A-C Approach		
	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0.3$	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0.3$
in-control	200.0	200.1	200.0	200.0	200.0	200.0
V_1	181.1	184.0	186.0	55.0	72.7	82.8
V_2	44.6	41.7	44.5	4.1	4.8	5.4
V_3	17.7	12.5	11.2	1.9	2.1	2.2
C_1	186.4	189.3	190.0	126.9	154.2	167.2
C_2	104.0	114.0	123.4	23.6	37.8	53.7
C_3	37.7	36.9	42.7	9.6	13.1	18.1
(V_1, C_1)	172.6	176.8	179.7	45.4	61.5	72.9
(V_2, C_2)	38.3	34.6	36.9	3.9	4.5	5.1
(V_3, C_3)	14.5	9.9	8.6	1.9	2.0	2.2
S_1	142.4	149.6	155.6	17.7	22.5	26.3
S_2	21.9	16.9	16.4	1.9	2.1	2.3
S_3	9.4	4.8	4.9	1.4	1.2	1.2
S_4	5.5	3.4	2.7	1.03	1.04	1.05

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